## CHAPTER X

## VOIDS AND OTHER CHARACTERISTICS OF CONCRETE AGGREGATES

In this chapter are given tables of the specific gravities and voids of different materials, and the method of determining them, also laws relating to the voids in concrete aggregates, and the effect of compacting such materials.
Laws of Volumes and Voids. The most important of these general laws relating to volumes of different materials, and to their voids, may be stated as follows:
(I) A mass of equal spheres, if symmetrically piled in the theoretically most compact manner, would have $26 \%$ voids whatever the size of the spheres, but by experiment it is found that it is practically impossible to get below $44 \%$ voids. (See p. 168.)
(2) If a dry material having grains of uniform shape be separated by screens into grains of uniform dimensions, the separated sizes (except when finer than will pass a No. 74 screen) will contain approximately equal percentages of voids; in other words, a dry substance consisting of large particles, all of similar size and shape, will contain practically the same percentage of voids as a substance having grains of the same shape but of uniformly smaller size. (See p. 170.)
(3) In any material the largest percentage of voids occurs with grains of uniform size, and the smallest percentage of voids with a mixture of sizes so graded that the voids of each size are filled with the largest particles that will enter them. (See p. 171.)
(4) An aggregate consisting of a mixture of coarse stones and sand has greater density - that is, contains a smaller percentage of voids - than the sand alone. (See p. I72.)
(5) By Fuller and Thompson's experiments, perfect gradation of sizes of the aggregate appears to occur when the percentages of the mixed aggregate passing different sizes of sieves are defined by a curve which approaches a combination of an ellipse and straight line. (See Chap. XI, p. 201.)
(6) Materials with round grains, such as gravel, contain fewer voids than materials with angular grains, such as broken stone, even though
the particles in both may have passed through and been caught by the same screens. (See p. 174.)
(7) The mixture of a small amount of water with dry sand increases its bulk. In the case of most bank sands the maximum volume and hence the smallest amount of solid matter per unit of volume, that is, the largest percentage of absolute voids - being reached with from $5 \%$ to $8 \%$ of water. (See p. 176.)

## CLASSIFICATION OF BROKEN STONE.*

Rocks which are commonly employed for concrete or for road making are commercially classified as (a) traps, (b) granites, (c) limestones, (d) conglomerates, and (e) sandstones
The trade term "trap" includes dark green to black, heavy, close textured, tough rocks of igneous origin, thus covering a variety of rock whose mineralogical names are diabase, norite, gabbro, etc. As shown in the table below, the traps usually range in specific gravity from 2.80 to 3.05 .

Granites, commercially so called, include the lighter colored, less dense rock, such as not only true granite, but syenite, diorite, gneiss, mica schist, and several other groups. Their specific gravities range from about 2.65 to 2.85 , averaging close to 2.7 . Although, as road metal, the traps are usually far superior to granites, for concrete there appears to be no great difference in the value of the two classes. The distinction, however, is worth keeping because a concrete stone is often purchased from road metal quarries.
Limestones of normal type range in specific gravity from 2.47 to 2.76 , averaging about 2.60 , although the very soft stones, which are not suitable for high class concrete, may fall below 2.0.
Conglomerate, or pudding stone as it is often termed, is essentially a very coarse grained sandstone, ranging in specific gravity from 2.50 to 2.80. It makes a good concrete aggregate.

Sandstones of compact texture, such as the Potsdam and Medina sandstones, and the Hudson River bluestone, may run as high in specific gravity as 2.75 , while the looser textured, more porous sandstones may fall as low as 2.10, a fair average being about 2.40 .
Shale and slate make poor concrete aggregates, because their crushing and shearing strength is low.
*The authors are indebted to Mr. Edwin C. Eckel for the material under this heading, which has been especially prepared by him for this Treatise.
Compled by Edwin C. Eckel.

| Locality. | Specific Gravity. | Locality. GRANITE. | Specific Gravity |
| :---: | :---: | :---: | :---: |
| Massachusetts |  | California |  |
| Boston | 2.78 | Penrhyn... | 2.77 |
| Minnesota |  | Rocklin .. | 2.68 |
| Duluth... | 3.00 | Connecticut |  |
| Duluth... | 2.80 | Greenwich.. | 2.84 |
| Taylors Falls | 3.00 | New London. | 2.66 |
| New Jersey |  | Georgia |  |
| Jersey City Heights | 3.03 | Stone Mt. | 2.69 |
| Little Falls ..... | 2.99 | Maine |  |
| New York |  | Hallowell. | 2.66 |
| Staten Island | 2.86 | Maryland |  |
|  |  | MASSACHUSETTS | 2 |
|  |  | Quincy. | 2.70 |
|  |  | New Hampshire Keene .......... |  |
|  |  | New York |  |
|  |  | Ausable Forks | 2.76 |
|  |  | Rhode Island |  |
|  |  | Westerly | 2.67 |
|  |  | VERMONT Barre | 2.65 |
|  |  | Wisconsin |  |
|  |  | Amberg . |  |
|  |  | Montello | 2.64 |
| LIMESTONE. |  | SANDSTONE. |  |
| Locality. | Specific <br> Gravity. | Locality. | Spavity, |
| Illinors |  | Colorado |  |
| Joliet | . 2.56 | Ft. Collins | . 2.43 |
| Lemont | . 2.51 | Trinidad. | . 2.34 |
| Quincy. | . 2.57 | Connecticut |  |
| Indiana |  | Portland ${ }^{1} \ldots$ | 2.64 |
| Bedford | 2.48 | Massachusetts |  |
| Salem . | . 2.51 | Longmeadow ${ }^{1}$ | 2.48 |
| Minnesota |  | Minnesota |  |
| Frontenac | . 2.63 | Fond du Lac | - 2.24 |
| Winona | . 2.67 | New Jersey |  |
| New York |  | Bellevill ${ }^{1}$ | 2.26 |
| Canajoharie | 2.68 | New York |  |
| Glens Falls . | . 2.70 | Albion ${ }^{2}$. | 2.60 |
| Kingston | . 2.69 | Medina ${ }^{2}$ | 2.41 |
| Prospect | . 2.72 | Potsdam ${ }^{3}$ | 2.60 |
| Sandy Hill | . 2.76 | Oxford ${ }^{4}$ | - 2.71 |
| Williamsville. | . 2.71 | Malden ${ }^{5}$ | - 2.75 |
|  |  | Oswego .. | 2.42 |
| France Soft Limestone |  | ОНІо |  |
| France <br> Caen $\qquad$ | . 1.84 | Cleveland | . 2.21 |
|  |  | Massillon. | 2.11 |
| ${ }^{1}$ Brownstone. |  | ${ }^{4}$ Bluestone. ${ }^{\text {a }}$ |  |
| ${ }^{2}$ Medina sandstone. |  | ${ }^{5}$ Hudson River Bluestone. |  |
| ${ }^{3}$ Potsdam sandstone. |  | ${ }^{6}$ Berea grit. |  |

## AVERAGE SPECIFIC GRAVITY OF SAND AND STONE

The specific gravity of a substance is the ratio of the weight of a given volume to the weight of the same volume of distilled water at a temperature of $4^{\circ}$ Cent. ( $39^{\circ}$ Fahr.). For ordinary tests of stone and sand, the water need not be distilled and may be at ordinary temperature.

A knowledge of the specific gravity of the particles of the sand and stone is important to the engineer as a ready means of determining the percentages of voids.
The uniformity in the specific gravity of different sands is very con venient for calculation. Different authorities who have tested large quantities of sand have reached almost identical conclusions as to the average specific gravity, and all state that it is practically a constant. Mr. Allen Hazen gives 2.65 , Mr. William B. Fuller, 2.64, Mr. R. Feret in France states that "one may without appreciable error adopt an average specific gravity of 2.65 for siliceous sands,"* while Mr. E. Candlot gives limits of 2.60 to 2.68 for sands which are not porous. $\dagger$ The specific gravity of calcareous sands averages about 2.69 by absolute determination, or about 2.55 if measured by the total volume of the particles having their pores filled with air.

Gravels also have quite uniform specific gravity. According to Mr. A. E. Schutté, who has tested gravel from more than forty localities in the United States and Canada, an average value is 2.66 .

The following table gives average values of various concrete aggregates. In every case, the specific gravity is the ratio of the weight of an absolutely solid unit volume of each material to the weight of a unit volume of water. Specific gravities of stone from various localities are given on page 162 .

*Bulletin de la Société d'Encouragement pour l'Industrie Nationale, 1897, Vol. II, p. 159r.
$\dagger$ Ciments et Chaux Hydrauliques, 1898 , p. 246
$\dagger$ Encyclopedia Britannica

## METHOD OF DETERMINING SPECIFIC GRAVITY

The specific gravity of a sample of material is determined by dividing its weight by the weight of water which it displaces when immersed.
The size of sample necessary for the accurate determination of a sand or stone of fairly uniform texture depends chiefly upon the delicacy of the apparatus employed. If scales reading to grams, and measures reading to cubic centimeters, are employed, a sample of 250 grams should give accurate results to two decimal places. With scales reading to $\frac{1}{4}$ ounce, a sample of 4 lb . is necessary for similar accuracy. The water must be maintained at $68^{\circ}$ Fahr. ( $20^{\circ}$ Cent.).
The sample should be taken by the method of quartering described on page 398 .
Before finding the specific gravity of siliceous sand, the sample should be dried in an oven at a temperature as high as $212^{\circ} \mathrm{Fahr}$. ( $100^{\circ}$ Cent.) until there is no further loss in weight. A porous stone, on the other hand, may be first moistened sufficiently to fill its pores, and then the surfaces of the particles dried by means of blotting paper. If this method is followed, the material should be in a similar condition when its voids are determined by the method given on page 165. The absolute specific gravity of the porous stone may be afterward found by drying in an oven and correcting for the moisture lost.
The apparent specific gravity of sand or stone may be determined with an apparatus consisting of scales reading to $\frac{1}{4}$ ounce or to 5 grams, and a tall glass vessel with a reference mark, such as a cylinder or a pharmacist's graduate. The method is as follows:

Make a mark at any convenient place on the neck of the vessel;
Fill the vessel with water at a temperature of $68^{\circ}$ Fahr. ( $20^{\circ}$ Cent.) up to this mark;
Take a known weight in grams or ounces of the material;
Pour material into vessel carefully, a few grains at a time, so that no bubbles of air are carried in with it;
Pour out the clear water displaced by the material (leaving water in the vessel up to the level of the mark), and weigh the water poured out.
Let
$S=$ Weight of material placed in vessel.
$W=$ Weight of water displaced.
Then

$$
\begin{equation*}
\text { Specific gravity of material }=\frac{S}{W} \tag{I}
\end{equation*}
$$

It is essential that the weight of water displaced be weighed to within $\pm 2 \%$. If the scales are not sufficiently sensitive, more material must be taken and a larger vessel used. With balances sensitive to I gr. or $\frac{1}{18}$ oz. the displacement of more than 3 ounces of water is necessary.

## METHOD OF DETERMINING VOIDS

The voids in sand, gravel, and broken stone may be obtained directly from the tables on pages 166 and 167 . Special determinations may be made as described below.
The percentage of voids in sand or fine broken stone cannot be accurately obtained by the ordinary method of placing in a measure and pouring in water, because it is physically impossible to drive out all the air. There may be enough of this held to amount to 1o\% of the volume of the sand, and thus cause a corresponding error in the percentage of voids.
The voids in coarse stone containing no particles under $\frac{1}{2}$-inch diameter may be determined by placing in a box or pail of known volume and pouring in water, but if the specific gravity is known, the method described below is simpler and more accurate.
The only apparatus required are scales of fair accuracy and an exact measure which contains not less than $\frac{1}{2}$ cu. ft . If a cubic foot measure is not available a 16 -quart pail will answer the purpose, although compactness of the sand is less easily adjusted because of the small diameter. Such a pail holds slightly over $\frac{1}{2} \mathrm{cu} . \mathrm{ft}$. and the exact measure is determined by weighing the pail, pouring in 3 I lb .2 oz . of water, and marking the level of the surface. The pail up to this mark contains $\frac{1}{2} \mathrm{cu}$. ft. of any material.
The method of determining the voids is as follows:
Weigh the measure;
Fill the measure to the required level with the material in the state in which the percentage of voids is required, that is, loose, shaken, or packed;
Weigh, and deduct the weight of the measure, calling the net weight of a cubic foot of the material, $S$;
If the material consists of, or contains, sand or fine stone, correct for moisture by taking an exact weight, - about io lb., - drying in an oven at a temperature of at least $212^{\circ}$ Fahr. ( $100^{\circ}$ Cent.) until there is no further loss in weight, and after calculating the percentage of moisture in terms of the weight of the original moist sand or stone, express the percentage as a decimal, $p$.

Select the weight of a cubic foot of absolutely solid rock＊from the table on page $\mathrm{I}_{3}$ ，and call it $R$ ．

$$
\begin{equation*}
\text { Per cent of absolute voids }=\left(1-\frac{S-S p}{R}\right) 100 \tag{3}
\end{equation*}
$$

The air voids are determined，if desired，by deducting the volume of moisture（its weight divided by the weight of one cubic foot of water）

Percentages of Voids Corresponding to Different Weights per Cubic Foot of Sand Gravel，and Broken Stone Containing Various Percentages
of Moisture．（See p． 168 ．）

|  | percentages of absolute voids in material containing moistures BY weight．$\ddagger$ |  |  |  |  |  |  | PERCENTAGES OF ABSOLUTE VOIDS IN material containing moistures by weicht．$\ddagger$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | \％ | \％ | \％ | \％ | \％ | \％ |  | \％ | \％ | \％ | \％ | \％ | \％ |
| 70 | 57.6 | 58.4 | $59 \cdot 3$ | 60.1 | 61.0 | I．I | 98 | 40.6 | 41.8 | 43.0 | 4.2 | $45 \cdot 3$ | ェ． 6 |
| 75 | 54.5 | 55.4 | 56.4 | 57．3 | 58.2 | I． 2 | 99 | 40.0 | 41.2 | 42.4 | 43.6 | 44.8 | т． 6 |
| 80 |  | 52.5 | 53.4 | 54.4 |  |  | 100 | 39.4 | 40.6 | 4 I .8 | 43.0 | 44.2 | 1． 6 |
| 8 I | 50.9 | 51.9 | 52.9 | 53－9 | 54.8 | 1．3 | 101 | 38.8 | 40.0 | 41.2 | 42.5 | 43.7 | 1． 6 |
| 82 | 50.3 | 51．3 | 52.3 | 53.3 | 54.3 | I． 3 | 102 | 38.2 | 39.4 | 40.7 | 41.9 | 43.1 | 1． 6 |
| 83 | 49.7 | 50.7 | 51．7 | 53.3 52.7 | 53.7 | 1.3 1．3 | 103 | 37.6 | 38.8 | 40.1 | 41.3 | 42.5 | 1． 6 |
| 84 | 49．1 | 50.1 | 5 I．I | 52.2 | 53.2 | I．4 | 104 | 37.0 | 38.2 | 39.5 | 40.8 | 42.0 | ． 7 |
| 85 | 48.5 | 49.5 | 50.6 | 51.6 | 52.6 | I． 4 | 105 | 36. | 37.6 | 38.9 | 40. | 41.4 | 1．7 |
| 86 | 48.5 | 48.9 | 50.0 | 51.0 | 52.0 | 1．4 | 106 | 35.8 | 37.0 | 38.3 | 39.6 | 40.9 | 1．7 |
| 87 | 47.3 | 48 | 49.4 | 0．4 | 51．5 | 1.4 | 108 | 34.6 | 35.9 | 37.2 | 38.5 | 39.7 | ． 7 |
| 88 | 46.7 | 47.7 | 48.8 | 49.9 | 50.9 | 1.4 | 109 |  | 35．3 | 36.6 | 37.9 |  |  |
| 89 | 46.1 | 47．1 | 48.2 | $49 \cdot 3$ | 50.4 | 1.4 | 110 | $\begin{aligned} & 33 \cdot 9 \\ & 33 \cdot 3 \end{aligned}$ | 34．7 | 36.0 | 37.3 37.3 | $\begin{aligned} & 39.2 \\ & 38.7 \end{aligned}$ | 1． 8 |
| 90 | 45.5 | 46.5 | 47.6 | 48.7 | 49.8 | I． 4 |  |  |  |  |  |  | 1.8 |
| 91 | 44.8 | 45.9 | 47.0 | 48.2 | 49.2 | 1.5 | 15 | 30.3 | 31.7 | 33.1 | 34.5 | 35.9 | 1.8 |
| 92 | 44.2 | 45.4 | 46.5 | 47.6 | 48.7 | 1． 5 | 120 | 27.3 | 28.7 | 30. | 31.6 | 33．1 | 1．9 |
| 93 | 43 | 44.8 | 45.9 | 7．0 | 48.1 | 1． 5 | 125 | 24. | 25.8 | 27.3 | 28.8 | 30.3 | 2.0 |
| 94 | 43.0 | 44.2 | $45 \cdot 3$ | 46.5 | 47.6 | 1.5 | 130 | 21. | 22.8 | 24. | 25.9 | 27.5 | 2.1 |
| 95 | 42.4 | 43.6 | 44．7 | 45.9 | 47.0 46.4 | 1.5 1.5 |  | 18.2 | 19.8 | 21 | 23.1 | 24 | 2.2 |
| 96 | 41.8 | 43.0 | 44．I | $45 \cdot 3$ | 46.4 | 1．5 | 135 |  |  |  |  |  |  |
| 97 | 41.2 | 42.4 | 43.6 | 44.7 | 45.9 | 1． 6 | 140 | 15.2 | 16.8 | 18.5 | 20.2 | 21．9 | 2.2 |

[^0]in a unit volume of the sand or stone，from the total voids．Expressed in percentages with notation same as above，

Per cent．of air voids $=$ Per cent．of absolute voids $-\frac{S p}{62.3} 100$
Example．－Given a sand whose loose weight per cubic foot is found to be $9^{2} \mathrm{lb}$ ．and its moisture $3 \%$ by weight．Find the percentage of voids in the loose sand
Solution by formula．－Since from the example $S=92$ and $p=0.03$ ， and，from table on page $163, R=165$ ，

$$
\begin{aligned}
\text { Percentage of absolute voids } & =\left(1-\frac{9^{2}-0.03\left(9^{2}\right)}{165}\right) 100 \\
& =45.9 \%
\end{aligned}
$$

This percentage includes the space occupied by the moisture．The net percentage of voids occupied by air alone is the difference between the absolute voids and the percentage of moisture by volume．Moisture is $92 \times 0.03=2.76 \mathrm{lb}$. ，or $\frac{2.76}{62.3}=0.044 \mathrm{cu} . \mathrm{ft}$ ．，corresponding to $4.4 \%$ voids by volume，hence air voids are $45.9 \%-4.4 \%=41.5 \%$ ．

Percentages of Voids Corresponding to Different Weights per Cubic Foot of Dry Broken Stone of Various Specific Gravities．（See p．168．）

| Weight <br> one cu． ft ．of <br> stone． | PERCENTAGES OF ABSOLUTE VOIDS CORRESPONDING TO SPECIFIC GRAVITIES OF STONE OF |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2．4＊ | 2.5 | $2.6 \dagger$ | $2.7 \ddagger$ | 2.8 | 2.98 |
|  | \％ | \％ | \％ | \％ | \％ | \％ |
|  | 53.2 | 55.0 | 56.8 | 58.4 | 59.9 | 6 r .3 |
| 75 | 49.8 | 51.8 | 53.7 | $55 \cdot 4$ | 57.0 | 58.5 |
| 80 | 46.5 | 48.6 | 50.6 | 52.4 | 54．1 | 55.7 |
| 85 | 43.2 | 45.4 | 47.5 | 49.5 | 51.3 | 53.0 |
| 90 | 39.8 | 42.2 | 44.5 | 46.5 | 48.4 | 50.2 |
| 95 | 36.5 | 39.0 | 41.4 | 43.5 | 45.5 | 47.4 |
| 100 | 33.1 | 35.8 | 38.3 | 40.6 | 42.7 | 44.7 |
| 105 | 29.8 | 32.6 | 35.2 | 37.6 | 39.8 | 41.9 |
| 110 | 26.4 | 29.4 | 32.1 | 34.6 | 36.9 | 39.1 |
|  |  | 26.2 | 29.0 | 31.6 | 34.1 | 36.4 |
| 120 | 19.8 | 23.0 | 25.9 | 28.7 | 3 I .2 | 33.6 |
| 125 | 16.4 | 19.8 | 22.8 | 25.7 | 28.3 | 30.8 |
| 30 | 13.1 | 16.6 | 19.8 | 22.7 | 25.5 | 28.1 |
| 135 | 9.7 | 13.3 | 16.7 | 19.7 | 22.6 | 25.3 |
| 140 | 6.4 | 10.1 | $\mathrm{I}_{3} .6$ | 16.8 | 19.7 | 22.5 |

＊Sandstone
nite and slate
STinstone．
${ }_{\delta}$ Trap．

Solution by table (p. 166.) - Opposite 92 lb . per cu. ft., interpolating between $2 \%$ and $4 \%$ moisture, is $46.0 \%$ of absolute voids. From last column $3 \%$ by weight corresponds to $3 \% \times 1.5=4.5 \%$ by volume. $46.0 \%-4.5 \%=41.5 \%$ air voids.

Tables of Voids. From the tables on pages 166 and 167 , the voids in sand, gravel, and broken stone may thus be determined simply by weighing the material and finding the percentage of moisture contained in it, as above described. Since the percentage of moisture by volume is always greater than its percentage by weight, and the two are not proportional to each other, the final column is inserted in the first table for convenience in calculating the moisture by volume.

## VOIDS AND DENSITY OF MIXTURES OF DIFFERENT SIZED MATERIALS

The term density as applied to mortar is defined on page $\mathrm{I}_{35}$. Similarly, in a dry material, such as a concrete aggregate, it is represented by the total volume of the solid particles entering into a unit volume of the aggregate. In dry materials the density is the complement of the voids, since a material which has, say, $40 \%$ voids will have a density of 0.60 ; but density is a more correct term to use than voids because it is applicable to concretes and mortars in which connection the term voids is somewhat ambiguous. The example on page $\mathrm{I}_{3} 8$ a illustrates the method of determining the density of a concrete or mortar.
The densities of dry aggregates of uniform specific gravity, or of mixtures in uniform proportions of materials with different specific gravities, are in direct proportion to their weights. For example, the densities of different dry sands may be compared by weight; or the densities of different mixtures of sand and broken trap in proportions, say, 2 parts sand to 4 parts trap may be compared by weight; but the density of sand and the density of trap screenings cannot be compared by weights unless the differing specific gravities are taken into account.
In the following discussion of the laws formulated on page 160 , both the terms density and voids are used in relation to the dry materials.
Voids in Masses of Similar Sized Particles. (i) The fact that the percentage of voids in a mass of equal spheres symmetrically piled in the theoretically most compact manner is independent of the actual diameter is simply a geometrical propositicn, evident without demonstration by inspection of Fig. 58.
In actual experiment it has been found that while the percentage of voids is uniform regardless of the size of the spheres, it is impossible to
pour spheres into a measure so that they will arrange themselves symmetrically, and the rather astonishing result has been reached by Mr. Fuller (see p. 185) that $44 \%$ is the smallest percentage of voids which can be obtained with equal perfect spheres, no matter what may be their actual diameters or the size of the receptacle.
The following simple demonstration,* which is of theoretical interest, proves that the percentage of voids in a mass of equal spheres symmetrically piled in the most compact manner is $26 \%$, and that the radii (and consequently the diameters) of the two next smaller spheres which can


Fig. 58.-Spheres of Equal Size. (See p. 168.)
be inscribed between the larger ones are respectively 0.4 I and 0.22 of the radius of the large spheres.

The circles in Fig. 58 represent a horizontal plan of two layers of spheres. The centers $A_{1} A_{2} B_{1} D_{1}$ form a regular tetrahedron.
Let edge be 2.
Altitude $=$ difference between level of centers A, B, C, and level of centers D, E is $\frac{2}{3} \sqrt{6}$
Let number of spheres in a layer be $m$, number of layers $n$.
*For which the authors are indebted to Dr. Harry W. Tyler.

Volume of one sphere is $\frac{4 \pi}{3}$
Volume of spheres in a layer, $\frac{4 m \pi}{3}$
Volume of all spheres, $\frac{4 m n \pi}{3}$ (approx.) $=V_{1}$
Cross-section of including space is $2 \sqrt{3} m$ (approx.)
Volume of including space is $\quad 2 \sqrt{3} m \times \frac{2}{3} \sqrt{6} n$ (approx.)

$$
=4 \sqrt{2} m n \text { (approx.) }=V_{2}
$$

Ratio $\frac{\mathrm{V}_{1}}{\mathrm{~V}_{2}}=\frac{4 m n \pi}{3 X_{4} m n \sqrt{2}}=\frac{\pi}{3 \sqrt{2}}=0.74$ (approx.) corresponding to about $26 \%$ voids.

Inscribed Spheres.
I. Sphere inscribed between spheres $A_{1} A_{2} B_{1}$ and $D_{1}$ :

Distance from any vertex $A_{1}$ of tetrahedron to center is $\frac{1}{2} \sqrt{6}$
Radius of small sphere $=\frac{1}{2} \sqrt{6}-1=0.22$ (approx.) or about $\frac{22}{100}$ of the radius of the large spheres.
2. Sphere inscribed between $A_{2} B_{1} B_{2}$ and $D_{1} D_{2} E_{1}$ :

Distance from $A_{2}$ to $E_{1}$ is $2 \sqrt{2}$
Radius of small sphere $=\sqrt{2}-1=0.41$ (approx.) or about $\frac{4 \mathrm{I}}{100}$ of the radius of the large spheres.
(2) The proposition that if a dry material such as sand, pebbles, or irregular broken stone, having grains of fairly uniform shapes, be separated by screens into grains of uniform dimensions, the separated sizes will contain approximately equal percentages of voids, is not so self-evident, but experiment proves that in portions of the same material screened to uniform sizes the percentages of voids will be substantially alike until very fine sizes are reached, such as will pass a No. 74 sieve; below this degree of fineness the particles are entangled by air. The authors have found by experiments given in the following table, that different lots of broken stone from the same quarry, each screened to uniform size, will contain substantially the same percentages of voids, but that lots of stone from different quarries screened to the same size may differ because of the structure of the rock. Published records usually show slight variations in the weight per cubic foot of different sized broken stone, but it is noticeable that some authorities give the heaviest weight,
which corresponds to the smallest percentage of voids, for the larger sizes, while others give the reverse. For example, Patton's Civil Engineering gives the smallest percentage of voids in the coarsest broken stone, while Butler's Portland Cement gives the smallest percentage in the finest stone. The variation in results is undoubtedly due to differences in methods of compacting and to the variations in the sizes of the stones of each lot.
Experiments by Mr. Feret in France, and Mr. Thomas F. Richardson in the United States, show that the percentages of voids in absolutely dry sand which has been screened to uniform size are almost identical. Mr. Feret, experimenting by shoveling dry sand loosely into a 50 liter ( $\mathrm{T} .8 \mathrm{cu} . \mathrm{ft}$.) box, - a measure large enough to eliminate errors of placing, - found that fine ( F ) medium (M) and coarse ( G ) sands each contained about $50 \%$


Loose stone is as thrown by a laborer into a measuring box or barrel.
Material rammed in 6 -inch layers.
voids, while mixing the sizes, which are defined on page 142 , in the best proportions, reduced the voids to $34 \%$.
Densest Mixture of Sand and Stone. (3) The fact that the densest mixture occurs with particles of different sizes is so evident as to require no proof, and this being recognized, it follows that the least density and hence the largest percentage of voids occurs when the grains are all of the same size. The converse of this proposition, that the smallest percentage of voids occurs in a mixture graded so that the voids of each size are filled with the largest particles which will enter them, is
*Mixed in proportions $444 \%$ No. 2, $33 \cdot 3 \%$ No. 3, and $22.2 \%$ No. 4 (dust).
$\dagger$ Another gravel tested, compressed, $8.5 \%$ on shaking, and $11.2 \%$ on hard ramming.


[^0]:    ＊The weight per c
    cubic foot of water．
    $\dagger$ Also applicable verages from 2.6 to 2.7 ．Table is based on specific gravity of of 2.65 ，
    $\ddagger$ The per cent．of absolute voids given in the columns include the space occupied by both the air and
     the weight of sand unde
    per cent．already found．

