doubt as to whether their formulæ are to be read in the only possible correct way, i.e., chemically, or whether they mean little or nothing in that sense, and are used merely as attempted abbreviations.
Slags which are properly adjusted as to relation between oxygen in $\mathrm{SiO}_{2}$ and in bases, may, to coin a term, be called "semiformulistic."
It is only when a definite and chemical relation exists between bases as well as between silica and bases, that we can speak of a slag as a chemical formula. Only in such are we entitled to attach the full chemical significance to each and every symbol.

When we have such a requirement, and wish to compute it, we may properly translate the formula at once into its numerical equivalent. Example:
$\mathrm{Fe}_{2} \mathrm{SiO}_{4}+\mathrm{Ca}_{2} \mathrm{SiO}_{4}$ becomes, when put into numerical expression:
\(\left.\left.$$
\begin{array}{r}\mathrm{SiO}_{2}=120 \\
2 \mathrm{FeO}=144 \\
2 \mathrm{CaO}=112\end{array}
$$\right\} \begin{array}{r}\mathrm{SiO}_{2}=15 \\
or, 2 \mathrm{FeO}=18 \\

2 \mathrm{CaO}=14\end{array}\right\}\) (or, in per cents.) | $\mathrm{SiO}_{2}=31.91$ per cent. |
| :--- |
| $\mathrm{FeO}=38.30$ per cent. |
| $\mathrm{CaO}=\underline{29.79}$ per cent. |
|  |
| 100.00 per cent. |

It is not always necessary to reduce the formula to the form of an analysis.
This will probably be done sooner or later, to enable the computer to get the comparative idea which long habit has based upon percentage. But so far as "laying out" the statistics for computation is concerned, the above example is a sufficient exposition of the fact that much simpler figures than the "per cents." of an analysis will serve to introduce the same ratios.

Frequently it happens that the ratios are very close to simple ones, although given by rather complex figures in the preparation of the data for caleulation. A case in point occurs among the examples to follow. It may little affect the practical outcome of the problem, and greatly facilitate the computation, to change the ratio over to the simple one at once. Judgment comes in here, and some perception of the extent to which the outcome will be affected. These are mental data hardly reducible to "rules."

## METHOD OF REPRESENTATIVE EQUATIONS.

In our so-called "representative" equations it will be found useful to adopt as a fixed rule 100 pounds of ore as the basis of calculation.

The analysis of the ore is always stated as totalling one hundred per cent. Analyses of all the other constituents of the charge will be similarly stated.

The "representative" feature of the equations is found in the adoption of the percentages of constituents, as the coefficients of the unknown quantities.

Before detailing the method, we make some general remarks on the "preparation" of the data for calculation. The item of reducing the number of bases by substitution of one for another by "conversion factors" has already been given. Another point is the fuel ratio, most important in iron, not always to be neglected in other metals. If the fuel charge be a fixity in relation to the ore, its ash, if it has been analysed both as to weight and composition, may be added directly to the ore analysis, so as to diminish the work of computation.

Example.- Suppose the case to be one in iron. Analysis of ore shows 8 per cent. silica. Analysis of coke shows 6 per cent. silica. Coke is to be seventy per cent. of weight of ore. Then for 100 lbs . of ore we have 70 lbs . of coke. The 100 lbs . of ore contain 8 lbs. silica; the 70 lbs. coke contain 4.2 lbs. silica ( 6 per cent. of 70 ). Add, and compute for 12.2 per cent. $\mathrm{SiO}_{2}$ in the ore.
The same can be done for any other constituent. In this way the subsequent statement is simplified.

Coke ash is not usually allowed for in lead and matte smelting, though in cases where high fuel ratio combines with high ash in the fuel it might well be made an element of calculation.

In "pyritic" smelting, i.e., theoretical pyritic work, where there is no coke at all, it vanishes. The case, however, is a trifle too ideal for discussion.
There are cases in iron smelting where the coke furnishes more silica than the ore. We shall sometimes include and sometimes neglect the coke ash in our examples. Its addition
presents no difficulty, as the few extra figures required are in the "preparation" and not in the actual calculation.
Having decided upon the composition of the slag desired, we may be limited by the obstacle of having no material at hand for the production of what we seek. One or two examples will be given, but we drop this complication for the moment.
We have now the ore and fuel prepared for calculation as hitherto explained both by conversion of bases, if necessary, and by addition of fuel ash in its proper ratio to the ore analysis. We have also the analyses of the fluxes. Weights to be used of the latter are as yet uncalculated.
Call weight of the flux $100 x$.
Call weight of a second flux, if there be one, $100 y$.
The simplification thus introduced will now be explained.
The analysis is of course stated in percentages. Suppose it is a limestone we are working into the charge. It contains, let us say, 10 per cent. silica and 50 per cent. lime (CaO). We have called the weight of limestone $100 x$. Then its silica is represented by $10 x$ and its lime by $50 x$.

It is not worth while to introduce fractions of percentages into the equations. Use the nearest whole number. If your lime was stated as 9.7 per cent., call it 10 per cent.

A little experience in working charges will soon show the student that it is often a matter of indifference whether to select the "excess" method or an equation, when there are no constituents but ore and one flux to consider.
But as the complexity rises, the relative simplicity of the "equation" comes out. In a calculated mixture of two ores and two fluxes, the computation and adjustment of "excess" is so troublesome that many an operator has been led into mere "fudge" figuring, trying one mix and another and checking out, until he finds one which will give results sufficiently near.

Very rarely are more than two "simultaneous" equations of the first degree called for. Although we are assuming that the reader is familiar with this very elementary algebra, we remind him that in solving by substitution (after getting value of one of the two unknowns), a small error in the value of the first unknown may produce a far greater one in the value of the second. Hence, although after the values are obtained, they may suffer the loss of their decimal figures without any danger to the fur-
nace charge, this curtailment is not safe until after the equations are solved. Here comes in the ever useful "sense of magnitude." It may be born. It may be learned. It cannot be taught.
It is important to remark that the equation method has regard to nothing but percentage or numerical ratio. Chemical proportion or "excess" does not figure in it.
The percentages for which you are figuring, however, may be and usually are the outcome of some strict formula, itself in turn derived from either experience or theory.
But to get a result on a "formulistic" slag by this method, it is necessary to first reduce the formula to "analytical" form ( 100 summation) or else to a rational form, i.e., one involving the chemical ratios reduced to their simplest figures.
Example.-We require a lime-iron slag, whose formula shall be:

$$
(\mathrm{CaO})_{2} \mathrm{SiO}_{2}+(\mathrm{FeO})_{2} \mathrm{SiO}_{2}
$$

By Problem III, this reduces to the analysis: .


These figures are cumbersome to "set up" as the ratios required for the constituents (see detail of the method in first example below). They would probably be changed to 30,30 , and $40(3,3,4)$, or to some convenient approximation.

However, it is not necessary to throw the formula into "analytical" form (see Problem I). Write out the formula, annexing the molecular weights, then reduce these to their simplest ratio. These numbers are in precisely the same ratios as their respective "percentages" in the "analytical" form, and are usually simpler.

Moreover, as they are the numbers which have to be used in getting to the "analytical" form, it is a much quicker process to use them instead of the latter. (We are, however, so accustomed to the 100 summation of ordinary analytical statement, that sooner or later every slag will be stated in that form.)

Take, then, the above formula and reduce it to molecular weight figures. Reduce the latter to their simplest ratio in whole numbers.

$$
\begin{array}{ll}
2 \mathrm{SiO}_{2}=120 . & 120 \div 8=15 \\
2 \mathrm{CaO}=112 . & 112 \div 8=14 \\
2 \mathrm{FeO}=144 . & 144 \div 8=18
\end{array}
$$

Now the requirement of the slag would be stated thus: "fifteen parts of $\mathrm{SiO}_{2}$ to fourteen of lime and eighteen of iron oxide."
This is merely by way of putting the requirements into the most convenient form for computation. Arithmetically, the above statement is exactly the same as the one which gives the three percentages under the analytical summation form, with the advantage of being far simpler. The molecular weights of the three most used constituents $\left(\mathrm{SiO}_{2}, \mathrm{CaO}\right.$, and FeO$)$ lend themselves rather kindly to simplification of ratios.

In nine out of ten cases in actual practice, all of this preparation is outside of the requirements. For in most cases, the requirement will be already stated in analytical form, with very simple numbers as percentages. Thus, slag required is:

| $\mathrm{SiO}_{2}$ |  | 40 per cent. |
| :---: | :---: | :---: |
| CaO . |  | 20 per cent. |
| FeO . |  | 40 per cent. |
|  | - | 100 per cent. |

In which, in "setting up" the equation the numbers may figure as 4,2 , and 4 .

Illustrative Case for Method by "Representative Equations."
In order not to complicate a first illustration we take a case without any adjustment for matte, and in which we do not even state analysis of ore, outside of its slag-forming constituents. We also start with a requirement "ready made" as to percentages in slag (i.e., no formula).
We need in this case an iron-lime slag, hence we shall assume an iron ore as well as a limestone flux. Analyses of all three follow:
$\left.\begin{array}{ll}\text { N: } & \begin{array}{c}\text { Ore, } \\ \text { per cent. }\end{array}\end{array} \begin{array}{c}\text { Iron flux, } \\ \text { per cent. }\end{array}, \begin{array}{c}\text { Limestone, } \\ \text { per cent: }\end{array}\right]$

Condition as to slag.-The slag is to be composed of silica, lime and iron oxide in equal weights, i.e., $33 \frac{1}{3}$ per cent. each.

We have first to remove the slight complication of the two bases in the limestone. As given in the "conversion factors," some pages back, we multiply the magnesia by 1.4, thus getting 14 per cent. of "conventional" lime. Now the analysis of our limestone, $i . e$. , the one which is to be used in calculations, is: Lhmestone.

| $\mathrm{SiO}_{2}$ | 6.0 per cent. |
| :---: | :---: |
| CaO | 54.0 per cent. |

The data being now "prepared" for calculation, we proceed to make the regular assignment of "unknowns" for the fluxes, that is, represent iron ore (lbs.) by 100x, and limestone (lbs.) by $100 y$.
We take 100 lbs . of ore as basis for calculation. Now assemble all the expressions for silica, iron-oxide and lime. Those derived from the ore will be in pounds direct. Those derived from the fluxes will also be in pounds expressed in terms of $x$ and $y$, and the coefficients of these letters will be simply the percentages as given in the analyses.

|  | From ore. | From iron flux | $\begin{gathered} \text { From } \\ \text { limestone. } \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| Silica | 40 | $8 x$ | $6 y$ |
| Iron oxide |  | $82 x$ |  |
| Lime ( CaO ) | 10 |  | $54 y$ |

The condition being that each of these constituents shall be present in the slag in equal weight, we construct the two equations at sight by equating equals. Take any pair, e.g.:

$$
\begin{array}{ll}
\left(\mathrm{SiO}_{2}=\mathrm{FeO}\right) & 40+8 x+6 y=20+82 x \\
(\mathrm{FeO}=\mathrm{CaO}) & 20+82 x \\
=10+54 y
\end{array}
$$

Multiplication of the first equation by 9 solves easily, giving:

$$
\begin{array}{ll}
x=0.3253 & (100 x=32.53) \\
y=0.6792 & (100 y=67.92)
\end{array}
$$

Answer.-For each 100 lbs . of ore, take 32.53 lbs . iron flux and 67.92 lbs . limestone.
Proof.-Taking the answers, we find the weights of all the constituents of the charge. By the conditions, they should come out exactly equal.

| $\mathrm{SiO}_{2}$, from ore. | 40.00 | FeO , from ore. 20.00 | CaO , from ore. 10.00 |  |
| :--- | ---: | :--- | :--- | :--- |
| $\mathrm{SiO}_{2}$, iron flux. | 2.60 | FeO , iron flux. 26.67 | CaO , iron flux. $\ldots .$. |  |
| $\mathrm{SiO}_{2}$, limestone | $\frac{4.07}{46}$ | FeO , limestone | $\ldots .$. | CaO , limestone $\frac{36.67}{46.67}$ |

In the examples following, we begin with cases of extreme simplicity. In fact we have avoided complicated cases for the most part, when they involved nothing more than very detailed work which demanded no knowledge of principles, but merely patience in picking out and arranging data. The solving of such problems is good practice in accuracy, but not particularly "instructive."
Example.-Two ores have as slagging constituents:

|  | I. | II. |
| :---: | :---: | :---: |
| $\mathrm{SiO}_{2} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$ | 10 per cent. | 20 per cent. |
| $\mathrm{CaO} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$ | 20 per cent. | $\ldots \ldots \ldots \ldots$ |
| $\mathrm{FeO} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$ | 60 per cent. |  |

Take 100 lbs . of No.1. Mix the two for slag of 30 per cent. silica. Solution.-Let lbs. of No. $2=100 x$. The weights of all the constituents will now be

$$
\text { Total } \mathrm{SiO}_{2}=10+20 x \quad \text { Total bases }=20+60 x
$$

By the condition, the silica is 30 per cent., the bases 70 per cent.

## Hence

$$
7(10+20 x)=3(20+60 x)
$$

$$
x=0.25 \text { and } 100 x=25
$$

Assembly and summation of the weights of each and every constituent will constitute the-
Proof:
$\mathrm{SiO}_{2}$ from No. $1=10 \mathrm{lbs}$.
$\mathrm{SiO}_{2}$ from No. $2=5 \mathrm{lbs}$. Total $\mathrm{SiO}_{2}=15 \mathrm{lbs}$.
FeO from No. $2=\ldots \ldots \ldots \ldots \ldots \ldots .15 \mathrm{lbs}$
FeO from No. $2=$ .5 lbs
.20 lbs.
50 lbs.
Since there are $15 \mathrm{lbs} . \mathrm{SiO}_{2}$ in a total of 50 lbs ., we have the 30 per cent. called for.

Example.-Take the same materials. With 100 lbs . of No. 1, add No. 2 to produce a "singulo" silicate slag.
Answer. Impossible. The $\mathrm{SiO}_{2}$ is "short."
Proof:
In No. 1. $2 \mathrm{CaO}: \mathrm{SiO}_{2}=1: 0.535=20: 10.70=\mathrm{SiO}_{2}$ required.
In No. 2. $2 \mathrm{FeO}: \mathrm{SiO}_{2}=1: 0.416=60: 24.96=\mathrm{SiO}_{2}$ required.
Since neither of the ores has enough silica to satisfy its own base on the "singulo" formula, we must use some outside flux to produce a "singulo."

Example.- Still assuming the same materials (ores Nos. 1 and 2), take 100 lbs . of No. 1, add No. 2 in such quantity as to make 2 molecules of iron oxide to 1 of lime (i.e., chemical ratio), then calculate the weight of pure silica to add to the mix to bring about a singulo silicate in the slag.
The formula required is $4 \mathrm{FeO}, 2 \mathrm{SiO}_{2}+2 \mathrm{CaO}, \mathrm{SiO}_{2}$.

$$
\mathrm{CaO}: 2 \mathrm{FeO}=56: 144=(7: 18)=20 ; 51.43
$$

Weight of the iron oxide corresponding to twice the CaO , i.e., twice as many molecules.

$$
60: 100=51.43: 85.72=\text { lbs. of ore No. } 2 \text { required. }
$$

As above, we know that the CaO calls for $10.70 \mathrm{SiO}_{2}$. In the mix as calculated we have 27.14 lbs . $\mathrm{SiO}_{2}$. (Prove this.)

$$
27.14-10.70=16.44 \mathrm{lbs} . \mathrm{SiO}_{2} \text { for } \mathrm{FeO}
$$

But $16.44 \mathrm{lbs} . \mathrm{SiO}_{2}$ calls for 39.46 FeO on the formula.
$51.43-39.46=11.97 \mathrm{FeO}$ excess in the mix; this calls for $4.99 \mathrm{lbs} . \mathrm{SiO}_{2}$. (Parts of the operations are omitted; the student should verify the results by making full calculations for himself.) Ans. 4.99 lbs . silica to be added.
Take the above problem from the point at which we have determined that we must use 85.72 lbs. of No. 2 ore in the mixture. We can calculate for the additional silica more quickly.
The analysis of the silicate, already formulated, is:

| Silica | 31.02 per cent. |
| :---: | :---: |
| Iron oxide (FeO) | 49.66 per cent. |
| Lime ( CaO ) | 19.31 per cent. |
|  | 99.99 per |

This formula is often calculated as $\mathrm{SiO}_{2}=3 ; \mathrm{FeO}=5 ; \mathrm{CaO}$ $=2$. Use this instead of the analysis, and call silica to be added " $x$."
Then we may write at once, $\mathrm{FeO}=\mathrm{SiO}_{2}+\mathrm{CaO}$. Or, in numbers:

$$
51.43=27.14+20+x \text { and } x=4.29
$$

(The iron oxide required and contained in the allotted weight of No. 2 is 51.43 , total silica in the ores alone 27.14, total lime is 20 , see above.)
Now check up by adding all the constituents. The weight of 4.29 checks with the " $3: 5: 2$ " proposition, though of course
not accurately with the former figure. In practice the difference would hardly be felt.
This is an illustration of the point mentioned in the remarks preceding this section, viz: the substitution of an approximate and very simple ratio for a complex one.
In examples like these we work to practically perfect checks. Decimals of a pound are not called for in practice, but this is no argument for loose approximations in treating the subject. Short cuts are dangerous for beginners.
In presenting the "analysis" of ores in these purely slagging examples it will be understood that only the elements are given that enter into the slag. In matting cases the complete analyses are necessary.
Example.-We have an ore, an "iron-ore flux" and a limestone, analyses as below:

|  | Ore, <br> per cent. | Iron flux, <br> per cent. |
| :---: | :---: | :---: |
| $\mathrm{SiO}_{2} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$ | 30 | 10 | | Limestone, |
| :---: |
| per cent. |

Take 100 lbs . of the ore. Requirement for the slag is:


Find weights of "iron flux" and limestone necessary.
Let $100 x=$ lbs. of iron flux and $100 y=$ weight of limestone
$\left(\mathrm{SiO}_{2}=\mathrm{Al}_{2} \mathrm{O}_{3}+\mathrm{CaO}\right) . \quad 30+10 x+5 y=10+5 x+53 y$
$\left(\mathrm{SiO}_{2}: \mathrm{FeO}=30: 40\right) . \quad 4(30+10 x+5 y)=3(5+76 x)$
$x=.609 . \quad y=.48 . \quad 100 x=60.9 . \quad 100 y=48$
Proof:

|  | $\mathrm{SiO}_{2}$. | FeO. | $\mathrm{Al}_{2} \mathrm{O}_{3}+\mathrm{CaO}$. |
| :---: | :---: | :---: | :---: |
| Ore. | 30.00 | 5.00 | 10.00 |
| Iron flux | 6.09 | 46.28 | 3.05 |
| Limestone | 2.40 | ... | 25.44 (CaO) |
|  | 38.49 | 51.28 | 38.49 |

These numbers are exactly as $3: 4: 3$.
Example.-Ore, iron-flux and limestone having analyses as below, mix for a slag containing 30 per cent. $\mathrm{SiO}_{2}, 40$ per cent. FeO , and 30 per cent. CaO .

Take, as usual, 100 lbs . of the ore, $100 x \mathrm{lbs}$. iron-flux and 100 y lbs. of limestone.

| Ibs. of limestone. |  |  |  |
| :---: | :---: | :---: | :---: |
| Ore, <br> per cent. | Iron ore, <br> per cent. | Limestone, <br> per cent. |  |
| $\mathrm{SiO}_{2} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$ | 40 | 10 | 7 |
| $\mathrm{Fe} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots .20$ | 80 | . |  |
| $\mathrm{CaO} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$ | 10 | $\ldots$ | 52 |

Operation omitted, the example being in line with those already given. The weight of the slag, per 100 lbs . of ore charged, is 172.47 lbs ., and it contains $51.74 \mathrm{SiO}_{2}$, the same of CaO , and 68.99 lbs . FeO , numbers which are as $3: 3: 4$.

Example.-We have two ores whose slagging ingredients are:


Mix the two ores for zine oxide tenor of 7 per cent. then for 100 lbs. of the mix find lbs. of iron flux and limestone to make, irrespective of zinc, a slag whose per cent. composition shall be:

| SiO | 30 per cent. |
| :---: | :---: |
| FeO . | 50 per cent. |
|  | 20 per cent. |

Also give analysis of the slag, zine included, under the supposition that there is no loss of zine in smelting.
Applying the "mixing" rule, we have $17.5-7=10.5$ and $7-4=3$. Take, therefore, 3 parts of No. 1 to 10.5 parts of No. 2, or 1 part of No. 1 to $3 \frac{1}{2}$ parts of No. 2. This gives the required 7 per cent. of zinc oxide, also the following for other percentages of the mixture: $\mathrm{SiO}_{2}=47.77 ; \mathrm{FeO}=8.44$. Let $100 x=$ iron flux, $100 y=$ limestone.
The equations are:
(1) $47.77+8 x+5 y=.6 \times(8.44+80 x)$
(2) $.4 \times(8.44+80 x)=50 y$

Iron flux $=116.8$, Limestone $=81.5 \mathrm{lbs}$.


These numbers are as $3: 5: 2$. Analysis of the slag, zine included:

| $\mathrm{SiO}_{2}$. | 29.02 per cent. |
| :---: | :---: |
| FeO . | 48.33 per cent. |
| CaO | 19.33 per cent. |
| ZnO | 3.32 per cent. |

Example.-For single equation. Slagging constituents of ore, $\mathrm{SiO}_{2}=20$ per cent., $\mathrm{FeO}=15$ per cent. Limestone, $\mathrm{SiO}_{2}=10$ per cent. Bases, 50 per cent. Mix 100 lbs. ore with limestone for 40 per cent. $\mathrm{SiO}_{2}$ in slag.
Operation omitted. One unknown quantity is needed.
Ans. Limestone $=42.857 \mathrm{lbs}$.
Analysis of slag:


Example (this is an arithmetical exercise, not a slag prob-lem).-We have used $1,000 \mathrm{lbs}$. of ore in a charge, containing 30 per cent. $\mathrm{SiO}_{2}$. Also charged 600 lbs . of iron-ore flux and 600 lbs. limestone, whose compositions were respectively as follows: Iron flux, $\mathrm{SiO}_{2}=15$ per cent., $\mathrm{FeO}=75$ per cent. Limestone, $\mathrm{SiO}_{2}=10$ per cent., $\mathrm{CaO}=50$ per cent.
The slag analyzes: $\mathrm{SiO}_{2}=30$ per cent.; bases, 70 per cent.
What was the percentage of slagging bases in the ore?

- Operation omitted.

Ans. 30 per cent.
Example (also a mere piece of arithmetic).-Use silicon $=28.4$ in the calculation.
An iron ore contains $\mathrm{SiO}_{2}, 15$ per cent.; $\mathrm{FeO}, 85$ per cent. The pig metal contains all of the iron from the ore, and analyzes: Carbon, 3.5 per cent.; silicon, 2.5 per cent.
What percentage of silica in the ore was reduced to silicon?

Operation omitted. Answer, 3.37 per cent. of the silica is reduced.

Some examples more complex will now be considered at length. A general method, however, is better set forth by the use of simple cases like the preceding than by problems of greater detail, whose complications often serve to mask the simplicity of the principles involved.

In order to illustrate how the equation method operates in cases where there are percentages of each element in each material (ore, iron flux, and lime ) we here give such a case, in which we are to suppose that the analyses as presented have been simplified by extraction of matting constituents, and reduction of the various bases to the two groups, iron and lime. As this is intended merely as an example of the adaptation of equations to the case mentioned, the coefficients have been made extremely simple.

Example.-Ore, iron flux, and limestone, reduced to simplest forms, and none but slagging bases given:

|  | Ore. | Iron ore flux. | Limestone. |
| :---: | :---: | :---: | ---: |
| $\mathrm{SiO}_{2} \ldots \ldots \ldots \ldots \ldots \ldots$ | 40 | 10 | 4 per cent. |
| $\mathrm{FeO} \ldots \ldots \ldots \ldots \ldots \ldots$ | 20 | 72 | 6 per cent. |
| $\mathrm{CaO} \ldots \ldots \ldots \ldots \ldots \ldots \ldots 10$ | 10 | 50 per cent. |  |

Conditions.-Slag to be: $\mathrm{SiO}_{2}, 40$ per cent.; $\mathrm{FeO}, 40$ per cent.; $\mathrm{CaO}, 20$ per cent.
Solution.-Call iron and lime fluxes as usual, $100 x$ and $100 y$. Ore, 100 lbs .

$$
\begin{aligned}
& \text { (1) } 40+10 x+4 y=20+72 x+6 y \\
& \text { (2) } 40+10 x+4 y=20+20 x+100 y
\end{aligned}
$$

(1) Reads: "Weight of silica equals weight of iron oxide."
(2) Reads: "Weight of silica equals twice weight of lime."
$x=.3169 \quad y=.1753 \quad 100 x=31.69 \quad 100 y=17.53$. Ans. Iron flux, 31.69 lbs ; limestone, 17.53 lbs .
Proof.-All $\mathrm{SiO}_{2}$ from ore $=40.00$, from $10 x=3.17$, from $4 y=.70$.
Total $\mathrm{SiO}_{2}=43.87 \mathrm{lbs}$.
All FeO from ore $=20.00$, from $72 x=22.82$, from $6 y=1.05$.
Total $\mathrm{Fe} 0=43.87 \mathrm{lbs}$.
All CaO from ore $=10.00$, from $10 x=3.17$, from $50 y=$ 8.765 .

Total $\mathrm{CaO}=21.935 . \quad 21.935 \times 2=43.87$.

The weights reduce to 40 per cent., 40 per cent., and 20 per cent.
This case would make a very complex problem if any method by "excess" were attempted. It illustrates the point that as conditions multiply, the "equation" method becomes relatively simpler.

Conditions which render a problem impossible are not always recognizable at first glance. Suppose we have the following analyses:

| Iron ore flux, <br> per cent. | Limestone, <br> per cent. |
| :---: | :---: |
| 10 | 10 |
| 80 | $\ldots$ |
| $\ldots$ | 50 |

The requirement is a slag which shall analyze $\mathrm{SiO}_{2}, 40 ; \mathrm{FeO}$, $40 ; \mathrm{CaO}, 20$ per cent. It looks as though no more iron should be added, since the proportion of iron oxide in the ore exactly balances the silica according to required formula. But the lime is lower than the required ratio, so we have to add limestone. If the limestone were absolutely pure-i.e., if it had no silica, we could use it alone, and get the slag called for. But in adding it, we add silica, thus disturbing the ratio of silica to iron. In short, we need some of the iron ore in our charge. Take, as usual, 100 lbs . ore.
Here we omit both statement and operation, giving only answer and proof.
Answer:
Iron flux required to 100 lbs . of ore, 1.613 lbs . ( $100 x$ ) Limestone required for same, 11.29 lbs . ( 100 y )

## Proof:

| $\mathrm{SiO}_{2}$, from ore $\ldots 30.00$ | FeO , from ore.. 30.00 | CaO, ore $\ldots$ | 10.00 |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| $\mathrm{SiO}_{2}, 10 x \ldots \ldots$. | .16 | $\mathrm{FeO}, 80 x \ldots \ldots$ | 1.29 | $\mathrm{CaO}, 50 y \ldots$ | 5.645 |
| $\mathrm{SiO}_{2}, 10 y \ldots \ldots$ | $\underline{1.13}$ |  | $\overline{31.29}$ |  | $\overline{3}$ |
|  | $\underline{31.29}$ |  |  |  |  |

"These weights check absolutely to the required ratio, making slag as required, $40: 40: 20$, for $\mathrm{SiO}_{2}, \mathrm{FeO}$, and CaO . Yet the problem looks hardly possible, to exact figures, on hasty inspection.
We might have a little more iron than silica in the ore, and still have to add iron flux. Thus, keeping all of the data of the last problem, except the per cent. of iron oxide in the ore if we raise
that to 31 per cent. (instead of 30 ) we shall find that an exceedingly minute portion of the iron flux still has to be added, though now it becomes so small as to be absurd as a practical addition ( 0.1613 lb .).
Again we raise the per cent. of FeO in the ore, this time to 32. State the conditions in the usual way ( $100 x$ for iron flux, $100 y$ for limestone). You will now get a small minus value for $x$, the usual "notice" that you are attempting an impossible problem.
Without illustrating by further examples, the general case may be thus stated, the materials given being understood to be (1) the ore, (2) an iron oxide flux, (3) a limestone. It is also understood that the flux and the limestone are never pure, but always contain some percentage of silica. But both the fuxing materials are to be basic-i.e., holding base in ratio far above silica. By "iron" and "lime" we understand the "groups" already given.
If, in the ore, both the bases are below the required ratio to silica, the problem is solvable.

If one is equal to the required ratio, and the other below it, the problem is still possible, but only under certain conditions as to analyses-i.e., solvable within certain limits.
If both happen to be in the required ratio, then the ore is charged as self-fluxing, needing no additions.
If one base is lower and the other higher than the requirement, then the problem is solvable only within certain limits (as shown above).
If both bases are higher in ratio than called for, the problem is impossible.

Impossible, that is, under the conditions imposed. Practically solvable by adding silica or highly siliceous material in proper proportion, to the charge, instead of the fluxes, which, as above, are supposed to be always basic.

The following problem, as to its data, is from Prof. Edward D. Peters's fine work on the "Principles of Copper Smelting," 1907.
"We take, for a final comprehensive illustration, a copper-gold-silver ore of such a nature that it will not, when smelted by itself, yield a proper slag. We will assume also that we have at our disposal certain suitable fluxes and fuel. From the analyses
of these substances we will determine how we must mix them so that they may, on smelting, yield a slag of the composition that we desire, and a matte of suitable grade." ("Principles of Copper Smelting," page 118.)

Professor Peters then proceeds to explain all the simplifications called for, the requirement for the slag being:

```
Silica..................................... . . }38\mathrm{ per cent
Iron oxide (FeO) . . . . . . . . . . . . . . . . . ...... }50\mathrm{ per cent
Earths. 50 per cent 12 per cent.
100 per cent.
```

It will be noted that all of the basic slagging constituents except iron oxide are thrown together under the general classification of "earths," a proceeding justified in the present case, as, without here repeating all the original analyses and simplifications, we may state in brief that owing to the small quantities of alumina and magnesia, it serves all practical purposes to add them together without troubling ourselves to "convert" them by the factors already discussed into their chemically "equivalent" lime basis. This is set forth by footnote, page 123 of the "Principles."
The calculation starts from a roasted ore, which is supposed to be the carrier of the chief values in precious metals. For fluxing material we have a hematite ore and a limestone.
We have also the coke, whose ash is included in the computation for slag. Thus we start with four elements of computation, requirement for slag composition as above being the condition by which we have to regulate the charge.
Professor Peters assumes 100 lbs . ore as basis of calculation.
As the present elucidation concerns only the calculation of the slag, we shall not go into all the physical reasoning and the details of supposed losses of copper and sulphur, which result in subtractions from the original analyses.
What we wish to show in the present discussion is chiefly the difference in method of slag calculations; and since there are no differences in the "simplifications". of data prior to such calculations, we shall omit the discussion in the text (pp. 117-124) and assume precisely the same simplified analyses of ore, hematite and limestone as are arrived at finally in Professor Peters' discussion, which he gives on page 125.

It will be noted that the analyses are given before computing the matte. It is at this very point that we make our first departure from the method in Professor Peters' text, as we take the iron for matte from the ore itself, instead of from the hematite.

But for this, we should present the analysis freed from even the matte subtractions. As it is, we adopt all of Professor Peters' simplifications up to this point, and then proceed with our "representative equations."
After then, adding MnO to FeO , and throwing all other basic constituents together as "earths," we arrive at the "simplified analysis." The ore is supposed to be roasted, and we here omit, as wholly outside of the scope of this illustration, the values of the precious metals in the original analyses.

They are, indeed, omitted by Professor Peters in the analysis annexed.

In short, as the simplification is along precisely the lines we have already set forth, it would be a waste of space for us to do otherwise than to start the slagging problem proper from any other point than where it is started in the text quoted.

|  | "Simplified Analyses." | (Peter | p. 125.) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{SiO}_{2}$ | Feo. | Earths. | Cu | s. |
| Silicious ore | 42.0 | 30.6 | 5.6 |  |  |
| Hematite. | 5.0 | 75.6 | 5.9 |  |  |
| Limestone.. | 3.3 | 3.9 | 49.1 |  |  |
| Coke ash. | 42.6 | 22.2 | 32.1 |  |  |

We have now to explain that the coke is assumed at 12 per cent. of the ore charge. We are to compute on the basis of 100 lbs. ore, consequently we shall calculate for 12 lbs . coke. Ash of the coke stated at 12 per cent.
At this point we part company with the text quoted, and proceed to calculate the required charge by "representative equations."
The calculation in the "Principles" goes upon chemical excesses, and as will be seen arrives at a result practically the same as ours. But this being a case where the composition of the required slag is given in percentages there is not the slightest occasion to consider any chemical relations. They are properly out of consideration as soon as we have gone to the "percentage" basis of calculation.

We must first get the matte "out of the way." This being done by the method of excess already explained, no further chemical question shall interfere from that point to the end.
As usual assume that $\mathrm{Cu}_{2} \mathrm{~S}$ will be the form of the copper in the matte. This disposes of copper and part of sulphur, as follows:

$$
4: 1=5.76: 1.44
$$

Then, adding the two last numbers, expressing copper and sulphur combined with it, we have for the copper component of the matte 7.20 lbs. per 100 of ore.
The "residual" sulphur is $4.8-1.44=3.36$.
Adjust this to as much FeO as is required for the formation of FeS.

$$
\begin{aligned}
& 32: 72=3.36: 7.56 \\
& \mathrm{~S} \quad \mathrm{FeO} \quad \mathrm{~S} \quad \mathrm{FeO}
\end{aligned}
$$

We are then to subtract 7.56 FeO from the ore, as allowance of what is to be taken out in the matte. This 7.56 Fe 0 corresponds to 5.88 Fe , too simple a calculation to be repeated here. We have then to add the residual sulphur to this iron, $5.88+3.36$ $=9.24=$ iron component of the matte.
Matte to 100 lbs . ore charge:


Since the coke, consequently its ash, is in fixed ratio to the ore, we may at this point combine its analysis with that of the ore, thus ridding ourselves of a complication at the outset. The coke has 12 per cent. ash, and is itself 12 per cent. of the ore charge. That is, coke ash per 100 lbs . ore charge is $12 \times .12$ $=1.44 \mathrm{lbs}$. Calculate from the analysis of ash above the lbs. or fractions of its several constituents, and add results in proper places in ore analysis.
Thus, using analysis and weight ( 1.44 lbs. ), we have-
Ash, distributed as to constituents to be added to ore analysis:

$$
\begin{aligned}
& \text { Earths............................. } .321 \times 1.44=0.5 \mathrm{lbs} .
\end{aligned}
$$

We have dropped decimals in second place, as too trivial to "bother" with in a constituent itself rather trivial.

Remembering that we had in our matte allowance to subtract 7.56 from Fe , we can now tabulate our final analysis, ready for calculation.
If we then drop all fractional parts of the percentages, as would certainly be done in practice, we shorten our work considerably. In order, however, to demonstrate how little difference such abbreviations make, we shall in this instance, after solving with simplified coefficients, return to the original analysis as here annexed, and taking in all the fractional numbers, construct the equation with all these fractions in it, arriving, as will be seen, at a result so close to the first that one result is as good as another.
However, the first analysis as here tabulated retains the fractions.
Silica: To 42 lbs. in ore analysis above, add 0.6 lbs. from coke ash.. 42.60 FeO: To 30.6 lbs. in same, add 0.3 (coke), subtract 7.56 (matte).. 23.34 Earths: To 5.6 lbs. in same add 0.5 from coke ash................. 6.1
This completes the "clearance," first for matte and second from the little complication of the coke ash. The latter would hardly be regarded in actual practice, but is retained and calculated in, both because it is well to understand such details, and because it is allowed for in the text we have started from.
We shall now state the full analysis, calling it, for reference, analysis "A." We shall then restate it, dropping fractional figures, and shall call the latter analysis "B."

| Analysis "A": | $\mathrm{SiO}_{2}$. | FeO . | Earths. |
| :---: | :---: | :---: | :---: |
| Ore (coke ash included) | 42.6 | 23.34 | 6.1 |
| Hematite. | 5.0 | 75.6 | 5.9 |
| Limestone. | 3.2 | 3.9 | 49.1 |
| Analysis "B": | $\mathrm{SiO}_{2}$. | Fe0. | Earths. |
| Ore | 43 | 23 | 6 |
| Hematite. | 5 | 76 | 6 |
| Limestone. | 3 | 4 | 49 |

In practice, analysis " $B$ " would be accepted, as simpler and close enough.

Find required weights of hematite and limestone.
Calculation of charge starting with analysis " $B$."
Call weight of hematite $100 x$.
Call weight of limestone 100 y .

The equations are all in "pounds" since the basis is 100 lbs . of ore.
Now get together the "representative" terms, $\mathrm{SiO}_{2}, \mathrm{FeO}$, and "Earths."

$$
\begin{aligned}
& \text { Silica }=43+5 x+3 y \\
& \text { FeO }=23+76 x+4 y \\
& \text { Earths }=6+6 x+49 y
\end{aligned}
$$

These come directly from the vertical columns of the analyses. For example, the $\mathrm{SiO}_{2}$ in the 100 lbs . ore is 43 lbs . Since $\mathrm{SiO}_{2}$ in hematiteg is 5 per cent. it is 5 per cent. of $100 x$, that is, $5 x$; similarly the $\mathrm{SiO}_{2}$ in the limestone is $3 y$.

Apply the same method to the other constituents, and we get the three expressions above.
Refer to the slag condition as given at the opening of this problem.
The silica is to be to the "earths" as 38 to 12. So state it:

$$
\begin{equation*}
38: 12=(43+5 x+3 y):(6+6 x+49 y) \tag{1}
\end{equation*}
$$

The FeO is to be to the "earths" as 50 to 12. So state it:

$$
\begin{equation*}
50: 12=(23+76 x+4 y):(6+6 x+49 y) \tag{2}
\end{equation*}
$$

As we need but two equations the statement is complete. Solve the equations:
$x=0.4821,100 x=48.21$
$y=0.1133,100 y=11.33$
Hematite for 100 lbs . ore charge 48.21 lbs .
Nothing remains as to solution. But if we desire to check up our results the procedure is easy, and is here given at length.

Proof.-Taking the weights as found, for hematite and limestone, we ascertain by multiplication the respective weights of $\mathrm{SiO}_{2}, \mathrm{FeO}$, and of "earths" in each, add them to already known weights of these constituents in the ore, and obtain total for each constituent in the slag.
If these prove to be in the required ratio $(38: 50: 12)$, the proof is absolute.

|  | Silica. |  | FeO. |  | Earths. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| From ore | 43.00 |  | 23.00 |  | 6.00 |
| From hematite (5x) | 2.41 | (76x) | 36.64 | (6x) | 2.89 |
| From limestone (3y) | 0.34 | (4y) | 0.45 | (49y) | 5.55 |
| Totals | 45.75 |  | 60.09 |  | 14.44 |

Total weight of slag per 100 lbs . of ore charged, with the above found weights of fluxing material, is 120.28 lbs.

Then, testing the ratios of the three weights as in the above proof, for the three constituents, too simple an operation to be inserted, we find:

$$
38: 50: 12=45.75: 60.09: 14.44
$$

The "proof" then is absolute, and the only error is in the slightly abbreviated coefficients, which is the same as a slight alteration in the analyses.

We shall now restate the equations, not because there is any difference in principle, but to show what an immaterial matter this dropping of decimals really is. Note that from start to finish of the "setting up" of the problem there has been no chemical adjustment whatever; none was needed.
Turn now to analysis " A " and write the two, equations as before.

$$
\begin{aligned}
& 38: 12=(42.6+5 x+3.2 y):(6.1+5.9 x+49.1 y) \\
& 50: 12=(23.34+75.6 x+3.9 y):(6.1+5.9 x+49.1 y)
\end{aligned}
$$

No difference except that we have now used as coefficients the original percentage figures, with their fractional parts.
Solving these as before we get:
$x=0.4752,100 x=47.52$ Hematite $=47.52 \mathrm{lbs}$. to 100 lbs ore

$$
y=0.1102,100 y=11.02 \text { Limestone }=11.02 \mathrm{lbs} . \text { to } 100 \mathrm{lbs} \text {. ore }
$$

Proof is accomplished just as before. We find that the total weight of the slag per 100 lbs . ore charged is 119.33 , but the percentages of $\mathrm{SiO}_{2}, \mathrm{FeO}$ and "earths" come out as before, viz: $38: 50: 12$.

The ordinary method of solution of such cases puts the ingredients into groups, after carefully adjusting the elements of each according to the slag requirement, and calculating "excess" of one element over another, in ore, hematite and limestone, first within themselves, i.e., finding excess of silica in ore, of iron in hematite, and of lime in limestone; and afterwards the adjustment of these ingredients thus prepared as to "available" elements, to one another.

It is a tedious and entirely useless proceeding. The assignment of any "percentage" (as here, $38,50,12$ ) throws out of consideration all necessity of further treatment of chemical "excess."
The "representative method" is perfectly general in its application. The above case can be deliberately figured from equation to answer in ten minutes.

