INDETERMINATE CASES.

An indeterminate case is one in which several mixtures may bring about the required condition.

Generally, a problem to which there are several answers, sometimes an infinite number of answers, may be called indeterminate.

Algebraically, an equation containing two unknown quantities (x and y), with no other condition or equation to assist the solution, is called an "indeterminate" equation. The same term may be applied to two equations involving three unknown quantities, etc.

There are cases in which, although but one answer is possible (at least but one rational answer), the statement is nevertheless indeterminate in form. These are not considered. They are wholly exceptional in metallurgical practice. Discussion of a single ordinary case, in which several mixes alike meet the general requirement, will be sufficient. In accordance with our precedents this is made extremely simple.

Example.—We have three ores and a limestone, whose analyses, as to slagging constituents, are as follows:

	SiO_2.	Bases.	Lime.
Ore No. 1	50	30	.. per cent.
Ore No. 2	40	20	.. per cent.
Ore No. 3	30	10	.. per cent.
Limestone	10	..	50 per cent.

The *ore* charge is to be 1,000 lbs. Requirement for the slag as below, viz:

Silica	50 per cent.
Bases from the ore mixture	25 per cent.
Lime from the limestone	25 per cent.
	100 per cent.

There are several ways of taking up the solution; we adopt the equation method as the most general.

Let $100x$, $100y$, $100z$, and $100v$ represent respectively the weights of ores Nos. 1, 2, 3, and of the limestone. The last

($100v$) is inserted to give generality to the discussion, as its value can be deduced without any formal calculation.

The constituents are now represented by the same figures as their respective percentages, all with the unknown quantities annexed, viz:

	SiO_2.	Bases.	Lime (CaO).
I	$50x$	$30x$	...
II	$40y$	$20y$	...
III	$30z$	$10z$	...
Limestone	$10v$	...	$50v$

There are three equations on the conditions:

$$100x + 100y + 100z = 1000 \text{ or } x + y + z = 10 \quad (1)$$
$$30x + 20y + 10z = 50v \quad (2)$$
$$50x + 40y + 30z + 10v = 30x + 20y + 10z + 50v \quad (3)$$

No. 3 reduces to $x + y + z = 2v$

Hence by (1) $2v = 10$ and $v = 5$. Limestone ($100v$) = 500 lbs.

The limestone therefore is constant for all possible mixes under the conditions. This, as already mentioned, might have been deduced without the formality of an equation.

The two equations remaining reduce to:

$$(a)\ x = 5 + z \text{ and } (b)\ y = 5 - 2z$$

Two equations with three unknown quantities: "Indeterminate."

Discussion of the maximum and minimum values of each of the unknowns is, however, exceedingly simple. Take z first, as it occurs in both equations:

(a) shows that 0 is a possible value of z (minimum).

(b) shows that with $y = 0$, z has its maximum value of 2.5.

Substituting in (a) we find x has a maximum value of 7.5, minimum of 5.

Substituting in (b) y has a maximum value of 5, minimum of 0.

Or, writing the values multiplied by 100, which express the weights of ore:

	Maximum.	Minimum.
Ore No. 1, lbs	750	500
Ore No. 2, lbs	500	0
Ore No. 3, lbs	250	0

(Limestone, always 500 lbs., for any mixture.)

We see that ore No. 1 must always be used, and with it some portion of No. 2 or No. 3, or both, according to weight of No. 1 taken. Any violation of the equations violates also the slag condition, as may be easily shown.

Take maximum of No. 1, No. 2 goes to its minimum (zero); No. 3 to maximum, 250.

Take minimum of No. 1, No. 3 goes to its minimum (zero); No. 2 to maximum, 500.

In all other cases, viz: No. 1 somewhere between maximum and minimum, all three of the ores enter the mix. Assume for x any value between limits, substitution in equations (a) and (b) gives values to y and z.

Examples under each case:

(a) Ore No. 1, take maximum weight of same. Calculate bases and silica.

No. 1, 750 lbs.; No. 2, 0; No. 3, 250; total, 1000 lbs.

	SiO_2.	"Bases."	CaO.
From I	375	225	... lbs.
From II	0	0	... lbs.
From III	75	25	... lbs.
Limestone	50	...	250 lbs.
Total	500	250	250 lbs.

(b) Ore No. 1, take minimum weight.

No. 1, 500 lbs.; No. 2, 500 lbs.; No. 3, zero.

	SiO_2.	"Bases."	CaO.
From I	250	150	... lbs.
From II	200	100	... lbs.
From III	0	0	... lbs.
From limestone	50	...	250 lbs.
Total	500	250	250 lbs.

(c) Ore No. 1, take weight intermediate between maximum and minimum.

No. 1, 700 lbs.; No. 2, 100 lbs.; No. 3, 200 lbs.

	SiO_2.	"Bases."	CaO.
From No. 1	350	210	... lbs.
From No. 2	40	20	... lbs.
From No. 3	60	20	... lbs.
From limestone	50	...	250 lbs.
Total	500	250	250 lbs.

Conditions satisfied in every case—*i.e.*, silica always equal to bases, and CaO always equal to bases from the ores. The problem is thus reduced to finding maximum or minimum which can be used of the three ores. It has found application in increasing or decreasing use of a certain ore, according to supply or cost.

Novices in algebraic statement often attempt to get as many equations as there are unknown quantities, in an indeterminate case. For instance, another equation which in reduced form would read $5x + 4y + 3z = 45$ could be taken to represent the condition that $SiO_2 = 2 \times CaO$. But this is already implied in the equations above, so that in eliminating x we merely regain the old form $y = 5 - 2z$.

Pyritic Smelting.

The calculations for slag in pyritic smelting are necessarily less definite than for other forms. This arises from the uncertainties of oxidation, as it is not possible to predict exactly what proportion of FeS_2 will be oxidized—*i.e.*, how much will go into matte and how much into slag.

In the absence of experimental data, the safe plan is to adopt a mixture that can hardly give a slag either very high or very low in silica. The conditions that modify oxidation are many, some of them under control, some to be learned only by experiment at the particular plant in question, with its limitations as to ores and fluxes.

Thus, the shape of the furnace; the relation of blast to fuel and charge; the pressure of the air, and especially its temperature; the proportion of finely divided sulphides to larger masses of quartz; the fusibility of the probable slag; all these and other considerations make the computation of the charge for pyritic or semi-pyritic smelting a very uncertain matter as compared with the perfectly definite estimates for mixtures already oxidized. This applies with especial force to the practice at a plant as yet untried. So far as relates to slag calculation the operation is a roasting and simultaneous fusion of ore, with the great disadvantage that there is no possible way to ascertain the composition of the roasted ore—*i.e.*, of the materials just as they separate into matte and slag.

Thus there are less of physical and arithmetical data than in any other case. So far as the computations go, however, they are the same as for cases already taken up. One example must suffice. Its data are close to an actual instance, but have been simplified by taking nearest whole numbers.

Example.—The ore analyzes:

SiO_2	40.00 per cent.
FeS_2	45.00 per cent.
CuS	9.00 per cent.
Bases	6.00 per cent.
Total	100.00 per cent.

Assume that two-thirds of the iron will go into the slag and one-third into the matte. The weight of the copper is assumed as six per cent., that being a close approximation, then the analysis of the ore by elements is:

SiO_2	40.00 per cent.
Fe	21.00 per cent.
Cu	6.00 per cent.
S	27.00 per cent.
Bases	6.00 per cent.
Total	100.00 per cent.

Under the assumption above, we have for the slag 40 lbs. SiO_2, 18 lbs. FeO, and 6 lbs. of other bases. Total, 64 lbs. ($FeS_2 \times .6 = FeO$.)

The matte should contain: FeS, 11 lbs.; Cu_2S, 7.5 lbs. ($Cu_2 : S = 4 : 1$).

Total matte 18.5 lbs. Its analysis:

Cu	32.43 per cent.
Fe	37.83 per cent.
S	29.73 per cent.
	99.99 per cent.

The slag would analyze no less than *65 per cent. of* SiO_2.

This being far too silicious to risk, especially upon an uncertain basis as indicated above, it would be necessary to add limestone. Suppose we have limestone of the composition SiO_2 10 per cent. and CaO 50 per cent. We calculate for *50 per cent. silica in the slag*. In the case here "set up," the equation and calculations are very simple.

Make the usual figures—*i.e.*, 100 lbs. ore, 100x lbs. limestone.

$$40 + 10x = 24 + 50x$$

$100x = 40$ lbs. Limestone required, 40 lbs.

Proof:

SiO_2 in 100 lbs. ore	40 lbs.	44 lbs.
SiO_2 in 40 lbs. limestone	4 lbs.	
FeO in 100 lbs. ore	18 lbs.	44 lbs.
Bases 100 lbs. ore	6 lbs.	
CaO in 40 lbs. limestone	20 lbs.	

Silica equals 50 per cent. as required.

Analysis of this slag:

SiO_2	50.00 per cent.
FeO	20.45 per cent.
CaO	22.73 per cent.
Other bases	6.82 per cent.
Total	100.00 per cent.

Some Well-Known "Formulæ."

The following are analyses of six formulated slags. They are all of the type of two base slags, here assumed as iron oxide and lime.

(1) Singulo with iron oxide and lime in equal chemical ratio. $(FeO)_2, SiO_2 + (CaO)_2, SiO_2$. Simplified ratios at right.

	Per cent.	Per cent.	Ratios.
SiO_2	31.91	32	15
FeO	38.30	38	18
CaO	29.79	30	14
	100.00	100.00	

The ratios 15 : 18 : 14 are exact, and easy to use in computation.

(2) Sesqui-silicate—same iron-lime ratio.

$$4FeO, 3SiO_2 + 4CaO, 3SiO_2$$

	Per cent.	Per cent.	Ratios.
SiO_2	41.28	41	5
FeO	33.03	33	4
CaO	25.69	26	3
	100.00	100.00	

Ratio 5 : 4 : 3 is sufficiently close for figuring.

(3) Bi-silicate, iron oxide and lime in equal ratio.

$$FeO, SiO_2 + CaO, SiO_2$$

	Per cent.	Ratios.
SiO_2	48.39	15
FeO	29.03	9
CaO	22.58	7
	100.00	

Ratio 15 : 9 : 7 is exact, being same as 120 : 72 : 56 or $2SiO_2$: FeO : CaO.

(4) Singulo, iron oxide twice lime, chemical ratio.

$$4FeO, 2SiO_2 + 2CaO, SiO_2$$

	Per cent.	Ratios.
SiO_2	31.03	3
FeO	49.65	5
CaO	19.31	2
	100.00	

(5) Sesqui-silicate, iron oxide twice lime.

$$8FeO, 6SiO_2 + 4CaO, 3SiO_2$$

	Per cent.	Ratios.
SiO_2	40.31	15
FeO	42.98	16
CaO	16.71	6
	100.00	

(6) Bisilicate, iron oxide twice lime.

$$2(FeO, SiO_2) + CaO, SiO_2$$

	Per cent.	Ratios.
SiO_2	47.37	6
FeO	37.90	5
CaO	14.73	2
	100.00	

Variations on these formulæ could be given almost ad infinitum. The above are six well known forms. The suggestions on the right of simpler ratios are meant to provide small numbers for multipliers in the statement equations, or in any other method of treatment of the computations. Their variation from the ratio of the precise analysis is not great in any case, and in some the ratios are identical. We repeat, that nothing that diminishes the chances of error is too trivial for attention.

Something has already been said as to the difference between assuming a fixed weight for ore alone, and calculating for the weight of the entire charge, as a fixed quantity.

Suppose the latter method be chosen, and we are to work it by equations. The fact that the entire weight is fixed in advance furnishes us with a third equation, and we shall have three simultaneous equations instead of two, to solve. We illustrate by simple data in the following:

Example.—Three ores analyze as follows:

	I.	II.	III.
Silica	10	40	.. per cent.
Iron oxide	20	..	20 per cent.
Other bases	..	10	15 per cent.

These are to be smelted without addition of other material. The charge is to be one thousand pounds.

The slag is to contain SiO_2, 50 per cent.; FeO, 30 per cent.; other bases 20 per cent.

Equations are as follows, taking $100x$, $100y$, and $100z$ as the weights.

$$100x + 100y + 100z = 1000 \text{ (or, } x + y + z = 10)$$
$$10x + 40y = 20x + 10y + 35z$$
$$2(20x + 20z) = 3(10y + 15z)$$

These give following weights for the mix:

No. I, 338 lbs.; No. II, 408.5 lbs.; No. III, 253.5 lbs. Total 1000 lbs.

Now assume one hundred lbs. for No. I., and $100x$ and $100y$ as others.

$$10 + 40x = 20 + 10x + 35y$$
$$2(20 + 20y) = 3(10x + 15y)$$

Weights: No. I, 100 lbs.; No. II, 120.83 lbs.; No. III, 75 lbs. Sum of weights is 295.83.

$$\frac{1000}{295.83} = 3.38+$$

Factor, 3.38. Multiply each weight as just found by this; we get the same results as before.

Usually it is easier to assume the fixed weight of one ore than to solve for the mix. We have also the less liability of error. In substituting values in the solution of simultaneous equations, a trivial error in value of one unknown may make quite a serious one in the value of another. As a rule, two simultaneous equations are sufficient, and it is better to limit them to this number if possible.

A few examples are annexed, with neither statements nor solutions. These are inserted as working examples to check on, answers being given.

Example:

	Ore.	Iron flux.	Limestone.
SiO_2	40	8	.. per cent.
FeO	10	80	.. per cent.
CaO	..	..	50 per cent.

Slag required $(FeO)_2\ SiO_2 + (CaO)_2\ SiO_2$, *i.e.*, singulo with iron oxide and lime in equal chemical ratio. Take 100 lbs. of ore.

(This slag analyzes to nearest per cents as follows: SiO_2, 32 per cent.; FeO, 38 per cent.; CaO, 30 per cent.; these may be taken as data for solution.)

Answers:—Ore, 100 lbs. Iron flux, 59.2 lbs. Limestone, 90.7 lbs.

Example.—Calcined ore, no fluxes. Take out the matte and calculate the slag, making the following suppositions as to combinations and losses, viz: All of the copper goes into matte. Three-fourths of the lead and one-half of the zinc also go into matte, taking precedence of the iron. All of the remaining sulphur combines with iron. (FeS in matte.)

One-half of the zinc goes into slag, and one-fourth of the lead. All of iron not put into matte goes into slag.

Take 100 lbs. of this ore, find weight and analysis of matte and slag. No allowances for losses of volatile material.

Analysis of the calcined ore:

SiO_2	27.3 per cent.
Cu	11.9 per cent.
Fe	37.5 per cent.
Zn	2.0 per cent.
Pb	1.0 per cent.
CaO	4.0 per cent.
S	7.0 per cent.
O, etc	9.3 per cent.
	100.00 per cent.

Answer: Weight of matte per 100 lbs. ore taken = 26.65 lbs.

Analysis of matte:

Cu_2S	55.91 per cent.
FeS	35.27 per cent.
ZnS	5.63 per cent.
PbS	3.19 per cent.
	100.00 per cent.

Weight of slag per 100 lbs. of ore taken = 73.35 lbs.

Analysis of slag:

SiO_2	37.24 per cent.
FeO	55.23 per cent.
CaO	5.45 per cent.
ZnO	1.71 per cent.
PbO	0.37 per cent.
	100.00 per cent.

Example.—Take a partly roasted product whose analysis follows:

SiO_2	62.00 per cent.
Al_2O_3	10.00 per cent.
Fe	14.42 per cent.
Cu	6.40 per cent.
S	4.51 per cent.

Assume the deficit of this analysis (2.67) as oxygen.

Take out the matte (Cu_2S + FeS), then, since the remaining constituents are highly silicious, fuse with a limestone having composition SiO_2 = 10 per cent. and CaO = 50 per cent., charging for *bisilicate slag*, no special ratio being required as between bases.

Find what the analysis of this bisilicate slag should be.

Answer:

SiO_2	56.38 per cent.
Al_2O_3	8.07 per cent.
FeO	9.68 per cent.
CaO	25.87 per cent.
	100.00 per cent.

Example.—Use the same data as in the last example. Instead of charging for a bisilicate slag, charge for a singulo of this formula:

$4FeO + 2CaO + 3SiO_2$, in which FeO and CaO are in true chemical ratio as shown by formula.

Treat the Al_2O_3, *however, as "outside" of this formula*, that is, take out from the ore the singulo silicate of Al_2O_3 before calculating the other fluxes. The slag, then, is to consist of a singulo silicate of Al_2O_3, *plus* the above formulated silicate of iron and lime.

For this charge, in addition to the limestone given, use an iron ore flux whose analysis gives SiO_2 = 10 per cent. and FeO = 81 per cent.

Take 100 lbs. of ore, $100x$ and $100y$ lbs. of iron flux and limestone.

Give weights of the latter to be added. Give analysis of the complete slag, *i.e.*, the constituents including the Al_2O_3.

Answers: Weight of iron flux = 135.8 lbs.; weight of limestone = 94.665 lbs.

Analysis of slag:

SiO_2	32.13 per cent.
Al_2O_3	3.82 per cent.
FeO	46.15 per cent.
CaO	17.90 per cent.
	100.00 per cent.

Example.—A partly roasted ore analyzes as follows:

SiO_2	26.00 per cent.
Cu	9.00 per cent.
Fe	41.35 per cent.
Zn	2.00 per cent.
Pb	4.00 per cent.
S	8.00 per cent.
O (deficit)	9.65 per cent.
	100.00 per cent.

Take out matte, assigning to it all the copper, half of the zinc, three-fourths of the lead, and as much iron as the "residual" sulphur permits on FeS formula. Consider the lead not entering the matte as "lost," making no allowance for it in the slag computation.

Find analysis of the matte, also the remaining constituents as slag.

Answer.—Analysis of the matte:

Cu	9	+ S 2.27	= 11.27	= 38.39 per cent.	Cu_2S
Fe	8.35	+ S 4.77	= 13.12	= 44.70 per cent.	FeS
Zn	1	+ S 0.50	= 1.50	= 5.11 per cent.	ZnS
Pb	3	+ S 0.46	= 3.46	= 11.80 per cent.	PbS
				100.00 per cent.	

Analysis of the slag:

SiO_2	37.33 per cent.
FeO	60.87 per cent.
ZnO	1.79 per cent.
	99.99 per cent.

This slag, however, formed simply from residual elements after matte has fallen, being considered too heavy, lime addition is desired. Limestone to be used has following analysis: SiO_2 = 7 per cent.; CaO = 52 per cent.

Condition.—Enough of this limestone shall be added to reduce the silica in the slag to 32 per cent.

Answer.—Weight of the limestone = 31.24 lbs.

Analysis of resulting slag:

SiO_2	32.00 per cent.
FeO	48.14 per cent.
CaO	18.43 per cent.
ZnO	1.42 per cent.
	99.99 per cent.

Example.—Resume the last example at the point where we have to compute the limestone necessary to add for production of 32 per cent. silica in the slag.

Instead of calculating this by either excess or equation method, we may use the "mixing" rule. But in doing so we must remember that 41 per cent. of the limestone is volatile matter, and 59 per cent. is "fixed," so the addition for this method must be treated as the non-volatile portion only. We may mix the "potential" slag we have already, with its 37.33 per cent. silica, *with the non-volatile portion of the limestone*, which, as a simple calculation will show, gives 11.86 per cent. silica ($7 \times 100 \div 59 = 11.86$).

The application of the "mixing" rule now becomes:

37.33 less 32 = 5.33. And 32 less 11.86 = 20.14.

Weight of the slag, which of course had to be found before getting its analysis, is 69.65 pounds. We have then:

$$20.14 : 5.33 = 69.65 : 18.43$$

Finally, $$\frac{18.43}{.59} = 31.24$$

or exactly the same result as calculated in the former example, as required weight of limestone to be added.

Burdening of the Iron Furnace.

The frequent change of the fuel ratio in the iron blast furnace has naturally brought about the custom of separate estimates of limestone charge for fuel and for ore. Thus, by allotting a certain weight of "stone" for each 100 lbs. of ore and for each 100 lbs. of fuel, the necessity of recomputing at every change of the fuel ratio is avoided.

It is the practice of many iron furnaces to consider alumina as an acid constituent of the slag. The alumina and the silica are simply added together, and called the "acid;" the same is done with all the bases, which are then computed "en masse" against the "acid."

The minimum figure adopted for "acid" is forty per cent.; the maximum is 50 per cent. To put it otherwise, the ratio of acid : base = 40 : 60 or in simplified ratio 2 to 3, shows the least proportion of acid, while equal weight of acid and base shows the highest allowable "acid." Usually a figure near equality is selected, especially if sulphur is present, as in that case some addition of limestone is common, in excess of the weight figured for slag.

These usages, far from introducing any new factor into the problem, reduce it to its most elementary form.

The data in the following example are taken from Robert Forsythe's work, "The Blast Furnace and the Manufacture of Pig Iron." (David Williams & Co., New York, 1908.) We drop from the tabulated analyses the sulphur, phosphorus, iron, and carbon, and confine computation to the slag-forming elements.

The problem is set forth on pages 168–171 of the above work.

	Fuel.	Ore.	Limestone.
SiO_2	5.30	7.49	0.64
Al_2O_3	3.00	0.81	0.32
CaO	1.00	0.15	54.20
MgO	0.70	0.12	0.35
Mn	....	0.45	....

As in the text we require a slag:

$$\text{acid} : \text{base} = 10 : 11$$

In order that our data may wholly coincide, we again adopt the text in subtracting 1.80 from the "acid" for silica reduced to silicon. We also assume that one-third of the manganese goes into the slag as MnO; this gives 0.2 MnO.

To compute "stone" required for the fuel, on basis of 100 lbs., call weight of limestone as usual, $100x$. Then,

$$SiO_2 + Al_2O_3 = 8.30 + 0.96x$$
$$\text{Bases} = 1.70 + 54.55x$$
$$10 : 11 = 8.3 + .96x : 1.7 + 54.55x$$

whence, $100x = 13.89$ = lbs. "stone" required for the fuel.

As to the ore we have (after taking out 1.80 acid):

$$SiO_2 + Al_2O_3 = 6.5 + .96x$$
$$\text{Bases} = .47 + 54.55x$$
$$10 : 11 = 6.5 + .96x : .47 + 54.55x$$

whence, $100x = 12.49$ = "stone" to 100 lbs. of ore.

These results are identical to the one-hundredth of a pound with those obtained in the text. In the latter, however, an addition is now made to the lime for "carrying" sulphur from the ore. This is omitted, as our purpose is confined to showing how readily the equation method can be applied to any case. Observe that in the above computation we take no note of "excess," nor of "flux efficiency," as the basic availability of the "stone" is sometimes called.

On page 250 we remarked that the equation method is relatively easier as the problem becomes more complex. However, the extreme limit of simplicity is reached in the above problem, and it will be found that fewer figures are used by this than by any other method.

With Al_2O_3 not over five per cent. in the slag, the proportion of MgO is not important. When Al_2O_3 rises to 10 per cent. or over, MgO in excess of twenty per cent. is found to cause viscosity.

It is usual, when sulphur is present, to add limestone *in excess of the slag requirement,* allowing seven *available* CaO to four sulphur (56 : 32). No lime addition, however, will entirely remove sulphur from the pig metal.

In the case above, suppose the fuel contained 0.75 sulphur, the addition of limestone would be 2.45 lbs. for each 100 lbs. fuel. This we leave the student to verify.

In closing this work we again call attention to the fact that we have written mainly for the student.

The method of slag computation by simultaneous equations, first taught by the author in 1883, will be found to be a great labor saver.

The more complex the requirements, the greater will be the *relative* complexity avoided.

The whole of the calculation of "excess" and residual quantities is eliminated.

It is hoped that the publication of this little treatise may prove serviceable to many who have been more or less confused by the irregularities of the methods in vogue.

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