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aggregate separately, every experiment was performed with a mixture of the aggregate and cement gaged with the water necessary to produce the proper consistency. The water was found necessary both in theory and practice. The cement and water actually occupy space in the mass, since many of the voids are too small for the grains of cement to fit into them without expanding the volume and the water also occupies actual bulk in the concrete. Besides this, a concrete mixed up with water is easier and smoother to handle than a mixture of dry materials alone which tend to separate when being placed.

Curve of Maximum Density. The Little Falls tests made by the writer indicated that the curve at greatest density was substantially a parabola. The Jerome Park tests based on a larger number of experiments define the curve still more accurately as a combination of an ellipse and a straight line.*

One of the most interesting developments was that a curve of substantially the same form would fit different materials whatever the maximum size of the stone. The $\frac{1}{2}$ -inch stone, for example, required but very slight change in curve equation from the $2\frac{1}{4}$ -inch stone.

The maximum density curve then was found to consist of a combination of an ellipse† and a straight line, the ellipse being first constructed with its

* Mr. Fuller's method of proportioning the materials so that their mixture will form a smooth, clearly defined curve appears, on its face, to conflict with Mr. Feret's conclusion (see p. 147) that the best mixture of sand and cement for mortar is made up of coarse and fine grains only, with no intermediate grains. For sand mortars, Mr. Feret's methods are undoubtedly more exact than Mr. Fuller's, but for a concrete mixture the conditions are different, and, as we have stated on page 172, more than two sizes of materials are theoretically necessary for obtaining the densest mixture. In practice, too, all classes of materials are more or less varied, and experiments show that the particles will best fit into each other if the sizes are graded. The best proof of the practical efficiency of Mr. Fuller's method lies in the fact that he has employed it day after day for determining the proportions of the aggregate for concrete used in constructing thin, water-tight walls. The proportions used by him for such work are about 1:3:7, whereas for water-tight construction where the materials are not scientifically graded 1:2:4 mixtures are commonly used.

The method is exact and scientific and not "rule-of-thumb." The nature of the materials and their variation from hour to hour makes great refinement unnecessary, so that an accuracy of, say, 2% or 3% in the percentages are all that is necessary in practice. Although further tests may show that for other materials the form of the curve varies from that indicated by Mr. Fuller, the general method of analyzing materials and combining the curves is undoubtedly applicable whatever the form of the curve, so that Mr. Fuller's general methods still hold

† In practice ellipses may be most readily plotted graphically by the Trammelpoint method as follows:

Plot the major and minor axes on the diagram. The major or horizontal axis in all cases is on a line 7% above the base. The minor or vertical axis is at a distance, a, to the right of the vertical zero ordinate of the diagram. Lay a strip of paper or a thin straight-edge upon the major or horizontal axis, and mark upon it two points to represent the length of the semi-major axis, calling one of these points—the point on the zero ordinate—0, and the other point A. Mark off on the strip or straight-edge, in the same direction from O, the length of the semi-minor axis, calling this point B. Now, swing the strip of paper or straight-edge little by little so that the outline of the curve may be marked off by the point O, while the points A and B are kept at all times upon the axes b and a respectively. The straight lines to continue the curves are drawn as tangents to them, or may be readily plotted from the data on the following page.

major axis coinciding with 7 per cent line of percentages, and the equation

of the ellipse, using the zero coördinates of the diagram, being $(y - 7)^2 = \frac{0^2}{\pi^2}$

 $(2ax - x^2)$. One of the ideal curves is illustrated in Fig. 73, page 207, showing the general form which it takes.

In practice it was necessary to raise the curve somewhat higher, that is, to use more sand than the very careful laboratory tests would indicate as the ideal mix.

The values of a and b for the different materials, including the cement for the Ideal Mix, based on the Jerome Park stone and Cowe Bay sand and gravel, which, as already stated, were fairly representative materials, are as follows:

Data for Plotting Ellipses in Curves of Ideal Mix.

Materials.	Ideal Mix Axes of Ellipse,		
	a 0.04 +0.16D 0.04 +0.16D	b 28.5+1.3D 26.4+1.3D	
ings	0.035 +0.14D	29.4+2.2D	

In this table, D = the maximum diameter of the stone, in inches.

For the Practical Mix the values of b must be greater so as to give a higher curve with more of the finer material. A quick and sufficiently accurate method of drawing the curves for the practical mix is to draw a straight line from the point where the largest diameter stone reaches the 100% line to the point on the vertical ordinate at zero diameter given in Column (1) in the following table.

Data for Plotting Curves of Practical Mix.

State State State State	Intersection of tangent with vertical	Height of	Axes of Ellipse,	
Materials.	at zero diameter (1)	tangent point (2)	a (3)	b + 7 (4)
Crushed stone and sand.	28.5	35.7	0.150D	37.4
Gravel and sand Crushed stone and screen-	26.0	33.4	0.164D	37.4 35.6
ings	29,0	36.1	0.147D	37.8

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Then mark the tangent point on this line where it is intersected by the vertical ordinate for one-tenth the maximum diameter stone. This mark should check with the values given in column (2) of above table. Then plot the location of minor axis of the ellipse from the values of a and b + 7, given in columns (3) and (4) in the above table. This point, together with the tangent point and the point at + 7 on the vertical ordinate at zero diameter where the curve begins, gives three points on the ellipse, which is usually sufficient for drawing the curve with the aid of an irregular curve. If more points are wanted, they may be plotted graphically by the trammel point method as given in the note on page 202.

RELATION OF DENSITY TO STRENGTH

Having determined the maximum density curve as just explained, it was important to know if the greatest strength coincided with the greatest density, and for this purpose a large number of beams, six inches square and six feet long, were made up and tested for transverse and crushing strength, for permeability and modulus of elasticity. Some beams were made using the proportions determined by the maximum density curve and other beams according to higher and lower curves to note if there were any decrease in these properties as the maximum density curve was departed from. The full results of the tests are given in the paper referred to,* but in general it may be said that a departure from the maximum density curve represented a reduction in all these properties except that when the curve was modified so as to use a uniform size of coarse stone instead of the graded stone it gave practically the same results as the graded. Any curving above the straight line in the coarse material decreased the density, and also the strength, indicating that the coarse aggregate should not have an excess of medium particles.

LAWS OF PROPORTIONING

From these experiments, laws of proportioning and also laws relating to strength and permeability which are outlined in full in the paper by Messrs. Fuller and Thompson* were evolved.

Those relating specifically to strength are given on page 000 and those relating definitely to permeability on page 000, and reference should be made to these for complete conclusions. The laws relating especially to the grading of the aggregates are as follows:

*See footnote p. 201

1.-Aggregates in which particles have been specially graded in sizes so as to give, when water and cement are added, an artificial mixture of greatest density, produce concrete of higher strength than mixtures of cement and natural materials in similar proportions. The average improvement in strength by artificial grading under the conditions of the tests was about 14 per cent. Comparing the tests of strength of concrete having different percentages of cement, it is found that for similar strength the best artificially graded aggregate would require about 12% less cement than like mixtures of natural materials.

2.-The strength and density of concrete is affected but slightly, if at all, by decreasing the quantity of the medium size stone of the aggregate and increasing the quantity of the coarsest stone. An excess of stone of medium size, on the other hand, appreciably decreases the density and strength of the concrete.

3.—The strength and density of concrete is affected by the variation in the diameter of the particles of sand more than by variation in the diameters of the stone particles.

4.-An excess of fine or of medium sand decreases the density and also the strength of the concrete, as will also a deficiency of fine grains of sand in a lean concrete.

5.—The substitution of cement for fine sand does not affect the density of the mixture, but increases the strength, although in a slightly smaller ratio than the increase in the ratio of cement.

6.-It follows from the foregoing conclusions that the correct proportioning of concrete for strength consists in finding, with any percentage of cement, a concrete mixture of maximum density, and increasing or decreasing the cement by substituting it for the fine particles in the sand or vice versa.*

7.-In ordinary proportioning with a given sand and stone and a given percentage of cement, the densest and strongest mixture is attained when the volume of the mixture of sand, cement and water is so small as just to fill the voids in the stone. In other words, in practical construction, use as small a proportion of sand and as large a proportion of stone as is possible without producing visible voids in the concrete.

8.—The best mixture of cement and aggregate has a mechanical analysis curve† resembling a parabola, which is a combination of a curve approaching an ellipse for the sand portion and a tangent straight line for the stone

* This very important law requires further tests for confirmation, outside of the limits of the present tests + For definition of mechanical analysis, see page 193.

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portion. The ellipse runs to a diameter of one-tenth of the diameter of the maximum size of stone, and the stone from this point is uniformly graded. 9.—The ideal mechanical analysis curve, *i.e.*, the best curve, is slightly

different for different materials. Cowe Bay sand and gravel, for example, pack closer than Jerome Park stone and screenings, and therefore require less of the size of grain which the authors designate as sand.

10.—The form of the best analysis curve for any given material is nearly the same for all sizes of stone, that is, the curve for $\frac{1}{2}$ -inch, 1-inch, and $2\frac{1}{4}$ inch maximum stone may be described by an equation with the maximum diameter as the only variable. In other words, suppose a diagram in which the left ordinate is zero, and the extreme right ordinate corresponds to $2\frac{1}{4}$ inch stone, with the best curve for this stone drawn upon it. If, now, on this diagram the vertical scale remains the same, but the horizontal scale is increased two and a quarter times, so that the diameter of 1-inch stone corresponds to the extreme right-hand ordinate, the best curve for the 1inch stone will be very nearly the one already drawn for the $2\frac{1}{4}$ -inch stone. The chief difference between the two is that the larger size stone requires a slightly higher curve in the fine sand portion.

11.—It follows from this last conclusion that from a scientific standpoint the term sand is a relative one. With $2\frac{1}{4}$ -inch stone, the best sand would range in size from 0 to 0.22 inch diameter, while the best sand for $\frac{1}{2}$ -inch stone would range in size from 0 to 0.05 inch diameter.

APPLICATION OF MECHANICAL ANALYSIS DIAGRAMS TO PRO-PORTIONING

The mechanical analysis diagram offers a very exact method of determining the proper proportions of any materials for concrete by sieving each of the materials, plotting their analyses and combining these curves so that the result is as near as possible similar to the maximum density curve.

Plot on the diagram the maximum density curve for the given materials to be used; if the equation for this material is not known use the practical equation previously given. Make a mechanical analysis of all of the materials which it is desired to mix together in the right proportions and plot the result of each analysis on the diagram on which the maximum density curve has been plotted.

The aim is to find a new curve representing the mixture of the materials, but which will conform as nearly as possible to the curve of maximum density. The proportions of different materials required to produce this curve will show the relative quantity of each which must be used in proportioning. The theory of the combination and complete discussion of the methods to be employed with different forms of curves are treated in Appendix IV.

A less exact method, but one which is convenient in practice, is by inspection and trial of different percentages. To illustrate this trial plan, the method of forming a curve of a mixture of several materials in stated proportions such as 1:2:4 will be given, then the curve for the mixture of the same materials which corresponds nearest to the curve of maximum density, and finally the application will be made to material like run of the bank gravel which may be separated into two or three parts.

In reading this discussion it must be borne in mind that the same principles will apply to mixtures of several aggregates, although for simplicity the principal part of the discussion refers to two aggregates. The same

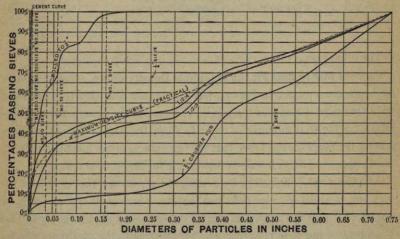


Fig. 73.—Curves of Fine and Coarse Crushed Stone and Mixtures. (p.207)

approximate plan may be used for the larger number of aggregates or the more exact method in the Appendix may be adopted.

Plotting Curve of Mix in Studying Proportions. In Fig. 73 we have $\frac{3}{4}$ -inch Shawangunk grit as one aggregate and the same material rolled to $\frac{1}{8}$ -inch maximum size as the other, giving the mechanical analysis curves shown in the diagram.*

In this diagram a curve of cement is also plotted so that the 1:2:4 curve represents the combination of the three materials. The curve marked 1:2:4 then represents the analysis of the mixture of cement, screenings

* This diagram and the ones which follow are made up from materials used in subsequent studies by the New York Board of Water Supply, and referred to in the Discussion by Mr. James L. Davis, Transactions American Society Civil Engineers, Vol. LIX, p. 144.

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and stone in these proportions. This curve is made up by plotting various points and connecting these by a smooth curve. To find the point, for example, where the curve cuts the ordinate corresponding to the No. 20 sieve, the sums of the percentages of the individual materials at this same ordinate are taken in the proportion which they bear to the concrete mixture. All of the cement is finer than the No. 20 sieve, and since the cement is one part of the seven parts in the mixture, one-seventh of 100 per cent represents the percentage of cement in the mixture at the given ordinate. Similarly, since there are two parts of sand in the seven parts, the sand percentage at the No. 20 ordinate, 61 per cent, is multiplied by two-sevenths, and the stone percentage, 6 per cent, by four-sevenths, thus giving as the point on the No. 20 sieve ordinate in the combined curve:

- $\frac{1}{7}$ X 100 per cent = 14.3 per cent for cement
- $\frac{1}{4} \times 61$ per cent = 17.4 per cent for sand $\frac{1}{4} \times 6$ per cent = 3.4 per cent for stone

Total..... 35.1 per cent for the point in the curve.

The other points in the curves are found in a similar manner.

Curve of Mix to Best Fit the Maximum Density Curve. Take the same two aggregates plotted in Fig. 73, but in this case disregard the cement or rather consider it a part of the sand. (Frequently the cement must be considered in the trial mixtures in order to study the part of the curve representing the fine material to see that the percentages of the finest particles are satisfactory). The slide rule is convenient for this proportioning.

Averaging the $\frac{3}{4}$ -inch stone by a straight line, we see that it crosses the 0.15 line at about 9%; we note also that the $\frac{1}{8}$ -inch sand crosses the same line at 98% and the maximum density curve crosses the line at 43%, that is, along this line it is 34% from the $\frac{3}{4}$ -inch stone to the maximum density curve and 55% to the $\frac{1}{8}$ -inch sand. The percentages to be used to obtain a 43% mixture would be an inverse ratio of these two numbers to their total, that is, $\frac{34}{89} = 38\%$ of fine material and $\frac{55}{89} = 62\%$ of the coarse material. With the slide rule take these percentages of each curve, add together and plot a new curve, and see if it conforms reasonably with the maximum density curve. If it does not, make another trial of percentages, the plot of the curve indicating by inspection the new percentages.

It must be remembered that the fine portion of the curve includes also the cement, so having decided on the amount of cement to use, say the equivalent of a 1:7 mix, which has $12\frac{1}{2}\%$ of cement, the actual proportions would be $12\frac{1}{2}$ parts cement to $38 - 12\frac{1}{2} = 25\frac{1}{2}$ parts fine aggregate to 62 parts coarse aggregate, or translated into the usual nomenclature, 1:2.04:4.95, or practically 1:2:5, showing that the ordinary mixture with this particu-

lar material is the best. Supposing, however, the equivalent of a richer mixture, say 1:2:4, is wanted. This would contain $1:6 = 14\frac{10}{2}$ cement and the proportions would be

 $14\frac{1}{2}:23\frac{1}{2}:62,$

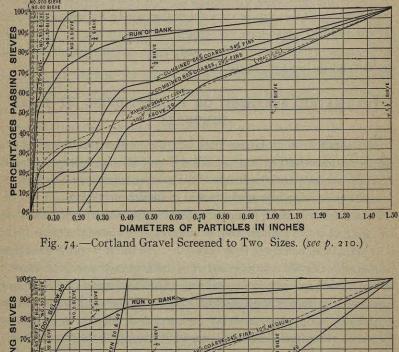
I: I.62: 4.27,

or practically

or

$$1:1\frac{2}{3}:4\frac{1}{4},$$

showing that for richer mixtures less fine materials is desirable.



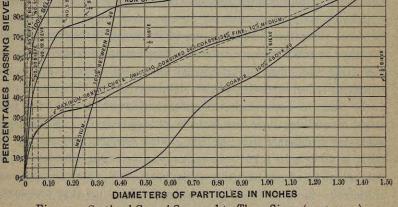


Fig. 75.-Cortland Gravel Screened to Three Sizes. (see p. 210.)

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Run of Bank Gravel. Gravel as it is found in the natural bank almost always contains too much fine material. In many cases screening this into two sizes produces a good curve which fits very closely to the curve of maximum density.*

Other gravels, especially where the sand is greatly in excess, require two screenings for the best result. Fig. 74 represents a common run of such gravel, showing that screening into two sizes will not permit a mixture fitting very near to the maximum density curve. The figure also shows how far away the original analysis of the run of the bank is from the ideal curve. In Fig. 75 the same sand is shown screened into three sizes, and illustrates the improvement that can be obtained in this case by the extra screening, the effect of which is to leave out some of the medium size particles which are too large to fill the voids of the coarse stones, and therefore decrease the density and the strength of the mixture.

VOLUMETRIC SYNTHESIS OR PROPORTIONING BY TRIAL MIXTURES

The density tests at Jerome Park and the relation there found of the strength to the density indicate a method of proportioning by trial mixtures, which in fact compared the density of the same materials mixed in different proportions or different materials mixed in similar proportions.

Having determined the particular sand and stone which are to be used on any piece of work, a simple and accurate way of determining proportions is by actual trial batches of fresh material. For this it is only necessary to have good scales and a strong and rigid cylinder, say, a piece of 10-inch wrought-iron pipe capped at one end. Carefully weigh out and mix together on a piece of sheet steel or other non-absorbent material all the ingredients, having the consistency the same as is intended to be used in the work. Place these in the pipe, carefully tamping all the time, and note the height to which the pipe is filled. Weigh the pipe before filling and after being filled, thus checking weight of material mixed. Throw this material away before it has time to set, and clean the pipe. Make up another batch, using the same weights of cement and water and the same total weight of sand and stone, but have the ratio of weights of the sand and stone slightly different from the first. Note whether, after placing, the height in the cylinder is less or more than was the height of the first batch, and this will be a guide to further similar mixes, until a proportion is found which gives the least height in the cylinder, and at the same time works

*An illustration of this is given by Mr. James L. Davis, in Transactions American Society of Civil Engineers, Vol. LIX, p. 145. well while mixing and looks well in the cylinder, all the stones being covered with mortar. This method, if carefully followed, will give very accurate results, but of course does not indicate, as does mechanical analysis, what other changes can be made in the physical sizes of the sand and stones so as to get the best available composition.

Mr. A. E. Schutté, in studying the proportions of materials for bituminous macadam pavement for the Warren Brothers Company, has very effectively developed the method of volumetric synthesis with dry materials. His experiments included various classes and sizes of stone, sand, and screenings ranging from 3 inches diameter down to that which passes a No. 200 sieve. He found that the best method for compacting dry materials, such as sand, gravel or broken stone, is to place them in a vessel the shape of a truncated cone, with the largest diameter at the bottom. The cone is filled with the coarsest material and taken by a laborer, who compacts it by repeatedly striking the cone against the ground, keeping the measure full by adding new material of the same kind. When it ceases to settle, the contents is emptied and mixed with a portion of a finer material, replaced in the measure and compacted as before. By repeated trials the exact size and maximum volume of successive finer materials, which may be added without appreciably increasing the bulk of the coarsest after thoroughly compacting, are determined. Mr. Schutté has found that for different shapes of particles the proportions of each size must be varied, but having determined the required percentages for a certain stone, that is, for a stone from a certain quarry, the proportions of the sizes from day to day need be varied but little.

Practical Proportioning During Progress of the Work. The above methods of mechanical analysis and volumetric synthesis are methods to be used in the office or laboratory in determining the relative values of all the aggregates available for the work. When the work is begun, however, and the same general character of aggregate is used day by day, it is only necessary to see that the material does not change or, if it does, simply to readjust the relation between the fine and coarse aggregate. To do this by the mechanical analysis method, it is only necessary to have a nest of about six 8-inch sieves: say, stone sieves with I inch, $\frac{1}{2}$ -inch and $\frac{1}{4}$ -inch diameter holes and sand sieves No. 8, 20, 50 and 100, together with a cover and pan. The shaking can be done by hand, and the sievings beginning with the finest emptied into a long glass tube. If a standard sample has been previously put in the tube in the same way and the points of division between the different sievings marked on a paper pasted on the outside of the tube, the difference between the standard and the sample under test can be quickly seen and modifications made in the mix accordingly.

212 Proportions in Actual Structures. Compiled by TAYLOR AND THOMPSON. or Portland Cement bbl. of 380 lbs Sand ft. base ninal mea Nominal Proportions. Sto ft. Loose Cu. /olumes on nom actual) uremen Authority. Reference. Structure. Loose cu. nominal Asst. Engineer New Brooklyn Bridge Piers 8.5 19.5 Sec. 1 Boston El. Ry. Column Foundations 1:22:5* 1 19.1 nominal G. A. Kimball our. A. E. S. 9.5 June '03, p. 353 N. Y C. & H R. R. R. actual W. J. Wilgus 26.2 Assn. of Ry. Supts. 13.0 I I 23.7 14.0 7.1 actual actual 12 2 1900, p. 207 I .7.0 3.5 actua I C. M. & S. P. Ry. Piers and Abutments 1:2:5 Culverts and Foundations 1:3:72 Assn. of Ry. Supts. 7.8 21.4 actual W. A. Rogers I I 28.5 10.5 actual 1900, p. 228 Or. R. R. & Nav. Co. nominal W. H. Kennedy Assn of Ry. Supts. Abutments, Piers and Culverts . 18.3 1:3:5 11.0 12.8 1900, p. 182 22.0 nominal I 14.7 25.7 nominal I actual A. S. Markley Assn. of Ry. Supts. C. & E. I. R. R. 1:2:5 9.3 26.7 1900, p. 245 Northern Pacific Ry. actual E. H. McHenry Assn. of Ry. Supts 11.2 20.2 I 11.2 20.2 actual 1900, p. 235 T actual Fred Eilers Assn. of Ry. Supts. C., B. & Q. R. R. 1:3:6 12.5 22.5 nominal Lewis Kingman Assn. of Ry. Supts. Mexican Central Ry. 1:3:6 13.5 27.0 N.Y.R.T Com. Spec. 1900, p. 212 Spec. 1900, p. 83 N. Y. Subway Roofs and Sidewalls not over 18 in. thick . . . 1:2:4 Sidewalls or Tunnel Arches . . 1:22:5 7.2 9.0 14.4 18.0 nomina I I nominal Wet Foundations not over 24 in. thick . . . 1:2:4 Wet Foundations nominal T 72 14.4 18.0 exceeding 24 in. thick . 1:21:5 90 nominal Boston Subway $1:2\frac{1}{2}:4$ 8.3 13.2 nominal H. A. Carson I Harvard University Stadium . . 1:3:6 Maine Fortifications Leveling for Foundations . . . 1:5:10 Walls and Masses 18.2 36.5 nominal S. W.Roessler Report Chief of Engrs. U. S. A. not exposed to fire 1:4:8 Walls and Masses 14.6 20.2 1001, D. 011 T . . . 1:3:6 11.0 22.0 nominal exposed to fire Masses for greater imperviousness 1:3:5 18.3 nominal 11.0 W. B. Fuller Little Falls 266 nominal 11.4 7.6 15.2 nominal C. Coleman Cement, Sept., 'oc, Duluth Ship Canal Piers 11.8 23.8 I p. 144 W. B. Fuller Boonton, N. J., Dam 1:23:648 10.5 23.8 nominal Geo. W. Rafter 11.4 36.8 Trans. A. S. C. E. Vol. LII, p. 102 Eng. News. Oct. nominal Emile Low Buffalo Breakwater 30 5 T 1 9.6±¶ 19.3±¶ nominal Specifications Pennsylvania Tunnel 1:212:5 15, '03, p. 337 Specifications, 1000 nominal H. A. Carson

* Mixture varied with loading from 1:1:3 to 1:3:6.

East Boston Tunnel $1:2\frac{1}{2}:4$

7.7 12.4

I

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The test by volumetric synthesis is one easily made in a modified way in the field and with care gives good results. Procure a galvanized tin pail and a spring balance graduated to half pounds; take a representative sample of concrete, being careful that it contains no more stones or mortar than the regular concrete; tamp it into the pail until level full and weigh. Any variation from the standard weight will show a change in the character of material, and this change can usually be detected and corrected by observing the materials and mixing. If not, then mechanical analysis methods will have to be used.

PROPORTIONS OF CONCRETE IN PRACTICE

The proportion of cement to the aggregate depends upon the nature of the construction and the required degree of strength or water-tightness as well as upon the character of the inert materials. Strength and impermeability are discussed in Chapters XX and XIX respectively, but the table which follows, compiled by the authors, giving the proportions adopted upon important structures, may in some cases be useful as an arbitrary guide. Actual measurement, that is, measurement of proportions as actually used, almost invariably shows leaner mixtures than the nominal proportions called for. This is largely due to the heaping of the measuring boxes in practise.

In general, as both strength and imperviousness increase with the proportion of cement to aggregate, relatively rich mixtures are necessary for loaded columns and beams in building construction, for thin walls subjected to water pressure, and for foundations laid under water.

* Pages 214 and 215 are omitted in this Edition.