SPECIFIC GRAVITY.

The **Specific Gravity** of a substance is the weight of any given volume of it, as compared with the weight of the same volume of a substance taken as the standard of comparison.

We reserve the term "Density" for gases. The standard substance adopted for comparison in solids and liquids is *water at its greatest density,* which is at 4° C. Barometric pressure of 760 mm. is also understood, though this has very little influence in ordinary determinations of specific gravity.

Determinations are usually made at ordinary temperatures, and if great accuracy is called for, reduction to standard conditions may be made afterward.

Another consideration comes in, for accurate work, viz: the fact that the substance under investigation has not the same specific gravity as the weights used for the determination. Hence the displacement of air is unequal. Theoretically, then, specific gravity determinations should be made "in vacuo." This not being practicable, another reduction should be made, allowing for the difference between the figures obtained, and what they would have shown had it been possible to perform the necessary operations in an actual vacuum. These differences are rarely taken into the account except in those exceptional cases where minute differences in specific gravity under slightly variant conditions are to be recorded.

Water is 773 times as heavy as air. It is evident that the correction would be greatest when the substance greatly differs in specific gravity from the weights used.

In practice it is usual to consider the comparison as made between weights with one gram as the weight unit and one cubic centimeter as the volume unit.

Since one cubic centimeter of water, under the conditions above specified, weighs one gram, this convention brings about the convenient identity *in figures,* of actual weight in grams with specific gravity.

Example.—One c.c. of water weighs one gram. Sp. gr. = 1. One c.c. of copper weighs 8.85 grams. Sp. gr. = 8.85.

Having agreed upon this convention, we might define specific gravity by the figures representing the *weight of one cubic centimeter of the substance.*

Elementary principles of physics show that the weight of a body is the product of its specific gravity by its volume. This finds its easiest practical expression in the metric system, owing to the connection by simple relations between linear and volumetric measures, and between the latter and weights, standard (water) having once been adopted.

We do not go into the definition of "mass" as distinct from "weight."

A few examples are appended illustrating the principle mentioned, viz: $W = V \times G$, where W = weight, V = volume, and G = specific gravity.

Special problems on gravity determinations follow in order.

Problems.—These substitutions are hardly "problems." Any two of the quantities W, V, G being given the third follows. We have only to remember:

$$W = V \times G \qquad V = \frac{W}{G} \qquad G = \frac{W}{V}$$

Any two, then, of the quantities in the following examples may be taken as the given or known data, the third one becomes the "unknown."

(*a*)	$V = 732.0$	$G = 0.89$	$W = 651.48$
(*b*)	$V = 650.0$	$G = 1.01$	$W = 656.50$
(*c*)	$V = 232.0$	$G = 1.84$	$W = 426.88$
(*d*)	$V = 38.56$	$G = 0.81$	$W = 31.2336$
(*e*)	$V = 62.7$	$G = 1.23$	$W = 77.121$
(*f*)	$V = 250.6$	$G = 0.98$	$W = 245.588$
(*g*)	$V = 36.7$	$G = 2.01$	$W = 73.767$
(*h*)	$V = 62.9$	$G = 0.97$	$W = 61.013$
(*i*)	$V = 611.0$	$G = 13.4$	$W = 8187.4$
(*j*)	$V = 722.0$	$G = 3.03$	$W = 2187.66$
(*k*)	$V = 13.75$	$G = 1.2$	$W = 16.50$
(*l*)	$V = 86.5$	$G = 1.32$	$W = 114.18$
(*m*)	$V = 11.6$	$G = 2.17$	$W = 25.172$
(*n*)	$V = 0.91$	$G = 1.25$	$W = 1.1375$
(*o*)	$V = 20.50$	$G = 0.90$	$W = 18.45$
(*p*)	$V = 14.00$	$G = 1.27$	$W = 17.78$
(*q*)	$V = 222.00$	$G = 4.19$	$W = 930.18$
(*r*)	$V = 321.00$	$G = 0.95$	$W = 304.95$
(*s*)	$V = 932.00$	$G = 0.86$	$W = 801.52$

Problem X.—Ordinary specific gravity determination, by weighing in both air and water. The substance, having been weighed in the usual manner, is suspended by a fine thread from the balance arm, lowered into water by suitable appliances, and "weighed" in water. *Apparent loss of weight* is then made the means of calculating specific gravity. Neglect correction for weight of the air. The substance in this operation must of course be heavier than water. (See later for substances lighter than water.) The substance manifestly displaces a volume of water equal to its own volume, so that we at once derive the rule: "*Divide weight in air by loss of weight in water.*"

It is obvious that the "loss of weight" is simply the *weight of a volume of water equal to the volume of the substance.*

Let G = sp. gr., W = weight in air, and W' weight in water. Then the apparent "loss of weight" will be $W-W'$. It is further evident that the weight of any volume of water is to unity (sp. gr. of water) as the weight of the same volume of any other substance is to its sp. gr., that is:

$$W - W' : 1 = W : G. \text{ Hence (as by rule above):}$$

$$G = \frac{W}{W - W'}$$

Examples:

(a) $W = 6.3245$	$W' = 5.0596$	$G = 5.00$
(b) $W = 37.8182$	$W' = 21.1582$	$G = 2.27$
(c) $W = 28.9344$	$W' = 20.4864$	$G = 3.4250$
(d) $W = 5.5748$	$W' = 4.3078$	$G = 4.4$
(e) $W = 3.9144$	$W' = 0.6524$	$G = 1.2$
(f) $W = 86.8371$	$W' = 82.7020$	$G = 21.0$
(g) $W = 25.92$	$W' = 17.92$	$G = 3.24$
(h) $W = 94.800375$	$W' = 86.373675$	$G = 11.25$
(i) $W = 37.638408$	$W' = 0.738008$	$G = 1.02$
(j) $W = 16.037$	$W' = 4.234$	$G = 1.3587$
(k) $W = 22.224$	$W' = 20.308$	$G = 11.599$
(l) $W = 21.424$	$W' = 19.848$	$G = 13.59+$

Problem XI.—To find the specific gravity of a substance soluble in water. In this case the substance may be weighed in a liquid in which it is not soluble. This liquid may be either lighter or heavier than water, but for convenience in operations it should be lighter than the substance. See next problem.

The specific gravity of the liquid must of course be known. It may be necessary to determine it, as in problem to be presently given for determination of sp. gr. of liquids.

Call the specific gravity of the liquid "a." Call the specific gravity *with reference to this liquid* ("apparent specific gravity") "g." Call as above the *true* specific gravity (*i.e.*, referred to water), "G." We now have (since sp. gr. of water = 1):

$$1 : a = g : G \text{ and } G = a \times g$$

Rule.—Multiply sp. gr. of the liquid by (apparent) sp. gr. of the solid in the liquid. The latter is deduced by the last rule.

Examples:

"a."	Wt. in air.	Wt. in liquid.	G.
(a) 0.8	2.752	2.064	3.2
(b) 0.85	3.393	2.262	2.55
(c) 0.9375	2.7232	1.8722	3.00
(d) 1.2	8.47	7.00	6.91+
(e) 0.9	21.6	15.6	3.24
(f) 0.86	10.5	8.2	3.92
(g) 1.6	12.2	10.6	12.20
(h) 0.92	14.2	11.6	5.02
(i) 0.88	6.4	5.8	9.39
(j) 1.33	12.42	8.82	4.59
(k) 0.82	20.6	14.2	2.64
(l) 0.92	6.0	3.0	1.84

Problem XII.—To determine the specific gravity of a substance lighter than water.

Attach the light body to a heavy one, and weigh the pair together in water. Also weight of the heavy body *alone* in water must be determined. As in all other cases, the weight of the body under investigation must be found, under usual conditions, viz: "in air."

A piece of metal, heavy enough for all probable cases, may be selected and weighed in water "once for all." Silver, as being practically unalterable by air and damp, is the best. *Note that it is unnecessary to determine weight of the heavy body in air.*

It is evident that the combined apparent weight of the pair, in water, must be less than the weight of the heavy body alone, in water. We may call the loss of weight in water (as compared with weight of the heavy body alone) the "buoyancy" of the light body. The weight of a bulk of water *equal in volume to the solid,* is the fundamental datum as in the first case. The weight of this bulk of water is the *sum* of the weight in air of the solid, *plus* its apparent *minus weight* (buoyancy) when attached to the heavy body.

Examples.—(a):

Weight of the heavy body in water................	6.3584
Weight of both bodies on water..................	5.8880
"Buoyancy" of the light body....................	0.4704
Weight of the light body in air..................	0.5096
Weight of volume of water equal to that of solid....	0.9800

Now we have the "data" of the original problem, viz: weight of the solid, and weight of an equal volume of water, so that the specific gravity is deduced at once as in the previous case.

$$\frac{.5096}{.9800} = 0.52 = \text{sp. gr.} \left(\frac{W}{V} = G\right)$$

The figure 0.98 is an expression for volume as well as weight; a point in metric convenience which the student may well note.

	Wt. of heavy body in water.	Both, in water.	Light, in air.	Sp. gr.
(b)	6.1342	5.8408	0.5216	0.64
(c)	4.3186	3.7570	1.7784	0.76
(d)	22.0632	17.0365	5.0267	0.50
(e)	6.42	5.90	2.60	0.83
(f)	16.40	15.94	1.60	0.777
(g)	10.2	8.8	1.4	0.50

Problem XIII.—To determine the specific gravity of a liquid. Special apparatus is used for this determination. For close work a tared flask holding a certain weight of water to the tip of its perforated stopper, may be filled with the liquid. The net weight gives sp. gr. directly, if the weight of the water is a power of ten (10 or 100). $t°$ C. is marked on flask.

The instruments known as "hydrometers" and by other names, which are dropped into the liquid placed in a cylinder, are well known. Those of smaller range are the more delicate. All alike have readings marked on their "stems."

These devices, however, being appliances for yielding results without calculation, need not here be dwelt upon.*

The sp. gr. of a liquid may be determined thus:

Weigh a solid (1) in air, (2) in water, (3) in the liquid. We may use the same solid for every determination, thus saving operations (1) and (2).

Call these three weights W, W', and W'', then we have:

$$W - W' : W - W'' = 1 : G = \text{sp. gr. of the liquid}$$

Examples.—(a):

Solid in air,	3.393		3.393
Solid in water,	2.063	In liquid,	2.262
	1.330		1.131

$$1.330 : 1.131 = 1 : 0.85 = \text{sp. gr. of the liquid}$$

	W.	*W'.*	*W''.*	*Sp. gr. of liquid.*
(b)	6.7672	5.8336	5.2020	1.676
(c)	14.0	12.0	11.2	1.4
(d)	10.2	6.7	8.45	0.5
(e)	30.0	25.0	20.0	2.0
(f)	4.6	3.2	3.8	0.57
(g)	8.8	7.6	6.4	2.00

Problem XIII (2nd case).—Having a liquid of given sp. gr. "*a*" (greater than 1), how much water must be added to bring it to sp. gr. "*b*"? (No allowance for condensation or expansion.) Let x = c.c of water to be added to 1 c.c of liquid.

$$a + x = b\,(1 + x) \qquad (\textit{i.e.}, W = G \times V) \qquad x = \frac{a - b}{b - 1}$$

(Figures representing wt. of 1 c.c. are same as figures for sp. gr.)

Examples.—(a) Acid, sp. gr. = 1.4 to be reduced to sp. gr. 1.2.

$$x = \frac{1.4 - 1.2}{1.2 - 1} = \frac{.2}{.2} = 1$$

Ans. Equal volume of water.

*Specific gravity of liquids may be derived from Baumé scale by adding 133 to Baumé reading and dividing 143 by the sum. *Example:* Linseed oil marks 20° Baumé.

Then $\dfrac{143}{133 + 20} = 0.934$ = sp. gr. referred to water as unity.

(*b*) Sp. gr. of liquid to be brought from 1.85 to 1.25. How much water?

$$\frac{1.85 - 1.25}{1.25 - 1} = \frac{.60}{.25} = 2.4$$

Ans. For each *c.c.*, add 2.4 *c.c.* of water.

Proof:

$$\text{Weight} = 1.85 + 2.40 = 4.25$$
$$\text{Volume} = 1 + 2.40 = 3.40$$

$$\frac{W}{V} = \frac{4.25}{3.40} = 1.25 \text{ as required}$$

In the same way formula for liquid lighter than water to be made up to intermediate sp. gr. may be used.

The student will find that in any problem where the statement is not at once evident, *the general principle* $W = G \times V$ will solve it.

Problem XIV.—Specific gravity of a solid by the "bottle" method. This depends upon displacement of water, method slightly differing from the above.

This will first be illustrated in full. In practice a "tare" weight accompanies the "bottle," which is constructed to contain an even number of c.c. of water at about 16° C. Multiples of ten are best for the number of c.c.

Examples.—(*a*):

Bottle full of water, weight...................... 32.026
Bottle with solid and filled with water, weight...... 63.526
Weight of the substance alone, "in air"........... 34.500

Solution.—Add together weight of water and bottle, and the substance itself, *weighed in air.*

	32.026
	34.500
Sum...............	66.526
Deduct weight of bottle with substance and filled with water.	63.526
Weight of water displaced is the remainder................	3.000

We have now the needed data, viz: weight of the substance and weight of an equal volume of water.

As before : $\frac{W}{V} = G$. That is : $\frac{34.5}{3.00} = 11.5$ = sp. gr. of the solid.

(For the weight of the water, 3 (grams) is also the number designating its volume in cubic centimeters, hence is taken as the "volume" of the water.)

Formulistically this is expressed as below:

$$\text{Specific gravity} = \frac{W}{(W + A) - K}$$

In which formula W = weight of substance taken; A = weight of bottle filled with water; K = weight of bottle and substance with bottle filled with water to the "mark."

Omissions of detail should be readily supplied by the student.

Since the apparatus provided for this method includes a "tare" weight for the bottle, the data are reduced in practice to weight in air, weight of the water, and weight of the substance + water remaining in the bottle—that is, the water originally present in the bottle *when full*, minus the volume of it displaced by the substance.

The operation then consists simply in weighing the substance, then placing it in the bottle, filling the latter with water and again weighing. Weight of water which fills the bottle by itself is a constant.

	Wt. of water.	Solid + water.	Solid in air.	*G.*
(*b*)	10	12.5	4.5	2.25
(*c*)	10	16.80	8.5	5.00
(*d*)	10	18.40	9.6	8.00
(*e*)	25	38.75	14.50	19.33
(*f*)	100	102.20	3.97	2.24
(*g*)	100	102.50	16.50	1.178
(*h*)	100	107.89	8.64	11.52
(*i*)	100	109.70	10.24	18.96
(*j*)	100	108.30	12.80	2.84
(*k*)	100	123.00	25.00	12.50
(*l*)	100	103.00	3.60	6.00

A great variety of cases may be based on the principle of specific gravity, and several will be found scattered among the miscellaneous problems. One special case is here annexed, with examples; all others are left to the device of the student.

Problem XV.—A mixture of two solids of known specific gravities being given, to deduce the respective weights of each, *given* the weight and specific gravity of the mixture.

This is often called the problem of Archimedes, and was probably first solved by him. It is usually applied to the estimation of free gold in quartz.

Examples.—(*a*) General solution. Let x and y be the required weights of the two solids. Let a and b be their respective specific gravities (known). Let c = weight of the mixture and d = its specific gravity.

The easiest solution is by two simultaneous equations, viz:

$$x + y = c \quad \text{and} \quad \frac{x}{a} + \frac{y}{b} = \frac{c}{d}$$

Refer back to the general principle, $W = V \times G$, and its simple transpositions. The first equation reads "weights of the two substances equal weight of the mixture." The second reads "volumes of the two substances equal volume of the mixture."

$$x = \frac{ac(d-b)}{d(a-b)} \quad \text{and} \quad y = \frac{bc(a-d)}{d(a-b)}$$

In these as in other problems in specific gravity assume 1 c.c. as unit of volume and 1 gram as unit of weight. Call the two solids "*A*" and "*B*."

	Sp. gr. of *A*.	Sp. gr. of *B*.	Wt. of mix.	Sp. gr. mix.	Wt. of *A*.	Wt. of *B*.
(*b*)	19.3	2.66	10.0	6.0	6.45+	3.55
(*c*)	12.0	6.00	12.0	8.0	6.0	6.0
(*d*)	8.0	2.0	10.0	2.5	2.667	7.333
(*e*)	20.0	1.0	10.0	1.5	3.5+	6.5
(*f*)	20.0	2.5	10.0	7.0	7.347	2.653+

Miscellaneous Problems Involving Principles of Specific Gravity.

1. Sp. gr. of silver is 10.5. What is the weight of a cube of silver whose edge measures 3.5 centimeters?

Ans. 450.1875 grams.

2. Sp. gr. of iron is 8. An iron rod of square section is one meter long, section $1\frac{1}{2}$ cm. square. What is its weight?

Ans. 1800 grams.

3. Sp. gr. of copper is 8.8. A hollow sphere of copper *just floats* in water. Its wt. = 1 kilo. What is its volume? What is volume of the copper?

Ans. Vol. of sphere = 1 liter. Of copper = 0.1136+ liters.

No allowance in (3) for air contained in the sphere.

4. Sp. gr. of lead = 11.35. A sheet is $\frac{1}{8}$ of 1 centimeter thick, and one meter square. What is its weight?

Ans. 14187.5 grams, or $14\frac{1875}{10000}$ kilos.

5. Sp. gr. of zinc = 7. A rod of circular section is 1 cm. in diameter, and 1 meter long. What is its weight?

Solution:

$$R = 0.5. \quad R^2 = 0.25. \quad \pi R^2 = 0.7854$$
$$0.7854 \times 100 \times 7 = 549.78 \text{ grams.}$$

6. A kilometer of copper wire weighs 2000 kilograms. (Sp. gr. = 8.8.) What is the diameter of the wire?

Solution: Remember that $\frac{W}{G} = V$. Having ascertained volume of the wire in c.c., find area of the base of cylinder (*i.e.*, the wire itself), thence deduce its radius.

Ans. $R = 0.85054+$. (Double for diameter.)

7. Cubes of 6 and 15 cm. edge, respectively, have the same weight. The smaller has sp. gr. = 21, what is sp. gr. of the larger? *Ans.* 1.344 sp. gr.

8. A metallic cube weighs 39.304 grams. Sp. gr. = 8. What is length of its edge? *Ans.* 1.7 cm.

9. A sphere of quartz (sp. gr. = 2.5) weighs 19.9382 grams. What is its radius? *Ans.* 1.1 cm.

10. A hollow sphere of copper (sp. gr. = 8.9) is filled with water; the whole weighs two kilograms. Exterior diameter = 0.12 meter. What is thickness of the copper? What is its weight?

Solution:

$$R = 0.06. \quad \text{Surface} = 4\pi R^2 = 0.04523904. \quad \frac{R}{3} = 0.02.$$

$$V = 4\pi R^2 \times \frac{R}{3} = 0.0009047808 \text{ cubic meters}$$

(904.7808 cubic centimeters.)

Let x = weight of the copper. Let y = weight of the water.

$$x + y = 2000. \quad \frac{x}{8.9} + \frac{y}{1} = 904.7808.$$

Hence $x = 1233.855$ grams. $y = 766.145$ grams.

Then, $\frac{1233.855}{8.90} = 138.635$ cubic centimeters of copper.

The surface has already been found, being 452.3904 square centimeters. Evidently cubic centimeters divided by square centimeters will give thickness of the metal.

$$\frac{138.635}{452.3904} = 0.306 \text{ centimeters} = \text{thickness of the copper.}$$

Weight of the copper, being value of x, has already been found.

11. Water at maximum density (4° C.) has sp. gr. = 1. Ice has sp. gr. 0.92. A liter of water at 4° becomes what volume of ice? (Sp. gr. is inversely as volume, for same weight.)
Ans. 1.087 liters.

12. An iceberg (sp. gr. 0.92) floats in sea water (sp. gr. 1.027.) What proportion of it will be above sea level?
Ans. 10.42 per cent.

13. Taking the above data for ice and sea water, how many cubic feet does an iceberg contain which shows 6000 cubic feet above water? *Ans.* 57,588.8 cu. ft.

14. Type metal contains: Lead, 70.00 per cent.; tin, 10.00 per cent.; antimony, 20.00 per cent. = 100.00 per cent. Take sp. gr. lead = 11.4. Sp. gr. tin = 7.3. Sp. gr. antimony = 6.7. Find the sp. gr. of the alloy, allowing neither expansion nor contraction. *Ans.* 7.53 sp. gr.

15. A liter of $POCl_3$ whose sp. gr. is 1.7 is deoxidized to PCl_3 whose sp. gr. is 1.6. What volume will the latter occupy?

NOTE.—Get the relative weight of the two—that is, from the known weight of the first, deduce stoichiometrically the weight of the second.

Then proceed with $V = \frac{W}{G}$ as in all similar cases.

Ans. 0.95175 liters.

16. A sphere of wood has 0.8 sp. gr. and 0.6 meter radius. What is its weight?

Find $\frac{4}{3}\pi R^3$. Multiply by 0.8

Ans. 723.82464 kilos.

CALCULATION OF ANALYSES.

Analytical determinations are made in a variety of ways, of which only the more important need be epitomized.

1. By separation and weighing of the element itself, if solid or liquid, or, if gaseous, measuring its volume. (Cupellation of gold and silver. Copper by the battery, and other familiar cases.)

2. By the isolation, usually by precipitation, of a compound containing the element "sought." The compound being weighed, weight of the element is determined by its stoichiometric relation to the compound. This is the commonest of gravimetric methods. (*Example.*—AgCl, used for determination of either silver or chlorine.)

3. By weighing or otherwise estimating mass of some substance which, though not containing the "sought," has a definite relation to it. This is a mere variant of (2), principle being the same. (*Example.*—Having obtained ammonium chloroplatinate, $(NH_4)_2PtCl_6$, ignite it and weigh residual platinum, whose relation to nitrogen in the now destroyed compound gives us required weight of the latter. Or again, we destroy the identity of the well known "yellow precipitate," and titrate for molybdenum, although phosphorus is our "objective.")

4. By expelling the element "sought" or a compound of it, and absorbing this in a previously weighed tube containing an appropriate absorbent. (Organic determination of carbon and hydrogen. For the former, as CO_2, the "potash bulb." For the latter, as water, the "calcium chloride tube." Increase in weight of bulb or tube gives the desired datum.)

5. By loss. (*a*) The "sought" is expelled by ignition, and the residual matter gives its quantity by weighing and subtraction from original weight. (*b*) The "sought" is dissolved out of the portion taken for analysis, by either water or some proper solvent, and the residue weighed. (*c*) In gas analysis the whole volume of the mixed gases is passed through the solvent, and the loss of volume indicates volume (hence weight if desired) of one of the gases in the mixture.