Having written the equation, the volumes of oxygen are mere "sight" problems. So are the volumes of CO_2 gas. It is nothing more than a comparison of the respective numbers of molecules.

6. $NH_4NO_2 = 2H_2O + N_2$. Ammonium nitrite decomposes into water and nitrogen gas. If we have 3.5 liters of nitrogen, what weight was taken of the nitrite?

$$64 : 22.4 = x : 3.5$$

Ans. 10 grams.

7. Phosphorus vapor is 4.42 times as heavy as air, which weighs 1.293 per liter. What would be inferred as to its molecular weight?

$1.293 \times 4.42 = 5.71$. (One step omitted.)

Ans. 128, nearly.

8. One liter of SO_2 gas weighs 2.86 grams. What is its molecular weight? Operation omitted. *Ans.* 64.

9. Air contains 79 per cent. nitrogen and 21 per cent. oxygen, by volume. We mix 600 c.c. of air with 50 c.c. CH_4 and explode by the spark.

Regarding the water as condensed, what are the compositions and volumes of the residual gases? (Normal temperature and pressure.)

$$600 \times .21 = 126 = \text{vol. of oxygen}$$
$$600 - 126 = 474 = \text{vol. of nitrogen}$$

The remaining part of the problem is "sight" work.

Ans.	Nitrogen..................	474 c.c.
	Oxygen in excess...........	26 c.c.
	Carbon dioxide (= CH_4)...	50 c.c.
	Residual gases, total.....	550 c.c.

10. $4NH_3 = 6H_2 + 2N_2$ Also : $6H_2 + 3\ O_2 = 6H_2O$

Four liters of ammonia gas are dissociated by the spark, and the resulting gas mixture exploded with six liters of oxygen.

What were the volumes of hydrogen and nitrogen after the dissociation? What are the final residual volumes, water aside? (Solve by inspection.)

Ans.	(1)	hydrogen	6 liters.
		nitrogen	2 liters.
	(2)	nitrogen	2 liters.
		oxygen	3 liters.

11. A liter of oxygen weighs 1.4296, what is its molecular weight?

Mol. wt. $: 22.4 = 1.4296 : 1$ *Ans.* 32.

12. 20 liters of hydrogen are burnt in 50 liters of air. Water aside, what are the residual gases and their volumes?

Ans. 50 liters air contain 39.5 liters nitrogen, and 10.5 liters oxygen. 20 liters hydrogen require 10 liters oxygen. Residual $N = 39.5$. $O = 0.5$.

13. Assuming that the units of molecular weights are equal (as actual weights) in all of the following reactions, give the volumes of all the gases which form their last terms. ("Unit" = 1 gram.)

$KClO_3 = KCl + \ldots\ldots O_3$* = 33.6 liters.
$MnO_2 + 4HCl = MnCl_2 + 2H_2O + \ldots\ldots Cl_2$ = 22.4 liters.
$HgO = Hg + \ldots\ldots O$ = 11.2 liters.
$2NaCl + H_2SO_4 = Na_2SO_4 + \ldots\ldots 2HCl$ = 44.8 liters.

14. Write out all the volumes of the following reactions, under the supposition that *one molecule* in formula indicates *two liters* in actual volume:

$2CO + O_2 = 2CO_2$.........4 liters + 2 liters, become 4 liters.
$H_2 + Cl_2 = 2HCl$.........2 liters + 2 liters, become 4 liters.
$C_2H_4 + Cl_2 = C_2H_4Cl_2$.......2 liters + 2 liters, become 2 liters.
$CO + Cl_2 = COCl_2$.........2 liters + 2 liters, become 2 liters.
$N_2 + 3H_2 = 2NH_3$.........2 liters + 6 liters, become 4 liters.
$2C_2H_2 + 5O_2 = 4CO_2 + 2H_2O$..4 liters + 10 liters, become 8 liters + 4 liters.

15. We have phosphorus 0.7557 gram. Find the volume of phosphine gas it will make (PH_3). *Ans.* 0.5546 liter.

PROBLEMS INVOLVING ONE OR MORE OF THE PRECEDING PRINCIPLES

(*a*) $2C_2H_2 + 5O_2 = 4CO_2 + 2H_2O$. Two liters of acetylene are exploded with ten liters of oxygen. Aside from water what are the volumes of the gases remaining, *i.e.*, both by combination and excess? (Sight solution.)

Ans. CO_2, 4 liters. Oxygen, 5 liters.

(*b*) $CO + Cl_2 = COCl_2$. One liter of CO and two of chlorine. What volumes and of what gases remain after the reaction?

Ans. Phosgene, 1 liter. Chlorine,1 liter.

* Strictly, according to proper theoretical form, we should not write O_3. Stoichiometrically, it can hardly be misinterpreted.

(*c*) Four liters of ammonia gas (NH_3), are dissociated by sparking, and then exploded with six liters of oxygen. Aside from water, what are volumes of the residual gases?

Ans. Nitrogen, 2 liters. Oxygen, 3 liters.

(*d*) Phosphine gas in burning produces HPO_3. Write the reaction. *Ans.* $PH_3 + 2\,O_2 = HPO_3 + H_2O$.

(*e*) Phosphorus pentachloride reacts with water, forming orthophosphoric and hydrochloric acids. Write this reaction.

Ans. $PCl_5 + 4H_2O = H_3PO_4 + 5HCl$.

(*f*) Chlorine gas passed into caustic potash (KOH) produces potassium chloride and potassium chlorate. (KCl and $KClO_3$.) Find an equation representing this reaction.

Ans. $6KOH + 3Cl_2 = 5KCl + KClO_3 + 3H_2O$.

(*g*) 16.7192 grams P_2O_5 was derived from what weight of phosphorus? *Ans.* 7.3 grams.

(*h*) $S + O_2 = SO_2$. (S = 32.) What volume of air will burn 1000 grams of sulphur? *Ans.* 3333 liters.

(Air estimated as containing 21 per cent. of its volume of oxygen.)

(*i*) $$S + O_2 = SO_2$$

1000 grams of sulphur are burnt. What volume of SO_2 is produced? *Ans.* 700 liters.

(*j*) $Hg + O = HgO$. 154 grams Hg make what weight of the oxide? *Ans.* 166.32 grams.

(*k*) $$S + H_2 = H_2S$$

Take 6.66 grams sulphur, what *volume* of hydric sulphide is produced? *Ans.* 4.662 liters.

(*l*) $$FeS + H_2SO_4 = FeSO_4 + H_2S$$

We want 100 liters of the gas, what weight of ferrous sulphide must be taken? *Ans.* 392.85 grams.

(*m*) $$Sb_2S_3 + 6HCl = 2SbCl_3 + 3H_2S$$
$$3FeS + 6HCl = 3FeCl_2 + 3H_2S$$

These are two ways of producing sulphuretted hydrogen. Suppose that the antimony sulphide costs eight cents a pound, and the iron sulphide ten cents, which produces the gas more cheaply? *Ans.* The iron sulphide.

(*n*) 3300 grams of gypsum are heated. What volume of steam at 300° is given off?

$$CaSO_4, 2H_2O = CaSO_4 + 2H_2O$$
$$172 : 22.4 = 3300 : 859.5$$
$$\text{And } \frac{859.5 \times 573}{273} = 1803.6 = \textit{Ans.}$$

(*o*) Lead is melted in a furnace and a current of air passed over it. $Pb_2 + O_2 = 2PbO$. If the air is admitted at 40° C. how many cubic feet will oxidize 100 lbs. of the lead? (Air has 21 per cent. oxygen by volume.)

$$413.8 : 22.4 = 1600 : 86.6 = \text{cubic feet of oxygen}$$

Lead taken at 206.9 atomic weight. For the proportion, the 100 lbs. become 1600 (ounces). The remaining calculations are omitted. *Ans.* 472.8 cubic feet air at 40° C.

(*p*) In the manufacture of sulphuric acid, we have, through several intermediate reactions, the general result:

$$S_2 + 3O_2 + 2H_2O = 2H_2SO_4$$

If we have two tons of sulphur, what volume of atmospheric air, in cubic feet, must be used? What weight of water? What weight of sulphuric acid will be produced? (Note—Use ounces. 32,000 oz. = one ton.)

Atmospheric air has 21 per cent. by volume of oxygen.

Ans. 1. 320,000 cubic feet of air.
2. 2250 lbs. water.
3. 12,250 lbs. sulphuric acid.

(*q*) $$2C_2H_2 + 5\,O_2 = 4CO_2 + 2H_2O$$

In this combustion six liters of acetylene gas take part. How many liters of atmospheric air (21 per cent. by vol. of oxygen) will be required?

What will be the volume of carbon dioxide? What will the latter weigh in grams? What will the water weigh?

Answers. (1) $71\frac{3}{7}$ liters of air. (2) Vol. of carbon dioxide, 12 liters. (3) Weight of CO_2, 23.57 grams. (4) Weight of water, 4.82 grams.

(*r*) A mixture of oxygen, carbon dioxide, carbon monoxide, and hydrogen, contains equal volumes of each gas. Find the analysis by weights.

(Note that the expressions for equal volume must be written with due regard to equal number of molecules, *i.e.*, O_2, CO_2, CO, and H_2.)

Ans.	Oxygen	30.19 per cent.
	Carbon dioxide	41.51 per cent.
	Carbon monoxide	26.41 per cent.
	Hydrogen	1.89 per cent.
		100.00 per cent.

(*s*) A mixture of gases has the following composition by weight:

Oxygen	59.02 per cent.
Hydrogen	4.92 per cent.
Carbon dioxide	36.06 per cent.
	100.00 per cent.

Deduce analysis by volume.

(Divide each percentage by either molecular weight or density, then reduce to volumes by percentage.)

Oxygen	36.0 per cent.
Hydrogen	48.0 per cent.
Carbon dioxide	16.0 per cent.—100.0 *Ans.*

(*t*) $$4KNO_3 + 5C = 2K_2CO_3 + 2N_2 + 3CO_2$$

If in this reaction four liters of nitrogen are evolved, what is weight in grams of the nitrate, and what is volume in liters of the dioxide?

Answers. Weight of nitrate, 36.07 g.; vol. of dioxide, 6 liters.

(*u*) Nitrogen tetroxide at 180° C. and upward is 1.57 times as heavy as air. (Air weighs 1.293 grams per liter.) What molecular weight does this show, and why is this supposed to indicate a different constitution from its greater density at less temperature?

Ans. Molecular weight would be $1.293 \times 1.57 \times 22.4 = 45.47$.

This is close to 46, theoretical molecular weight of NO_2.

At less temperatures, the greater density indicates approach to the double formula N_2O_4.

(*v*) $KClO_3 = KCl + O_3$. KCl remaining weighed 32.78 grams. Give weights of $KClO_3$ and oxygen, also volume of the latter.

Answers. $KClO_3$, 53.9 grams. Oxygen, 21.12 grams, 14.784 liters.

(*w*) A liter of ferric chloride vapor, reduced to 0° and 760 mm., weighs 14.56 grams. What is the molecular weight?

$$\text{Mol. wt.} : 22.4 = 14.56 : 1 \text{ (liter)}$$

Ans. 326+.

(*x*) $$P_2 + 5Cl_2 + 8H_2O = 10HCl + 2H_3PO_4$$

Using 20 grams phosphorus, what volume of chlorine at 15° C. was used? *Ans.* 38.114 liters.

What volume of HCl gas produced (at sight from last answer)? *Ans.* 76.228 liters.

Same equation: Liters of chlorine used, measured at 17° C. = 1.3433.

What was weight of the phosphorus? *Ans.* 0.7 gram.

(*y*) One liter each of hydrogen, methane, ethylene, acetylene, carbon monoxide, and phosphine, are burnt. State volumes of oxygen respectively necessary for complete combustion.

$2H_2 + O_2 = 2H_2O$	Oxygen = 0.5 liter.
$CH_4 + 2O_2 = CO_2 + 2H_2O$	Oxygen = 2 liters.
$C_2H_4 + 3O_2 = 2CO_2 + 2H_2O$	Oxygen = 3 liters.
$2C_2H_2 + 5O_2 = 4CO_2 + 2H_2O$	Oxygen = 2.5 liters.
$2CO + O_2 = 2CO_2$	Oxygen = 0.5 liter.
$2PH_3 + 4O_2 = P_2O_5 + 3H_2O$	Oxygen = 2 liters.

(This is a "sight" problem. The coefficients have been so adjusted that no rule of molecular symbolism is violated. We have only to remember that each symbol to the left indicates one liter, concretely, the oxygen appears at once.)

(*z*) $$NH_4NO_2 = 2H_2O + N_2$$

What volume of nitrogen from 7 grams of the ammonium nitrite?

$$64 : 22.4 = 7 : x = 2.45 \text{ liters}$$

(*a**) What does one liter of silicon chloroform weigh at 180° C? (Exact wts.)

$$SiHCl_3, \text{ molecular weight} = 135.758$$

$$\frac{V \times \text{m. w.}}{22.4} = 6.0606 \text{ grams, at } 0° \text{ C. } (180° \text{ C.} = 453° \text{ absolute.})$$

$$273 : 453 = 1 : 1.659$$

That is, one liter at 0° becomes 1.659 liters at 180° C. Finally:

$$1.659 : 1 = 6.0606 : 3.65 = \text{weight of one liter at } 180° \text{ C.}$$

(*b**) $$Br_2 + H_2S = S + 2HBr$$

217 grams of bromine dissolved in water, and sulphuretted hydrogen is passed into the solution. What volume of the gas at 15° C. is required? What would be the volume of the HBr, as gas, at the same temperature?

$$160 : 22.4 = 217 : 30.38$$
$$273 : 288 = 30.38 : 32.049 \text{ liters } H_2S \text{ at } 15° \text{ C.}$$

Read volume HBr at sight from data and first answer.

(*c**) $$10\ I_2 + 2P_2 + 8KI + 16H_2O = 4K_2HPO_4 + 28HI$$

We want 10 liters of gaseous hydrogen iodide (hydriodic acid) at 15° C. Find necessary weights of iodine, potassium and phosphorus.

In this as in all similar cases we must first get the volume of the gas at normal temperature and then calculate the equation.

$$288 : 273 = 10 : 9.479$$

In calculating the relation by gas volume, it makes no difference which of the solids in the first member we employ. Let us take the iodine.

$$2540 : 22.4 \times 28 = x : 9.479 \qquad x = 38.38 \text{ grams iodine}$$

We then get the others by simple ratio of the proper "weights," *i.e.*,

$$10\ I_2 : 2P_2 = 2540 : 124 = 38.38 : x = 1.87 \text{ phosphorus}$$
$$10\ I_2 : 8KI = 2540 : 1328 = 38.38 : y = 20.06 \text{ K iodide}$$

(*d**) $$2P_2 + 3KOH + 3H_2O = 3KH_2PO_2 + PH_3$$

This reaction evolved 143.5 liters of phosphine at 37° C. What weight of phosphorus was used? *Ans.* 70 grams.

(*e**) 135 c.c. gas at 17° C. and 600 mm. pressure, become 90 c.c. at 0° C. What is the pressure? *Ans.* 847 mm.

(*f**) 840 c.c. gas at 27° C. and 500 mm. become 720 c.c. at 760 mm. pressure. What is the temperature?

Ans. 117.9° C.

(*g**) 0.2726 liter air weighs 0.3 gram at 37° C. What is the pressure?

First reduce to 0° C.; this gives 0.2401 liter (weighing 0.3). Then:

$$0.2401 : 1 = 0.3 : 1.249 \text{ gram}$$

(Air at "normal" temperature and pressure weighs 1.293 grams per liter.)

$$1.293 : 1.249 = 760 : 734$$

Ans. Pressure = 734 mm.

(*h**) 100 liters of air at 760 mm. weigh 124 grams. What is the temperature? (Densities are inversely as temperatures.)

$$124 : 129.3 = 273 : 284.7 \text{ (absolute)}$$

Ans. 11.7° C.

(*i**) At what temperature does one liter of air weigh one gram? *Ans.* 80° C. (nearly)

(*j**) $$CaCO_3 + 2HCl = CaCl_2 + H_2O + CO_2$$

This yields 20 liters of CO_2 gas at 18° C. and 740 mm. What weight of the calcic carbonate was taken? The 20 liters make 18.27 at "normal," then:

$$100 : 22.4 = x : 18.27 \qquad x = 81.56 \text{ grams}$$

N. B.—Some of the above problems indicated by * involve Boyle's and Charles' laws, and should be passed over until the latter have been explained.

Density of Gases Compared with Air as Unity.

This method of stating gas density is now little used. By fixing the weight of a liter of air in the memory, also the density of air on the scale $O_2 = 32$, any desired transformation may be readily made.

Weight of one liter of air = 1.293 gram (at 0° C. and 760 mm.).

Density of air, on scale $O_2 = 32$, is 28.95.

Given density of a gas on scale air = 1, the weight of one liter of the gas is found by multiplying density by 1.293.

Given density of a gas on scale $O_2 = 32$, the weight of one liter is found as in the above rules, by dividing density by 22.4. (More closely, 22.38.)

Given density on scale air = 1 to find density on scale $O_2 = 32$, multiply by 28.95.

Given density on scale $O_2 = 32$ to find density on scale air = 1, divide by 28.95.

Examples.—1. Nitrogen referred to air has density 0.967. What is the weight of one liter?

Ans. $0.967 \times 1.293 = 1.250$ gram.

2. Carbon dioxide referred to $O_2 = 32$, has density 44. What is weight of one liter? *Ans.* $44 \div 22.4 = 1.964$ gram.

3. Ammonia (air = 1) has density 0.588. What is its density on scale $O_2 = 32$? *Ans.* $0.588 \times 28.95 = 17.02$.

4. CO gas ($O_2 = 32$) has density of 28. What is its density on scale of air = 1? *Ans.* $28 \div 28.95 = 0.967$.

Charles' Law.

"The volume of a mass of gas varies directly as the absolute temperature to which it is exposed."

Problem VIII.—Given the volume of a stated mass of gas, at a certain temperature, to find its volume under a given change of temperature.

It was long ago observed that if a volume of gas *at the temperature of zero Centigrade*, were subjected to changes of temperature, *pressure remaining constant*, it changed its volume *one two hundred and seventy-third of its volume at zero for each degree of temperature.*

For example, assuming 273 volumes at zero, if the temperature be raised to 8°, the volume will become 281. Similarly, if the temperature drops to −10°, the volume contracts to 263.

This fact suggested the "absolute zero" convention. On this thermometric scale "zero" is placed at a point 273° below the Centigrade zero, *the size of the degree remaining the same.*

Ice melts at 273° "absolute." Water boils at 373°. In short, the *interval in degrees* between any two points of observed temperature remains unchanged.

Thus, instead of saying (as we have to in using the Centigrade scale) that "a gas expands or contracts one two hundred and seventy-third of its volume at zero for each change of one degree in temperature," we reduce the terminology to the simplicity of the law, by saying merely that gases expand or contract "directly as the temperature."

In all questions, then, involving new gas volumes under changed temperature, we first convert Centigrade into "absolute." This is easy, as it consists in simply adding 273 to the Centigrade degrees.

65° C. = 338° Abs. 0° C. = 273° Abs. −40° C. = 233° Abs., etc.

Hence rule for the solution of Problem VIII.

"First convert temperature into degrees absolute, then write the proportion, placing volumes and degrees in equalized ratio."

Any of the four quantities involved may be the unknown, *i.e.*, present volume or new volume of the gas, present or new temperature to which it may be subjected.

Example.—Volume of gas at 4° C. is 100, what will it be at 281° C.?

Add 273 to each of the Centigrade temperatures, making them "absolute."

$4 + 273 = 277$ for the first. $281 + 273 = 554$ for the second.

$$T : T' = V : V'$$
$$277 : 554 = 100 : 200$$

Ans. New vol. = 200.

(*b*) Same data inverted. Starting from 4° C. what temperature is required to double the volume of the gas?

$$4 + 273 = 277. \quad \text{And } 277 \times 2 = 554 \text{ (absolute)}$$

If the answer is to be in Centigrade degrees, then, $554 - 273 = 281°$ C.

(*c*) What temperature will raise 100 vols. at 15° C. to 140 vols.?

First: $15 + 273 = 288$ "absolute" degrees corresponding to 15° C.

$$100 : 140 = 288 : 403.2 \text{ (Abs.)} \qquad 403.2 - 273 = 130.2° \text{ (C.)}$$

(*d*) A liter of oxygen weighs, at 0° C., 16. What will it weigh at 91° C.?

$$273 : 364 = 1 : 1\tfrac{1}{3}$$

The mass of course has not changed, so one liter of the gas at new temperature, being $\frac{3}{4}$ of $1\frac{1}{3}$, the weight of a liter of oxygen at the new temperature will be $\frac{3}{4}$ of 16, or twelve. *Ans.* 12.

(What is the unit of weight, may be asked. It matters not.)

Throughout this section we are speaking of change in temperature under constant pressure.

(*e*)

If	volume of gas at	0° Centigrade	= 1000
Then	volume of gas at	10° Centigrade	= 1036
	volume of gas at	20° Centigrade	= 1073
	volume of gas at	30° Centigrade	= 1110
	volume of gas at	40° Centigrade	= 1146
	volume of gas at	50° Centigrade	= 1183
	volume of gas at	60° Centigrade	= 1220
	volume of gas at	70° Centigrade	= 1256
	volume of gas at	80° Centigrade	= 1293
	volume of gas at	90° Centigrade	= 1330
	volume of gas at	100° Centigrade	= 1336

Generally, any continuation of this table may be obtained by:

$$\frac{1000 \times (273 + x)}{273}$$

For example, required volume at 30° we have 1000 × (273 + 30) = 303,000.

And $\frac{303{,}000}{273} = 1{,}110$ as in the table.

This merely illustrates the "273" fact, without direct resort to the "absolute" zero convention.

By taking volumes or temperatures from various places in the table the student may construct any number of little problems to be used as check work. They should be done by the "absolute" temperature method.

Examples are not extended, as many are scattered through the text.

Boyle's Law.

"The volume of a mass of gas varies inversely as the pressure on it."

Problem IX.—Given the volume of a certain mass of gas, to find the volume under a given change of pressure.

Pressure is usually expressed in barometric millimeters.

Since the volume is inversely as the pressure, if we designate the given volume as V, and the given pressure as P, while the new volume is V' and the new pressure is P', the proportion is

$$V : V' = P' : P$$

Rule.—Place volumes and corresponding pressures in inverse ratio.

(We use inches in barometric readings in the first examples.)

Examples.—(*a*) Volume of gas at 30 inches barometer is 1. What will it be at 24 inches?

$$V : V' = P' : P$$

$$1 : x = 24 : 30 \qquad \textit{Ans. } x = 1.25.$$

(*b*) Volume at 15 inches being 75, at what pressure will volume become 60?

$$75 : 60 = x : 15 \qquad \textit{Ans. } x = 18.75.$$

(*c*) Find volume of gas, now at 30 inches barom., which at 12 inches had volume 3750.

$$x : 3750 = 12 : 30 \qquad \textit{Ans. } x = 1500.$$

(*d*) Volume of gas is now 30. It was 75 at 22 inches, what is now the pressure?

$$30 : 75 = 22 : x \qquad \textit{Ans. } x = 55.$$

Or, generally,

$$a : x = b : c \text{ and } x = \frac{a \times c}{b}$$

Any quantity may be the unknown, as under last rule.

Millimeters are used in the following example.

(*e*) A closed vessel, *displacing one liter** is tared, barometer being at 760 mm. Barometer falls to 710 mm.

What weight will now restore the equilibrium, and on which side of the balance will it go? (Temperature constant at 0° C.)

Note.—1 liter of air under "normal" conditions, *i.e.*, 760 mm. pressure and 0° C. temperature, weighs 1.293 gram.

$$760 : 710 = 1.293 : 1.208$$

that is, a liter of air, pressure 710, weighs 1.208 gram.

$$1.293 - 1.208 = 0.085 \text{ (85 milligrams)}$$

The air that is being "weighed" here is outside of the vessel. It is, in fact, the volume of air which the vessel "displaces." For the vessel being closed, the weight of its contents has not changed.

But the apparent weight of the vessel increases, because the medium surrounding it has become lighter. The proportionate weight of the atmosphere (density) is a function of the pressure.

The weight, then, must be placed on the scale pan *opposite* the vessel.

* This indicates that the *exterior* size of the vessel is equal to one liter, instead of its contents. Hence, as it stands on the balance, the vessel may be said to "displace" one liter of the atmosphere which surrounds it.

Several similar cases will be found among the "Miscellaneous Problems" at the end of the text.

Laws of Charles and Boyle Combined.

It is rare, in the application of the rules for changes in volume, that we do not have to apply them for both temperature and pressure, in other words, for variations in both thermometer and barometer.

Ordinarily these rules are used separately, *i.e.*, the change of volume is figured from one change only, the result is then taken as an initial volume, and its new volume figured by the application of the rule for the other physical variation. It makes not the slightest difference whether the correction be applied first for pressure and then for temperature, or in the reverse order. This is too "rationally" obvious to demand any proof.

Example.—(*a*) 320 cubic centimeters of gas, pressure 950 mm., temperature 91° C., to be reduced to volume at 760 mm. pressure and 0° Centigrade.

First apply the rule for temperature (Charles).

$$91 + 273 = 364. \quad 0 + 273 = 273. \quad \text{(Abs. degrees, 364 and 273.)}$$
$$364 : 273 = 320 : x \qquad x = 240$$

Now take this volume, 240 c.c, and treat it as initial volume. Compute its change in passing from 950 to 760 mm. pressure (Boyle).

760 : 950 = 240 : 300 *Ans. 300 cubic centimeters*
new : old = old : new. "Vols. *inversely* as pressures"

(*b*) 100 vols. at 27° C. and 720 mm.

What vol. at 42° C. and 756 mm.? (Begin with rule for pressure.)

756 : 720 = 100 : 95.23895 (vol. under changed pressure)

Absolute degrees are 27 + 273 = 300, and 42 + 273 = 315.

300 : 315 = 95.2389 : 100 *Ans. No change in volume.*

Formulistic Combination of Charles' and Boyle's Laws.

We have shown how to apply the formulæ separately, getting correction for both temperature and pressure. It is quite easy, however, to apply a single formula for the double correction, although little is gained by so doing.

Suppose V, P, T to represent the original volume, pressure and temperature of a certain mass of gas. Let v, p, t represent

the volume, pressure and temperature after both computations have been made, *i.e.*, after allowing for changes in *both* temperature and pressure.

Let us first go through the "successive" operations, beginning with the change in temperature, seeking the new volume *under this change alone.*

We have, of course:

$$T : t = V : x$$
$$x = \frac{Vt}{T}$$

This is the volume of gas under change of *temperature* alone, and it is this volume that we are now to submit to computation for change in *pressure*. Under this final change, remember that the volume is assumed as "v," the final pressure being "p." By the rule as given, substituting value of "x":

$$p : P = \frac{Vt}{T} : v \qquad \text{Hence, } v = \frac{VPt}{Tp}$$

We may regard any of the quantities v, t, p, as unknown, and several of the problems hereafter given are so constructed. We shall have:

$$v = \frac{VPt}{Tp} \qquad t = \frac{Tvp}{VP} \qquad p = \frac{VPt}{Tv}$$

This formula, however, does not diminish the arithmetical work. Looking, *e.g.*, at the fractional value of "v," we see in it all of the operations of the "successive" applications of the corrections for both physical changes.

Any of these expressions may be readily reduced to the general form:

$$\frac{vp}{t} = \frac{VP}{T}$$

This is a good "mnemonic" form, on account of its perfect symmetry. Value of any of the factors in terms of the others can be derived from it by a simple transformation.

Examples.—(*a*) 120 c.c. of gas at 0° C., and 740 mm. pressure, become 125.4 c.c. at temperature of 20° C. What is the new pressure?

(Remember that the temperatures are 273° and 293° "absolute.")

$V=120$. $v=125.4$. $T=273$. $t=293$. $P=740$. Find "p."

$$p = \frac{VPt}{Tv}$$

$$(120 \times 740 \times 293) \div (273 \times 125.4) = 760$$

Ans. 760 mm.

Vary this and the following problems by changing about the data, making first one and then another the "unknown."

(*b*) A volume of gas at 60° C. and 30 inches barometer has volume of 109.57 when temperature is 5° C., pressure 24 inches. What original volume?

NOTE.—Take the latter three figures as V, P, T, and the first two as t, p. This avoids transformation of the formula.

$V=109.57$. $T=278$. $t=333$. $P=24$. $p=30$. Find "v."

$$v = \frac{VPt}{Tp}. \quad (109.57 \times 24 \times 333) \div (278 \times 30) = 105 \textit{ original vol.}$$

(*c*) One liter of air at 30 in. barometer and 136.5° C. becomes what volume at 20 in. bar., and 0° C.? (Absolute temperatures, 409.5 and 273.)

Formula as in (*b*):

$$(1 \times 30 \times 273) \div (409.5 \times 20) = \frac{8190}{8190} = \text{unity}$$

Ans. No change in volume.

The student is recommended to abandon the strictly "formulistic" solution of these cases when he has become thoroughly accustomed to the logic. There is less liability to error when the "rationale" is followed than when a formula is blindly applied. Treat the whole matter as a case of multiplication by fractions, *e.g.*:

A liter of gas at 7° and 740 mm. has what volume at 0° and 760 mm.?

$$1 \times \frac{273}{280} \times \frac{740}{760} = 0.9494$$

We know that the less temperature will *decrease* the volume, so we put the temperatures into a "proper" fraction. The greater pressure will also decrease the volume, so we put pressures into a proper fraction.

Evidently all the facts and formulæ are involved in this simple procedure. Experience has shown it to be "safer" also.

Examples.—(*a*) 0.700 liters at 27° and 1000 mm. have what volume at 0° and 760 mm.?

$$0.700 \times \frac{273}{300} \times \frac{1000}{760} = 0.838$$

(*b*) We have 1.110 liters at 0° and 760 mm., at what pressure will it reduce to a liter?

$$p = 760 \times 1.110 = 843.6$$

(*c*) We have 0.750 liter at 27° and 700 mm., at what temperature will the volume become one liter?

$$\frac{300}{.75} = 400 \text{ (\textit{i.e.}, 127° Centigrade)}$$

In fact, the blunder against which we chiefly warn is forgetting to transform centigrade into absolute degrees, for the operation, and back again, in the answer. (Add in the first case, subtract in the second.)

WEIGHTS OF GASES UNDER VARIATIONS IN TEMPERATURE AND PRESSURE.

Problems involving weight of any given volume of gas must include in the data, the weight of a liter of that gas under "normal" conditions. Variations due to temperature and pressure are the same for all gases, but *density* is a special factor for each gas. Carefully avoid the fallacy of changed weight for the *same mass* of gas, a precaution not so absurd as it looks.

It will aid the solution especially by short cut method, to remember that *density is directly as pressure.* If a liter of gas weighs a gram, then at double the pressure it will occupy volume of half a liter, and its density will be doubled. A liter at the new pressure would weigh two grams.

Examples.—(*a*) What will a liter of nitrogen weigh at 30° and 640 mm.? (A liter at 0° and 760 mm. weighs 1.25 grams.)

Reduce first to "normal" conditions:

$$1 \times \frac{273}{303} \times \frac{64}{76} = 0.7587 \text{ liter}$$

Now we have only to multiply by the "normal" weight per liter. For the mass remains the same. $0.7587 \times 1.25 = 0.9484$ weight of the gas.

(*b*) Suppose some other than volume to be the unknown, *e.g.*, a liter of air at 17° C. weighs 1.180, what is the pressure?

(A liter of air weighs 1.293 under "normal" conditions.)

Let us do this "rationally" first, and then use the formula. The volume of this air (pressure unchanged) at 0° is $\frac{273}{290}$ or 0.9414 liter: its weight is unchanged, but a liter *under the same conditions* would weigh 1.253, that is, $\frac{1.18}{.9414}$, so that the problem has now become a simple proportion:

$$1.293 : 1.253 = 760 : p. \qquad (p = 736+)$$

The factors which have been used for this result being now assembled, we see that

$$p = \frac{290 \times 760 \times 1.18}{273 \times 1.293}$$

If now P, T, and W indicate "normal" data, and p, t, w the new data (W and w standing for weights), we have:

$$p = \frac{tPw}{TW}$$

W being weight of one liter under "normal" conditions.

Compare with formulistic value of p in previous article, where we had

$$p = \frac{VPt}{Tv}$$

"V" and "v" are no longer present, but we have reciprocal factors of weight-for-volume (density). Being reciprocal, they take respectively reverse places in the fraction, density corresponding to V appears in the denominator, while that corresponding to v is now in the numerator.

The student will easily learn to use these data without formal substitution. The remarks already made as to a "sense of magnitude" apply here. Errors creep into computations of every kind, but they will be multiplied if the worker has not with him a corrective sense which raps his brain at a false move.

The following two examples and their inversions are carried out in detail and without formulæ, as type cases.

(*c*) What is the weight of one liter of ammonia gas, at 30° C. and 900 mm.?

The normal weight of the gas per liter (0° and 760 mm.) is 0.76. First we find weight at 0° of 1 liter at 900 mm. pressure:

$$760 : 900 = 0.76 : 0.90 = \text{wt. of 1 liter at 900 mm.}$$

Next apply correction for temperature:

$$273 : 303 = x : 1.11 = \text{vol. at 30° and 900 mm.}$$

This 1.11 is merely a suppositious volume, *i.e.*, *if* we had a liter at 900 mm. and 0°, it would have volume of 1.11 at 30°. But by the data we have vol. of 1 liter under the conditions named, hence the proportion.

$$1.11 : 1 = .9 : x \qquad \textit{Ans.}\ x = 0.8109 \text{ gram.}$$

By inverting the problem, we get both a new problem and a check on the above, viz:

(*d*) We have 0.8109 gram of ammonia gas, what will be its volume at 30° C. and 900 mm.?

First find volume of this weight at "normal" condition.

$$.76 : .8109 = 1 : x \qquad x = 1.067 \text{ liter.}$$

Apply temperature.

$$273 : 303 = 1.067 : 1.1842 = \text{vol. at 30°}$$

Apply pressure.

$$900 : 760 = 1.1842 : x \qquad x = 1 \text{ liter.}$$

(*e*) Pressure on 1 liter of HCl gas is 1000 mm., and its weight is 1.8 gram. What is the temperature C.? (Normally, 1 liter HCl weighs 1.628 gram.)

$$760 : 1000 = 1 : x \qquad x = 1.316$$
$$1.628 : 1.8 = 1 : 1.1057, \text{ and } 1.1057 : 1.316 = 273 : 325$$

Subtract 273 from this "absolute" temperature.

$$\textit{Ans.}\ 325 - 273 = 52° \text{ C.}$$

(*f*) Inversion and proof of (*e*). One liter of HCl being at 52° C., and 1000 mm. pressure, what is its weight?

$$760 : 1000 = 1 : 1.316 = \text{vol. at 760 mm. and 52° C.}$$
$$325 : 273 = 1.316 : 1.1057 = \text{vol. at 0° C.}$$
$$V : v = W : w, \text{ that is, } 1 : 1.1057 = 1.628 : 1.8 \quad \text{Ans.}$$

Some of the steps are purposely left without explanation.