From very numerous experiments such as those tabulated on page 136 Mr. Feret evolves the approximate formula

$$P = K\left(\frac{c}{1 - s}\right)^2 \qquad (1)$$

By suitably changing the value of K the formula may be adapted to either the English or the metric system of measurement.

As a proof of this formula Mr. Feret plots on a diagram, shown in Fig. 49, values of $\left(\frac{c}{1 - s}\right)^2$ from column (12) in the table on pages 136 and 137 for abscissas, and the average compressive strengths of the various mortars, from column (22), for ordinates. Since, in formula (1), K is equal to P divided by the square of the quantity in brackets, the value of K is the tangent of the straight line passing through the points. In Fig. 49

$K = 1965$, if the strength is in kg. per sq. cm.

or

$K = 28\,000$, if the strength is in lb. per sq. in.

This particular value is applicable only to the cement used by Mr. Feret in his experiments and to specimens at the age of five months, but the principles involved are of general application.

The most practical application of this formula is in the determination of the relative compressive strengths of various mortars made from the same cement, with sand in differing proportions and of different compositions. Mr. Feret calls attention also to its possible use in laboratory experiments and specifications. A cement, for example, may be required to furnish, when mixed with any sand, a definite value of K, since the value of K is independent of the choice of the sand and of the composition of the mortar.

Experiments by the authors tend to show that the formula does not apply strictly to specimens of different consistency, but that the general law of the increase of strength with the density is applicable except in extreme cases. The formula is inapplicable to tensile tests, although here, too, the general principle appears to hold good.

This subject as related to concrete is discussed on pages 355 to 362

GRANULOMETRIC COMPOSITION OF SAND

Feret's Three-Screen Method of Analyzing Sand.

The determination of the physical characteristics of the sand, which, mixed with a cement, will produce the densest mortar, has been the object

of a large number of experiments by Mr. Feret, which are recorded in Annales des Ponts et Chaussées, 1892. In America Messrs. William B. Fuller and Sanford E. Thompson have extended the researches, by a different method, to the investigation of the properties of concrete. The mechanical analysis of sand and stone is discussed in Chapter XI, and the results of earlier experiments are tabulated on page 376.

Mr. Feret, in studying any sand, separates it by screening into three sizes. He then recombines these three sizes in varying proportions, so as to obtain results which are applicable to any natural or artificially mixed sand. He distinguishes sand from gravel as consisting of grains which will pass through a screen having circular holes of 5 millimeters diameter (0.20 in.). The three sizes of sand he then calls G, M, and F, representing, respectively, the large (*gros*), medium (*moyens*), and fine (*fins*) particles as defined by sifting through metallic sieves with circular holes, or wire cloth of definite mesh, as follows:

Large grains, G, passing circular holes	5 mm.	(0.20 in.)	diameter.
Retained by circular holes	2 mm.	(0.079 in.)	"
Medium grains, M, passing circular holes	2 mm.	(0.079 in.)	"
Retained by circular holes	0.5 mm.	(0.020 in.)	"
Fine grains, F, passing circular holes	0.5 mm.	(0.020 in.)	"

These sizes, Mr. Feret states, are nearly equivalent to sand screened through sieves of wire cloth as follows:

Large grains, G, passing screen of	4	meshes per sq. cm.	(5 meshes per linear inch.)
Retained on "	36	" "	(15 " " ")
Medium grains, M, passing "	36	" "	(15 " " ")
Retained on a "	324	" "	(46 " " ")
Fine grains, F, passing "	324	" "	(46 " " ")

Sometimes, for experimental purposes, he divides each of the sands, G, M, and F, into three intermediate sizes.

The granulometric composition of any sand is represented by its relative proportions, expressed either in weights or absolute volumes, of G, M, and F. For example, a sand containing by weight 50% of the largest grains, 30% of the medium, and 20% of the fine grains, has a granulometric composition of g = 0.50, m = 0.30, f = 0.20.

The granulometric composition of a sand which has been mechanically analyzed, and plotted on a diagram similar to that shown on page 194, may be ascertained readily by drawing three ordinates corresponding respectively to screens of 5, 15, and 46 meshes per linear inch, and determining by the length or the difference in length of these ordinates the proportions which pass and which are retained by the screens of these three meshes. These three proportions or percentages represent the granulometric composition. An illustration of this method of transforming mechanical analysis to granulometric composition is shown in Fig. 57 on page 151.

Feret's Triangles. To simplify the tabulation of results, and arrange them so that they may be understood at a glance, Mr. Feret has used a graphical arrangement which is exceedingly ingenious. In nearly all his writings we find little triangles with the apexes labeled G, M, and F. Curves or contours in these triangles, representing the various properties of the sands or mortars, are based on a system of three instead of two

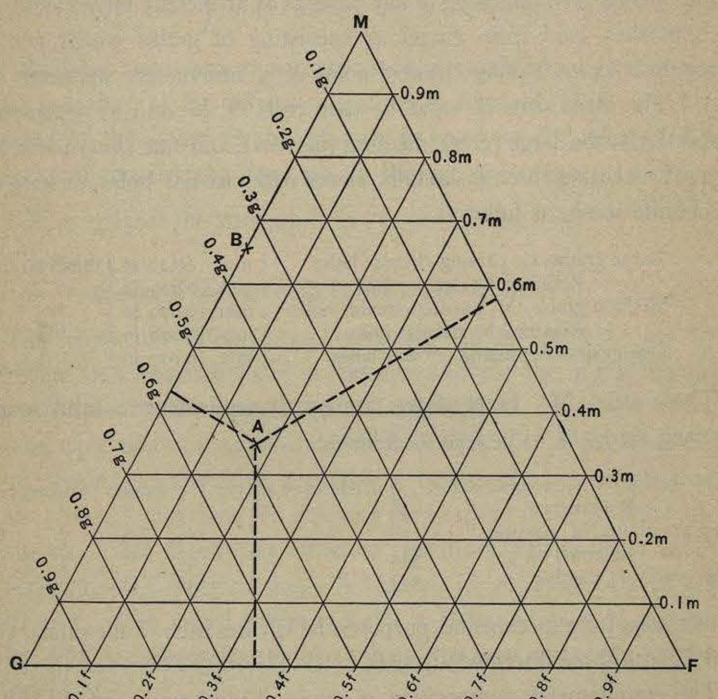

FIG. 50.—Feret's Three-Screen Method of Analyzing Sand. (*See p.* 144.)

co-ordinates, that is, each curve is the loci of points measured from 3 axes placed at angles of 60° with each other. A full discussion of the theory of this is given in his paper "Sur la Compacité des Mortiers Hydrauliques" in Annales des Ponts et Chaussées, 1892, II, but the principles may be understood by reference to Fig. 50. The apexes of the triangle are labeled G, M, and F, corresponding to the three sizes of sand described on page 143. The granulometric composition of any sand is plotted as a single point in this triangle. The proportion of each of the three sizes in the sand is represented by its perpendicular distance from the side opposite each apex.

For example, exactly at the apex G, the granulometric composition is g = 1.00, m = 0, f = 0. A sand represented by the point "*A*" in the triangle has for its granulometric composition, g = 0.48, m = 0.35, f = 0.17. Sand, *B*, whose point is on the line G M is a mixture of G and M with no fine particles. It can be readily proved by geometry that if the altitude of the triangle is 1.00, the sum of the three perpendicular distances from any given point in the triangle to the three sides equals 1.00. Also, that any combination of G, M, and F is contained in the triangle or else on one of its sides. To use Mr. Feret's language, "any sand will be represented by a point in the triangle and by one alone, and, reciprocally, one granulometric composition of sand, and only one, will correspond to a given point on the interior or sides of the triangle." If the altitude of the triangle

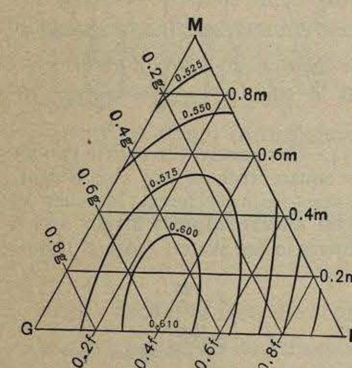

FIG. 51.—Absolute Volumes of Sand per Unit Volume of Sand not Shaken. (*See p.* 147.)

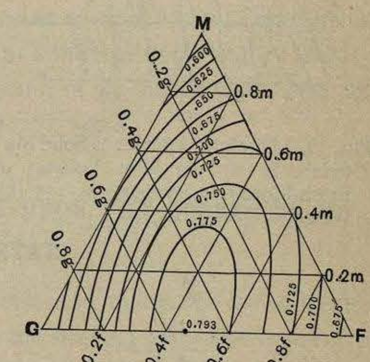

FIG. 52.—Absolute Volumes of Sand per Unit Volume of Sand Shaken to Refusal. (*See p.* 147.)

is considered 1.00, any point, *A*, in the triangle is readily plotted by locating it at perpendicular distances from each of the three sides corresponding to each component of its granulometric composition. For example, suppose that the granulometric composition of a sand, *A*, is g = 0.48, m = 0.35, f = 0.17. As the apex G represents a sand containing only coarse grains, and the line opposite to it, M F, all sands containing no coarse grains, the locus of a sand containing coarse grains (g = 0.48) will lie somewhere upon a line parallel to M F and at a distance 0.48 from M F. By similar reasoning it will also lie on a line parallel to G F and at a distance 0.35 from it. The intersection of these two lines is the locus of the sand *A*, and it will be seen that this intersection is at a perpendicular distance of 0.17 from the line M G (the side opposite F), which checks the plotting, since f = 0.17.

For comparing a special property of different sands, or of mortars composed of different sands, each sand employed in the tests is plotted and labeled with its value, — which may be in units of strength, weight, or volume, — and "contour lines" are sketched in by the eye, as one would draw contours from elevations on a topographical drawing.

Any point on the same contour line represents a sand made up of the

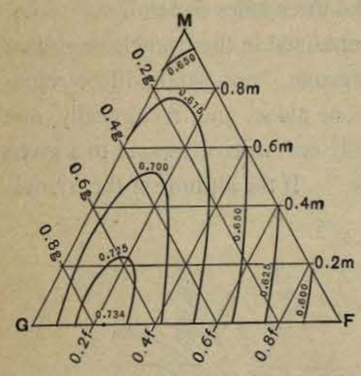

FIG. 53.—Absolute Volumes of Solid Materials (c+s) per Unit Volume of Fresh Mortar in Proportions 1:3 (by Weight). (*See p.* 147.)

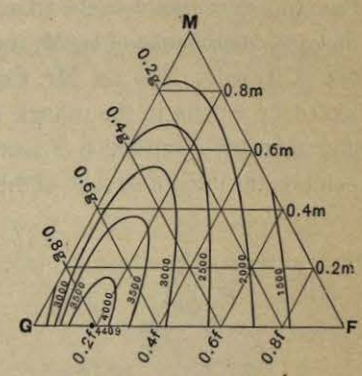

FIG. 54.—Compressive Strength in Pounds per Square Inch of 1:3 (by Weight) Mortars with Different Mixtures of Sand, after 9 Months in Air and 3 Months in Sea Water. (*See p.* 148.)

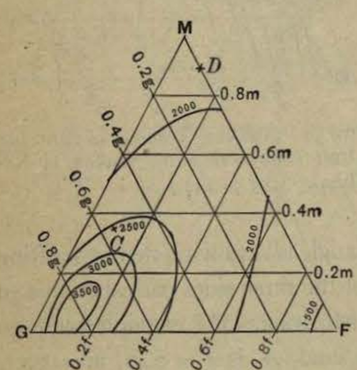

FIG. 55.—Compressive Strength in Pounds per Square Inch of Mortars with Various Mixtures of Sand, after One Year in Fresh Water. Proportions 100 lb. Portland Cement to 3.2 cu. ft. Mixed Sand. (*See p.* 148.)

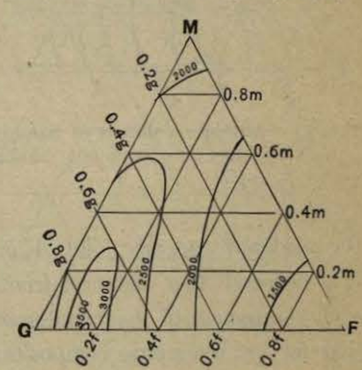

FIG. 56.—Compressive Strength in Pounds per Square Inch of Mortars with Various Mixtures of Sand, after One Year in Air. Proportions 100 lb. Portland Cement to 3.2 cu. ft. Mixed Sand. (*See p.* 148.)

different sizes, G, M, and F, in proportions corresponding to its perpendicular distances from the sides opposite each apex, but having the same strength, weight, volume, humidity, or whatever special function may be represented, as every other point on the same line.

Figs. 51 and 52, page 145, illustrate the use of the triangle for showing the volumes of sands composed of different sizes of grains. Any sand, for example, whose granulometric composition is represented by any point on the contour line labeled 0.575, in Fig. 51, has, when measured loose, 0.575 of its volume, or $57\frac{1}{2}\%$, of absolutely solid matter, or, taking the complement, $42\frac{1}{2}\%$ of voids. In Fig. 51 it will be seen that the greatest solid volume of loose sand is obtained by mixing G and F in proportions 60% G and 40% F by weight. The amount of solid matter in this mixture of maximum density is 0.61 of the unit volume; in other words, the sand contains 39% voids. By interpolating between the contour lines we may see that a sand consisting of equal parts of the three sizes, which would be represented by a point at the geometrical center of the triangle, has about 0.597 solid matter, or 40.3% voids. In sands shaken to refusal, Fig. 52, the mixture of maximum density consists of sands G and F alone, in proportions about 55% G and 45% F, and the total solid matter, that is, the absolute volume of sand, in a unit volume of the shaken sand of maximum density, is 0.798, corresponding to 20.2% voids.

EFFECT OF SIZE OF SAND UPON THE STRENGTH OF MORTAR

As a matter of fact, the actual size of a sand, that is, the size of its grains, is subordinate, in its influence upon the strength and other qualities of a mortar, to the density of the mortar produced from it. One naturally would suppose that the densest sand, that is, the sand which contains, when dry, the fewest voids, when mixed with a given proportion of cement, would make, inevitably, the densest and therefore the strongest mortar. Such, however, is not necessarily the case, for the addition of both the cement and water change the mechanical composition. A mixture of fine sand and cement, for example, requires a larger percentage of water in gaging than a mixture of coarse sand and the same cement. The total volume of a mortar of plastic consistency is affected by the quantity of water used, as well as by the volumes of the dry materials. Hence, a mortar consisting of fine sand and cement will be less dense than one of coarse sand and the same cement, even though the fine and coarse sands, when weighed or measured dry, each contain the same proportions of solid matter and voids.

Fine sand has more grains in a unit measure and therefore a greater number of points of contact of the grains. The water forms a film (see Fig. 63, p. 175,) and separates the grains by surface tension.

The fact is graphically illustrated in Feret's triangle, Fig. 53, page 146, in which the contour lines show the combined absolute volumes of the cement and sand in 1:3 mortar (proportioned by weight) made from sand of various compositions. It will be noticed that the point of maximum absolute volume, which is labeled 0.734, is much farther to the left than in Figs. 51 and 52, showing that for a mortar of maximum density, a sand is required containing more large particles, G, in proportion to the fine particles, F, than for maximum density with the same sand in its dry state.

From such experiments Mr. Feret* derives the law that:

The plastic mortars, which, per unit of volume, contain the greatest absolute volume of solid materials (c + s), are those in which there are no medium grains, and in which coarse grains are found in a proportion double to that of fine grains, cement included.

Figs. 54, 55, and 56, page 146, show the strength in compression, converted to pounds per square inch, of mortars made from various mixtures of the three sizes of sand.

Comparing these with Fig. 53 it will be seen that the curves of strength follow the same general direction as the curves of density. This is in conformity with the general laws stated at the commencement of the chapter and with the principles upon which Feret's formula (page 142) is based.

There is one point which must be noticed when studying these and other similar triangles of Feret, namely, that his results, as shown by the curves on his triangles, apply exactly only to sands and cements, and not to mixtures of sand and coarse stone. In all the triangles, sands for maximum density are composed of a mixture of fine and coarse grains with no medium grains. It is shown on page 172 that a denser mixture can be obtained with stone and sand and cement, that is, with three sizes of materials, than with sand and cement, and it is consequently probable that Feret could have obtained greater densities by making the size of G larger (that is, employing for G gravel or broken stone) and the size of F smaller, and that with this arrangement a portion of the medium grains would have been absolutely necessary to obtain the maximum density. In this connection, however, it must be remembered that Feret's experiments were intended to cover, as far as possible, practical combinations of sizes of sand for mortar. It is noticeable, even with the sizes of sand which he uses, that the curves in Fig. 53 run sharply upward, and that mortars from mixtures of three sizes of sand are therefore very nearly as dense and strong as those made from two sizes. Furthermore, when the three sizes

*Annales des Ponts et Chaussées, 1896, II, p. 182.

G, M, and F are mixed together, a graded mixture is formed in which there are particles ranging from 0.2 inch down to fine dust.

Experiments indicate, as stated on page 206, that sand for concrete requires for best results more fine material than mortar sand.

TESTS OF DENSITY AND STRENGTH OF MORTARS OF COARSE VS. FINE SAND

The application of Mr. Feret's tests is shown in the table on pages 136 and 137, and to illustrate its practical use in comparing the quality of different sands the following table is presented, giving the density and strength of three natural bank sands as tested in the laboratory of one of the authors.*

Compressive Strength and Elementary Volumetric Composition of 2-inch Cubes of Portland Cement and Bank Sand

By Sanford E. Thompson

Sand	Proportions by Weight	Proportions by Volume (nominal)	Percentages Passing Sand Sieves: ¼" Sieve	No. 8 Sieve	No. 20 Sieve	No. 50 Sieve	No. 200 Sieve	Elementary Volumes: Cement c	Sand s	Density $c + s$	$\left(\frac{c}{1-s}\right)^2$	Actual Average Compressive Strength, Age 7 days	Estimated Compressive Strength at 6 months, K=28,000
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Coarse	1 : 2.6	1 : 3	100	84	62	28	3	0.171	0.518	0.689	0.126	715	3530
Fine	1 : 2.6	1 : 3	100	84	100	77	6	0.154	0.466	0.620	0.083	405	2320
Very Fine	1 : 2.6	1 : 3	100	84	100	92	27	0.149	0.451	0.600	0.074	330	2070

PRACTICAL APPLICATIONS OF THE LAWS OF DENSITY

It is probable that many who read this chapter will question the practical use of it all. Sand from the same bank usually varies largely in different places, and even when sands of a uniform character are to be obtained, it is considered impracticable to mix two or more sizes on account of the expense involved. In other cases, only one quality of sand is obtainable, and consequently there is no opportunity for choice.

In answer to such critics, we outline below several conditions under which the investigation of the physical properties of the sand is not only interesting but essential from the standpoint either of quality or of maximum economy.

(*a*) The variation of the sand in different portions of the same bank may be utilized by requiring the contractor to mix two sizes without exact

* From paper by Sanford E. Thompson on "Sand for Mortar and Concrete," Bulletin No. 3, Association American Portland Cement Manufacturers, 1906.

measurement, so that the material as delivered shall contain not less than a certain percentage of sand coarse enough to be retained on a certain sieve.

(*b*) If two sands are available, a study of their physical characteristics will determine which is better suited to the work in hand as *the sand which produces the smallest volume of plastic mortar, when mixed with cement in the required proportions by dry weight, furnishes the strongest and least permeable mortar.*

(*c*) A good sand brought from a distance at a high price may be more economical than a poor sand from a neighboring bank.

(*d*) The relative value of crusher dust or of sand in a given locality may be determined by comparing their densities or the densities of mortars made from them.

(*e*) Frequently, a mixture of a fine and coarse sand, or of sand and crusher dust, proportioned according to their relative granulometric compositions or analyses, may be shown to produce a better mortar than either material alone.

(*f*) To produce impermeable mortar or concrete, it may be economical to screen a mixed gravelly sand into different sizes, and remix these in proportions which will produce a mortar of greater density.

(*g*) The value of "sand cements" for use in mortar and concrete under certain conditions may be made evident.

The use of mixed sand, as described in (*a*), was adopted by Mr. Thomas F. Richardson, Engineer, for the 1:2 Natural cement mortar employed in the stone masonry of the Wachusett dam of the Massachusetts Metropolitan Water Works, after an exhaustive study of the comparative tensile strength and permeability of mortars made with different sands. He required the contractors to furnish sand so coarse that at least 50% would be retained on a sieve having 30 meshes per linear inch. The sand was excavated by scrapers, and the condition was readily complied with, whenever the sand in one section was shown by samples to be running too fine, by taking alternate scraper loads of coarse sand from another place in the bank.

Mixed or graded sands are specially advantageous when concrete is made at a central plant such as a block manufactory. By using graded screenings, instead of the fine stone as it came from the crusher, and by slightly increasing the size of the coarse aggregate, Mr. Thompson obtained a strength two and one-half times as great with the same proportions of cement and, on the other hand, maintained equal strength with 40% less cement.

Comparative Tests of Different Sands. One of the most important applications of the laws of density is in the comparison of different sands. Void determinations of sand are valueless because of variations in moisture and compactness, but if equal dry weights of each of the sands to be compared are mixed with the same cement in the proportions required on the work, and then gaged to plastic consistency as described on page 138a, the best sand, provided it does not contain vegetable loam or other impurities to affect it chemically, is that which produces the smallest volume of mortar.

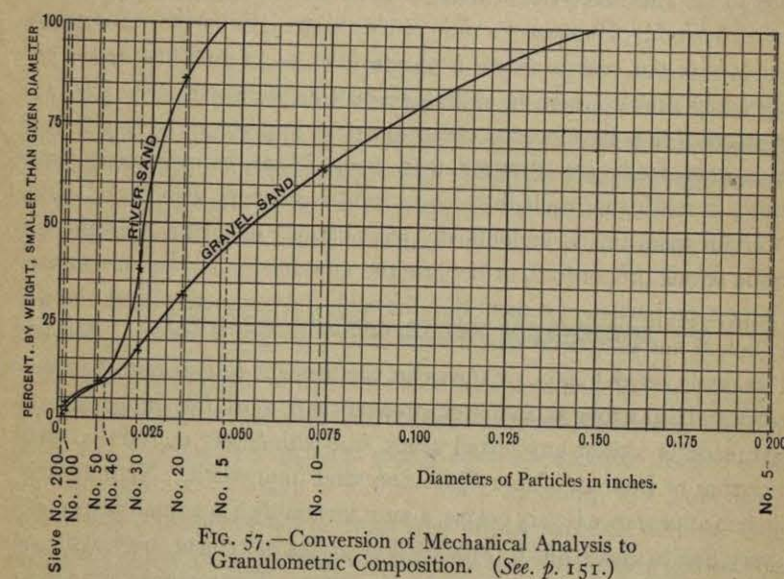

FIG. 57.—Conversion of Mechanical Analysis to Granulometric Composition. (*See. p.* 151.)

CONVERSION OF MECHANICAL ANALYSIS TO GRANULOMETRIC COMPOSITION

As an illustration of methods of contrasting two different sands and of making practical use of Feret's researches, we may compare tests made by Mr. R. L. Humphrey* in connection with the construction of the Pennsylvania Avenue Subway, Philadelphia. He found the tensile strength at the age of one year, of 1:3 mortar made with sand screened from gravel, to be about 50% stronger than that made with sand dredged from the Delaware River. The mechanical analyses† of the two sands are plotted by

*Transactions American Society of Civil Engineers, Vol. XLVIII, p. 558.

†Mechanical Analysis Curves are described in Chapter XI, page 190.