horizontal 60% line at the ordinate corresponding to a diameter of 0.117 inch, and the 10% horizontal line at ordinate 0.023 inch. Its uniformity coefficient and similarly the uniformity coefficients of the other sands are as follows:

			Uniformity Coefficient
Coarse sand	$\frac{0.117}{0.023}$	=	5.1
Medium sand	$\frac{0.038}{0.009}$	=	4.2
Fine sand	$\frac{0.018}{0.008}$	=	2.2

In general, it may be said that a sand with a uniformity coefficient above 4.5 is a good coarse sand for concrete work, and in comparing different natural sands the one having the highest uniformity coefficient may be considered the best.

As in ordinary bank sands the size of the particles at the 10% line (which is termed the effective size,* e. s.) does not greatly vary, the diameter at the 60% line alone is a very good indication of the coarseness of the sand. A knowledge of the effective size and the uniformity coefficient of any sand enables one accustomed to mechanical analysis diagrams to form a picture of its character.

Mr. Allen Hazen,† who first used these terms in the examination of filter sand, states with reference to the percentage of voids or "open space" in compacted sand corresponding to different coefficients:

> A rough estimate of the open space can be made from the uniformity coefficient. Sharp-grained materials having uniformity coefficients below 2 have nearly 45 per cent. open space as ordinarily packed; and sands having coefficients below 3, as they occur in the banks or artificially settled in water, will usually have 40 per cent. open space. With more mixed materials the closeness of packing increases, until, with a uniformity coefficient of 6 to 8, only 30 per cent. open space is obtained, and with extremely high coefficients almost no open space is left.

For loose sand at least 10 should be added to these percentage values.

* The effective size itself is of considerable value for comparison of sand for filters, but not for concrete.

† Twenty-fourth Annual Report of State Board of Health of Massachusetts for 1892.

CHAPTER XI

PROPORTIONING CONCRETE

By William B. Fuller*

IMPORTANCE OF PROPER PROPORTIONING

The proper proportioning of concrete materials increases the strength obtainable from any given amount of cement, and also the water-tightness. Conversely, it permits, for a given requirement of strength and water-tightness, a reduction in the amount of cement, thereby reducing the cost.

Upon large or important structures it pays from an economic standpoint to make very thorough studies of the materials of the aggregates and their relative proportions. This fact has been seriously overlooked in the past, and thousands of dollars have sometimes been wasted on single jobs by neglecting laboratory studies or by errors in theory. Since cement is always the most expensive ingredient, the reduction of its quantity, which may very frequently be made by adjusting the proportions of the aggregate so as to use less cement and yet produce a concrete with the same density, strength and impermeability, is of the utmost importance.

As an example of such saving, the ordinary mixture for water-tight concrete is about 1 : 2 : 4, which requires 1.57 barrels of cement per cubic yard of concrete. By carefully grading the materials by methods of mechanical analysis the writer has obtained water-tight work with a mixture of about 1 : 3 : 7, thus using only 1.01 barrels of cement per cubic yard of concrete. This saving of 0.56 barrels is equivalent, with Portland cement at $1.60 per barrel, to $0.89 per cubic yard of concrete. The added cost of labor for proportioning and mixing the concrete because of the use of five grades of aggregate instead of two was about $0.15 per cubic yard, thus effecting a net saving of $0.74 per cubic yard. On a piece of work involving, say, 20 000 cubic yards of concrete such a saving would amount to $14 800.00, an amount well worth considerable study and effort on the part of those in responsible charge.

Proper proportioning is also important for reinforced concrete so as to give the uniformity and homogeneity which cannot be obtained without careful attention to the proportions and grading of the aggregates.

* The authors are indebted to Mr. Fuller for the material for this chapter.

METHODS OF PROPORTIONING

It is recognized generally that for maximum strength a concrete should be as dense as possible, that is, that it should have the smallest practicable percentage of voids. The various methods of aiming toward this result have been outlined as follows:*

(1) Arbitrary selection; one arbitrary rule being to use half as much sand as stone, as 1 : 2 : 4 or 1 : 3 : 6; another, to use a volume of stone equivalent to the cement plus twice the volume of the sand, such as 1 : 2 : 5 or 1 : 3 : 7.

(2) Determination of voids in the stone and in the sand, and proportioning of materials so that the volume of sand is equivalent to the volume of voids in the stone and the volume of cement slightly in excess of the voids in the sand.

(3) Determination of the voids in the stone, and, after selecting the proportions of cement to sand by test or judgment, proportioning the mortar to the stone so that the volume of mortar will be slightly in excess of the voids in the stone.

(4) Mixing the sand and stone and providing such a proportion of cement that the paste will slightly more than fill the voids in the mixed aggregate.

(5) Making trial mixtures of dry materials in different proportions to determine the mixture giving the smallest percentage of voids, and then adding an arbitrary percentage of cement, or else one based on the voids in the mixed aggregate.

(6) Mixing the aggregate and cement according to a given mechanical analysis curve.

(7) Making volumetric tests or trial mixtures of concrete with a given percentage of cement and different aggregates, and selecting the mixture producing the smallest volume of concrete; then varying the proportions thus found, by inspection of the concrete in the field.

The most practical method known to the writer for accurately determining the proportions of each material is by mechanical analysis of the aggregates, as described on page 211.

Volumetric synthesis, or proportioning by trial mixtures (p. 210) is another method which is sometimes useful, and produces fairly scientific results.

Since in many cases the proportions for a concrete must be selected more or less arbitrarily, after outlining the principles of proper proportioning, some of the less exact methods which are frequently used in practice will be

* From "Proportioning Concrete," by Sanford E. Thompson, Journal Association Engineering Societies. Vol. XXXVI, Apr. 1906, p. 185.

taken up before referring to the more scientific ones, and some of the causes for inaccuracies of these approximate methods discussed.

PRINCIPLES OF PROPER PROPORTIONING

The principles underlying the correct proportions of the materials of concrete are the same as those for mortar, namely, that the mass when compacted shall have the greatest possible density. In order, therefore, to obtain a knowledge of correct proportioning it will be best to first study the general conditions which are known to affect density.

Perfect spheres of equal size piled in the most compact manner theoretically possible leave but 26% voids. If the spaces between such a pile of equal-sized perfect spheres were filled with other perfect spheres of diameter just sufficient to touch the larger spheres, it would take spheres having relative diameters of 0.414 and 0.222 of the larger spheres, and the voids in the total included mass would be reduced to 20%. Using in this same manner smaller and smaller perfect spheres, it is conceivable that the voids could be reduced to so low a per cent of the total mass and to a size so small as to be only in a capillary form, and thus prevent the passage of water. This is assuming that every particle is placed exactly in its assigned place, but it is inconceivable that such an arrangement should take place under practical conditions, and in fact numerous trials by the writer with large masses of equal-sized marbles have demonstrated that they cannot be poured or tamped into a vessel so as to give less than 44% voids.

If equal quantities of spheres of, say, three sizes are mixed together, the per cent of voids in the total mass immediately increases, becoming about 65%, due probably to the smallest spheres getting between and forcing apart the largest. If, however, the containing vessel is continually shaken and the spheres stirred around, the smallest spheres will gradually all gravitate to the bottom and the largest to the top and the amount of voids in the total mass will again approach 44%. If a large number of different sized spheres are used, employing an increasingly large number of the smaller sizes so that each larger size may be said to be wholly surrounded by the next smaller size, the voids remain the same, no matter what the shaking, and will in some cases reach as low as 27%.

With ordinary stones and sands the same law holds as with perfect spheres except that they do not compact as closely, and the percentage of voids under comparable conditions is larger, varying with the degree of roughness and other features of the stones and sands used for the experiments.

When dry cement is added to a dry aggregate of stone and sand it acts in the same manner as fine sand, and for obtaining the greatest density with dry cement, the cement must replace an equivalent amount of fine sand.

The theory of a concrete mixture is well stated by Mr. Feret* as follows:

> The problem of making the best concrete is thus reduced to the selection of a mixture of materials whose granulometric composition† corresponds to the maximum of density, since when this composition is known absolute volumes of cement may be substituted for equal absolute volumes of fine sand and vice versa, so as to vary the strength as desired while the density remains the same.

In other words, having mixed dry, inert materials in proportions necessary for greatest density, a portion of the grains of the very finest aggregate (that is, the finest particles of sand or dust) may be replaced by a corresponding quantity of cement to the extent required for the desired strength. This is not strictly true for concrete mixtures, because, when water is added to dry cement, the cement particles are separated from each other by the surface tension of the film of water, and it is no longer possible to obtain as dense a mixture as is theoretically possible with the dry mixture.

The density of concrete therefore has been found to depend upon the varying degree of roughness of the stone and sand, the relative sizes of the diameters of the stone, sand and cement, and the amount of water used.

The fineness of the cement particles and the amount of water to be used are determined by questions discussed elsewhere, and we have to deal here only with the proportioning of the sand and stone.

DETERMINATION OF THE PROPORTION OF CEMENT

The most difficult question to decide with accuracy in proportioning is the proportion of cement to use. This is to a considerable extent a matter of mature judgment, depending upon the nature of the construction, the degree of strength required within a certain limit of time, the required watertightness, the character of the aggregates, and many other matters which must be considered in direct connection with the work to be done and the available materials. An engineer experienced in concrete construction and tests can estimate approximately the strength of concrete made with certain materials, and select the proportions accordingly. The surest plan after selecting and grading the aggregates is to make up specimens of concrete and test its crushing strength, but this is usually impracticable for lack of time. The next best plan is to have the tensile strength determined of mortar made from the sand to be used and by comparing

*Chimie Appliquée 1897, p. 523.
†Proportioning of sizes.

this with the strength of the mortar of standard sand an idea can be formed of the proportion of cement to select. If a sand is fine, a richer mortar must be used, frequently instead of a 1 : 2 selecting a 1 : 1½ or even 1 : 1, and the amount of coarse aggregate also reduced to accord with this.

An experimental plan which has been followed to determine the minimum quantity of cement which will produce a concrete practically free from air voids is to mix the aggregates in the correct proportions as described in the pages which follow, compact them by ramming or hard shaking, and then determine their voids by weighing and correcting for specific gravity.* The sand should be in the natural state of moisture found in the interior of the bank, not because this is the condition in which it will be mixed in the concrete, but because it may be assumed in the natural state to contain a quantity of moisture varying with its fineness. If gravel is used it may be taken in the same way, while coarse broken stone should be dry, and dry broken stone screenings may be mixed with about 4% of water by weight. Correction must be made for this moisture after weighing the mixed material, so that the voids calculated will be simply air voids.

In determining the quantity of cement to fill these air voids it may be assumed without appreciable error that 100 lb. of cement will make 1.0 cu. ft. of neat paste. This is a larger volume than would result with ordinary plastic paste, but makes a slight allowance for the additional moisture required for the sand and stone. To the quantity of cement thus determined 10% may be added, *i. e.*, 10% of the cement, not of the total mixture, to provide for imperfect mixing.

PROPORTIONING BY ARBITRARY SELECTION OF VOLUMES

The common custom of specifying arbitrarily the proportions of cement, sand and stone in parts by volume, while convenient in construction, causes wide discrepancies in results because of different methods of measuring the materials. A concrete called a 1 : 2 : 4 mixture by one man may not contain any more cement than a concrete termed a 1 : 3 : 6 mixture by another.†

Notwithstanding this, if the units of measurement and the methods of measuring are stated definitely, arbitrary selection of proportions may give good results in practice, although necessitating a larger quantity of cement with consequently a greater net cost than more scientific proportioning would require.

The percentage of volume of sand required for ordinary gravel or broken

*See page 165.
†These variations are discussed more fully by the authors on page 218.

stone from which the finest material has been screened may be taken between the limits of 40% and 60% with an average, which is suitable under many conditions, of 50%. If the cement is taken as additional, which is not strictly correct, this ratio corresponds to proportions 1 : 1½ : 3, 1 : 2 : 4, 1 : 2½ : 5, and 1 : 3 : 6, which are suggested by the authors in Chapter II as standard mixtures for the use of those who are inexperienced in concrete work.

In cases where the coarse material contains a good many small particles, as does crusher run, broken stone or graded gravel, or the sand is so fine as to flow readily into the voids of the stone, the proportion of sand should be slightly less than half the volume of stone. Since the cement also increases the bulk of mortar and hence assists to fill the voids in the stone, it is suggested that with such aggregates the volume of the stone be made equal to the cement plus twice the volume of the sand. This would give proportions 1 : 1½ : 4, 1 : 2 : 5, 1 : 2½ : 6, and 1 : 3 : 7 for these special conditions.

Proportions adopted by various authorities and tabulated on page 212 may serve as a guide to arbitrary selection.

It is a good plan on work which will not warrant special tests and for which there is no choice of aggregates, to use at first twice as much stone or gravel as sand and then vary the relative proportions of the sand to the stone as the work progresses, governing this by the way the concrete works into place. Too much sand will be indicated by the harsh working of the concrete, while if there is too little sand, stone pockets are apt to occur on the surface of the concrete, and it will be difficult to fill the voids of the stone.

Screened vs. Unscreened Gravel or Broken Stone. Unscreened gravel is often used alone for the aggregate, but there is scarcely any case where the cost of screening and re-mixing the materials will not be less than the saving in the cement by using screened aggregates. The quantity of sand in different parts of the same gravel bank always varies greatly and the run of the bank rarely contains sufficient coarse stone to make a dense concrete. If, as is sometimes the case, the quantity of material coarser than ¼ inch is about the same as that which passes a ¼-inch sieve, then, if used without screening the same quantity of total aggregate must be used as would otherwise be specified for the coarse aggregate; that is, instead of 1 : 2 : 4 proportions, the unscreened gravel would require 1 : 4.

Broken stone as it runs from the crusher will contain considerable dust, and may sometimes be used economically by simply adding sand without screening. However, there is apt to be a separation of the coarse particles from the fine as they roll down the pile so that less homogeneous proportions can be attained. Consequently the writer is in favor of separating the aggregate into as many parts as is consistent with economy for the work in hand. Even on small work he believes it preferable to screen out the sand or dust and re-mix it in the specified proportions.

PROPORTIONING BY VOID DETERMINATION

The determination of proportions by finding the volume of water which may be poured into the voids of a unit volume of stone and selecting a volume of sand equal to this volume of water is one which gives no better results in practice than arbitrary selection of the proportions, as described in the preceding paragraphs, and varying the relative proportions of sand to stone when placing. The determination of the proportion of cement to sand by void measurement is still more misleading; in fact, for reasons discussed below, it is so inaccurate that no consideration will here be given to it.

The theory of proportioning by voids is that if the stone or gravel contains, say, 40 per cent voids as measured by the contained volume of water, the required volume of sand is theoretically 40% of the volume of the stone, and supposing the ratio of cement to sand to be as 1 : 2, the relation of parts of sand to parts of the coarse aggregate would be as 2 : 5, thus making the proportions 1 : 2 : 5. Because of the inaccuracy of this method of procedure, as discussed below, it is necessary in most cases, even although the cement and water will still further increase the bulk, to take a volume of sand, say 5% to 10% in excess of the voids; that is, for gravel with 40% voids to use 45% to 50% of its volume of sand, thus making the proportions 1 : 2 : 4½. If the coarse material is screened broken stone of large size, say 1½ or 2-inch, the volume of sand may be taken equal to the volume of voids instead of in excess of them, because the particles of sand will all be small enough to fit into the voids of the stone without appreciably increasing its bulk. Such stone usually has about 45% to 50% voids, so that we should have proportions 1 : 2 : 4½ or 1 : 2 : 4, the same as for the gravel concrete.

The irregular distribution of the materials by imperfect mixing may usually be disregarded, because the volume of gaged mortar is always in excess of the volume of sand from which it is made.

Care must be exercised in any case to guard against a larger excess of sand than is absolutely necessary, because the voids in a concrete are lessened by using stone in place of sand. Take, for instance, sand having 45% voids and stone having 40% voids. With the sand just filling the voids of the stone it is easily calculated that the resultant mass has 18%

voids; but supposing an excess of 10% of sand, there would be 10% of the material having 45% voids, which means there would be 2.5% more voids in the resultant mass.*

Authorities differ as to whether the stone should be loose or shaken when determining the voids. Loose measurement is generally considered preferable because it corresponds more nearly to the final volume of the concrete, and more sand is always necessary than will just fill the voids of rammed stone, since the sand and cement separate the stones and prevent their lying close together in concrete. In determining, however, the quantity of cement required for the mixture of aggregates the materials should be compacted as described on page 211.

The chief inaccuracy of this method of basing the proportions of the finer materials of a concrete mixture upon the water contents of the voids in the larger is due to the difference in compactness of the materials under varied methods of handling, and to the fact that the actual volume of voids in a coarse material may not and usually does not correspond to the quantity of sand required to fill the voids, and that therefore the common method of proportioning by basing the volume of sand or of mortar upon the volume of water which can be poured into the broken stone leads to false conclusions. The reasons for this inaccuracy are chiefly because the grains of sand thrust apart the particles of stone, and because with most aggregates a portion of the particles of sand or fine screenings are too coarse to enter the voids of the coarsest material.

Even in a mass of stones of uniform size many of the separate voids are much smaller than the particles. If we have, then, a mass of gravel ranging from fine to coarse or a mass of crusher-run broken stone, even with the finest sand or the dust screened out of them, the individual voids are many of them so small that a large number of the particles of natural bank sand will not fit into them, but will get between the stones and increase the bulk of the mass. On account of this increase in bulk, even with thorough mixing more sand is required than the actual volume of the voids in the coarse material. The separation of the particles of stone by the sand is illustrated in the mixture shown in Fig. 2, page 15.

To illustrate this important principle, an extreme example may be cited. Suppose that we have a mixture in equal parts of 1-inch stone and $\frac{7}{8}$-inch stone. By the usual method of reasoning employed in proportioning concrete, if the 1-inch stone has 50% voids, we should require a volume of $\frac{7}{8}$-inch, equal to 50% of the volume of the 1-inch stone, in order to fill

* See discussion by the writer in Transactions American Society of Civil Engineers, Vol. XLII, p. 142.

the voids in the latter. The absurdity of this is apparent, because the two stones are so near a size that the smaller cannot fit into the voids of the latter, and the bulk of the mixture is inappreciably less than the sum of the separate volumes, that is, the mixture still has nearly 50% voids. The principle is just as true, although the total effect is less, if we consider it with reference to the finer particles of the gravel or the crusher-run broken stone and the sand or fine screenings which are to be introduced to fill the voids. The sizes of many of the particles of the latter are so nearly equal to the sizes of the smallest particles of the coarse material that they increase the total bulk instead of reducing the voids. They also get between the surfaces of the stone particles and prevent the stones touching each other.

We might conclude from the above that the best concrete can be made with a coarse stone of uniform size and a sand whose particles are all small enough to fit into its voids; in fact, this is the conclusion reached by the advocates of broken stone of uniform size in preference to crusher-run stone.

Our experiments indicate that while this may be true in theory, in practice in making concrete the graded materials give about the same density and work rather smoother in handling and placing.

The point, however, which is to be emphasized is the inaccuracy of determining the exact volume of sand or mortar by simply measuring the water contents of the voids in the coarse aggregate.

The selection of the proportion of cement by determination of the water contents of the voids in sand is even more inaccurate than the proportioning of sand to stone by void measurement. The varying effect of moisture on the sand so influences the volume of the voids that their determination is chiefly important as an aid to the judgment; and as a matter of fact, although in practice the quantity of cement is supposed to depend upon the volume of voids in the sand, it is customary to select a definite relation of cement to sand varying according to the character of the construction from 1 : 1 to 1 : 3, recognizing, however, that fine sand—and fine sands in an ordinary state of moisture will almost always have the distinguishing characteristic of a lighter weight per cubic foot than coarse sands and a consequently larger percentage of voids—requires more cement for equivalent strength.

As already stated, if the work is too small to warrant a thorough study of the materials by mechanical analysis or volumetric synthesis, or some other scientific method, it is evident from the above discussion that it is nearly as accurate to determine the proportions by arbitrary selection (see p. 186) as by a study of voids.