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CHEMICAL ARITHMETIC.

SOME FUNDAMENTAL LAWS OF CHEMISTRY.

Conservation of Mass.—Matter is not destroyed by any chemical action; its mass remains constant, whatever its change of physical condition.

Definite Proportions.—In every chemical compound the constituents occur in definite unalterable proportions by weight.

Multiple Proportions.—When two elements unite in several proportions, and we regard fixed weights of one element, the quantities of the second element will bear a simple ratio to each other.

Avogadro's Rule.—The number of molecules in a unit volume of all gases is the same, temperature and pressure being equalized.

Gay-Lussac's Law.—When gases unite, the volumes both of the constituent parts and the product bear to each other some simple ratio.

Boyle's Law.—The volume of a mass of gas varies inversely as the pressure on it.

Charles' Law.*—The volume of a gas varies directly as the absolute temperature to which it is exposed.

Dulong and Petit's Law.—The product of the atomic weight of an element by its specific heat is a constant, substantially the same for all the elements.

*Gay-Lussac's name is retained for the law of gas union. Charles' name is used more for distinction than as a matter of history; probably Gay-Lussac made the first definite enunciation of the law.

DEFINITIONS.

An atom is the least mass of an element that can take part in a chemical reaction.*

A molecule is a particle of matter representing the limit of divisibility so far as similarity goes. ("Physical identity" limit.)* (A molecule may be either simple or compound. There are probably monatomic molecules. It is not necessary to connect the idea of "indivisibility" with the chemical atom.)

Atomic Weights.—The proportions in which or in simple multiples of which the elements unite among themselves.*

Molecular Weight.—Sum of all the atomic weights contained in a molecule.*

Specific heat of a substance is ratio of the quantity of heat required to raise a given weight of it one degree, to quantity required to raise an equal weight of water one degree. (Practically, the specific heat of water being assumed as unity, the specific heats of all other elements are represented by numbers bearing the proper ratios to unity as deduced from experimental data.)

Atomic heat of an element is the product of its atomic weight by its specific heat. Under the supposition naturally arising from Dulong and Petit's law, all atoms should have the same capacity for heat.

Specific Gravity.—Weight of a substance as compared with the weight of a standard substance of equal volume. (Illustrations of this property will be found in detail under the proper heading.)

Density.—This term is reserved for the specific gravity of gases, with $O_2 = 32$ as basis of comparison. This brings molecular weight and density of a gas to the same figures, instead of the 2 : 1 ratio.

Atomic Volume.—Quotient of atomic weight divided by specific gravity. Since (see "Specific Gravity")

$$V = \frac{W}{G}$$

*These are definitions sufficient for "stoichiometric" calculations.

then on the assumption that the atomic weights are the *relative* weights of atoms, their *relative volumes* will be indicated by

$$\text{at. vol.} = \frac{\text{at. wt.}}{\text{sp. gr.}}$$

If atomic weight be taken *in grams* then atomic volume will indicate volume in c.c. *Example.*—Lithium, at. wt. = 7.00, sp. gr. = 0.59, at. vol. = 11.86. Then 7.00 grams will occupy 11.86 c.c. in volume.

Absolute Temperature.—Temperature measured on a thermometer whose "zero" is fixed at *minus* 273° Centigrade. The degrees themselves are of the same value as degrees Centigrade, *i.e.*, the interval between degrees indicates the same difference in temperature. (See section on "Thermometers," and "Charles' Law.")

Pressure.—Pressure in chemical and physical experiments is almost invariably measured by the barometer. In English measures the barometric column is read in inches, and 30 inches is usually taken as the "normal" pressure. In barometers divided by metric linear units, the readings are in millimeters. "Normal" is taken as 760 millimeters. The two do not quite correspond, as 30 inches = 762 millimeters, absolutely, 761.99 mm.

Many laws and definitions are omitted in this summary, because we shall not extend our treatment of chemical problems into their domain.

PRELIMINARY REMARKS.

"The molecular weight of a compound is the sum of the atomic weights of its elements." This may be taken as an independent law, or, assuming the truth of the atomic theory, as a necessary sequence.

In "manganese pyro-phosphate" formula is $Mn_2 P_2 O_7$. Knowing atomic weights of the elements we deduce "molecular" weight of the compound, thus:

ManganeseAtomic weight of manganese = 55...	$Mn_2 = 110$
PhosphorusAtomic weight of phosphorus = 31...	$P_2 = 62$
OxygenAtomic weight of oxygen = 16...	$O_7 = 112$
Sum, or "molecular" weight of $Mn_2 P_2 O_7$	= 284

In using chemical equations, always "balance" them, *i.e.*, see that atoms (symbols) of an element on one side equal those of same element on the other side.

A common blunder in calculating weight of a compound from given weight of another one is to forget the doubling of a symbol or some other change involved in the change of formula.

Example.—What weight of K_2O can be derived from one gram of KCl ?

The student well used to the arithmetical handling of formulæ will see at once that the weight of the K_2O must be less than that of the KCl . If a novice he is liable to the blunder of stating his proportion thus:

$$KCl : K_2O = 74.5 : 94 = 1 : 1.26$$

not seeing that he has "created" an atom of potassium and half an atom (!) of oxygen. The true solution requires the equalization of the atoms at the start; for instance, call the " KCl ," " $2KCl$," then we get the true proportion:

$$2KCl : K_2O = 149 : 94 = 1 : 0.63$$

For only one molecule of K_2O can be derived from two molecules of KCl .

Similarly, if the percentage of MgO should be required in $Mg_2P_2O_7$, we should calculate for " $2MgO$," not for " MgO ,"

The ratio between the formulæ of "sought" and "found" is sometimes less simple than in the above examples.

Suppose that we have the weight of uranium phosphate, $(UO_2)_2P_2O_7$. It is required to find the factor by which this weight must be multiplied to obtain the corresponding weight of the oxide U_3O_8 .*

Direct comparison of the molecular weights will not serve, because the formulæ show, respectively, two and three atoms of uranium. If, however, we multiply the molar weight of the phosphate by three, and that of the oxide by two, we shall have in each product six atoms of uranium, or two molecules of the oxide, U_3O_8 . In other words, three molecules of the phosphate are shown to contain two molecules of the required oxide. By division under the rules to be given in the text we now readily deduce the factor 0.7865.

The careless use of symbols is an error to which many are liable. It is erroneous to write the symbol as a mere abbreviation of the name, convenient though it may often seem. *The symbol carries with it, understood, the atomic weight, and it stands for one atom.*

Some further mention of this will be found in the metallurgical part of the work. Generally the use of the symbol should be discouraged except in such connections that it expresses exact relations as denoted by its numerical value.

The reader unaccustomed to the metric system should attack it at once. If he imagines that it presents difficulties because it is not what he is used to, he is reminded that he has figured all his life in ordinary numeration and in U. S. money, which are decimal, as is the "metric" system.

Much of the mistaken notion as to the difficulties of the system may be traced to the prevalent methods of instruction concerning it, especially in the Public Schools, where it is overloaded with nomenclature, obscured by comparison with the ordinary system and tables, and, in short, made as difficult for the struggling infant as could be desired by his worst enemy. See, however, under the section devoted to this topic for details.

Every person should carry with him a "sense of magnitude" for the correction of gross errors. Of these the most common is

*Weight of U_3O_8 that can be actually obtained from the "known" or "given" weight of $(UO_2)_2P_2O_7$.

the misplacing of the decimal point. This, however, can be only "learned," not "taught." A person who could work the proportion $3 : 6 = 0.15 : x$, and find x equal to 3, instead of 0.3, may have merely made a slip in the application of the rule, and that is excusable. However, not to see the error by "instinct" would be indeed unpardonable. The retention in the mind of the "order of magnitude" for the answer to a simple proportion is an easy habit to acquire, and may save many a bothersome error. Take the case above. The computer should perceive as he sets down his figures that the answer must be less than unity.

Under the definitions above of "atom" and "molecule," also of atomic and molecular "weights," the reader trained in chemical theory will not fail to note the defects of these provisional definitions. They are nevertheless retained, for the reason indicated in the footnote, viz: they suffice for the purposes of stoichiometric calculations.

Strictly (using the old convention of unity for the atomic weight of hydrogen) the "atomic weight" of an element is the number of times which the mass of the least portion of it that can take part in a chemical reaction is greater than the least portion of hydrogen that can take part in a chemical reaction. The "molecular weight" of any substance (elementary or compound) would then be the number of times that the mass of the smallest portion of it that can exist in the free state, is heavier than the atom of hydrogen (or "the least portion of hydrogen that can take part," etc.).

Many of the explanations in the text will strike the expert as puerile. They are meant to be. Nothing more astonishes the writer than the inability of many technical men to attack the simplest chemical problems. Graduates in chemistry, with mathematics enough to compute bridge strains, turn pale at the idea of "tackling" an elementary case of stoichiometry. "Please assume, to begin with, that I have forgotten all that, if I ever knew it," would be a fair average estimate of their attitude. Excepting, then, the already excepted "fundamenta," all else is explained where there seemed to be the slightest chance for misapprehension.

THERMOMETERS.

Two thermometric scales are in common use, the "Centigrade" and the "Fahrenheit." The "Absolute" scale to be presently explained is convenient for the calculation of gas volume variations under heat, but has no other usual application.

Zero of the Centigrade scale is fixed at the melting point of ice. The boiling point of water (at 760 mm. pressure) fixes the 100° point.

Zero of the Fahrenheit scale is fixed at 32° (F.) below the melting point of ice. The boiling point of water is at 212°.

Thus between temperatures of melting ice and boiling water we have 100 degrees Centigrade, and 180 degrees Fahrenheit.

Since 100 degrees C. are equal to 180 degrees F., $1^\circ \text{C.} = 1.8^\circ \text{F.}$, which is usually expressed by saying that one degree C. equals nine-fifths of a degree of F., or conversely one degree F. equals five-ninths of a degree C.

To Convert Fahrenheit Degrees into Corresponding Centigrade.—"Subtract 32 and take five-ninths of the remainder."

Example.—What degree C. corresponds to 239° F.? $239 - 32 = 207$. Five-ninths of 207 is 115.—*Answer.*

To Convert Centigrade Degrees into Corresponding Fahrenheit.—"Take nine-fifths of the C. degrees and then add 32."

Example.—What degree F. corresponds to -10°C. ? Nine-fifths of -10 is -18 . $-18 + 32 = 14$.—*Answer.*

NOTE.—In starting from F. make the 32° correction first. In ending with it, make it last. This is merely mnemonic, but is sometimes useful.

Examples in Conversion of Degrees from one Thermometric Scale to Another.—(a) What temperature is expressed by the same number of degrees in either Fahrenheit or Centigrade scale?

Solution. $\frac{5}{9}(x - 32) = \frac{3}{5}x + 32$. *Ans.*, $x = -40$.

(b)	275° F =	135° C.	(i)	0° C. =	+ 32° F.
(c)	23° F =	- 5° C.	(j)	100° C. =	+ 212° F.
(d)	0° F =	- 17.78° C.	(k)	- 10° C. =	+ 14° F.
(e)	- 40° F =	- 40° C.	(l)	- 40° C. =	- 40° F.
(f)	- 112° F =	- 80° C.	(m)	35° C. =	95° F.
(g)	212° F =	100° C.	(n)	- 71° C. =	- 95.8° F.
(h)	73° F =	22.78° C.	(o)	- 16° C. =	+ 3.2° F.

"Absolute Scale."—To pass from Centigrade to "absolute" add 273. To pass from "absolute" to Centigrade, subtract 273.

Zero degrees "absolute" equals *minus* 273° Centigrade.

Zero degrees "absolute" equals *minus* 459.4° Fahrenheit.

It must not be forgotten that the "degree" is the same in the Centigrade and "absolute" scales, that is to say, the size or amount of difference in temperature indicated by any number of degrees is identical.

Hence, should it become necessary to convert Fahrenheit into "absolute," or the reverse, *first convert into Centigrade*, and then proceed.

THE METRIC SYSTEM.

We have assumed that any student in a technical school who may use this work is familiar with the metric system.

There are many, however, whose acquaintance with it is slight and to whom, because they were never rationally taught, it seems not only difficult but even "unnecessary."

For those who may now for the first time meet the system, a few observations are appended. They are intended less as expository than as hints for the best method of acquisition. Indeed, they are largely negative—"how *not* to attack the subject."

First: Do *not* bother about the relation of the metric system to the common system of weights and measures. That relation has nothing to do with any calculation. One might in fact make perfectly correct calculations in the metric or any other system without having the slightest idea of the magnitude of the concrete units figured upon.

Practise, then, with the aid of the examples given; do not ask, "What are the equivalent magnitudes expressed in every-day units?"

If it be desired to translate results in one system into the units of the other, it becomes a mere matter of reference to the tables showing the relation. That, however, is simply the application of a factor. In time, as one becomes accustomed to refer to both systems, some mental approximation will present itself without effort. But in the first acquisition, surely it is useless and confusing to stop at every step to consider what the numbers you are using may mean in another set of magnitudes.

Second: Do *not* try to memorize the tables. Especially, do *not* try to learn *all* the names of the multiples and submultiples of the various units. The names are logical enough, and any one using the system will soon find that they will come naturally to him, if he wants them. But in truth they are very largely unnecessary in practice. The reason is that, after all, the subdivisions are nothing more than tenths, hundredths and thousandths, or the multiples are ten times, a hundred or a thousand

times. So, having the names of the *units* of length, volume or weight, the "names" of the subdivisions are of exceedingly little importance.

It is true that certain submultiples are so frequently used that it is *convenient* to acquire and use their names. But the really important thing is to know the nature of the system, and it would be perfectly easy to use it with no names for the subdivisions except the inevitable ones of tenths, etc.

Third: If the student (in school or out of it) will get it fixed in his head that he has *been using the metric system all his life*, that will be the last of any "difficulty" in the case. Now we are addressing some one who has little or no knowledge of the system in practice, and we adopt the familiar form of remark and answer. Remarks in italics, answers in Roman.

"*You understand already how to compute in the metric system.*"

"How so?"

"*Because you have never computed in any other system in pure numbers, or in United States money. Because the "method" of calculation in the metric system is absolutely and entirely the same as calculation in ordinary numbers and their decimals. Here is a problem for you:*

"Add 3.52 to 9.67."

"Why, 13.19, of course."

"*Correct. It was a problem in the metric system and you have solved it.*"

"But in what denomination? Weight, length or volume?"

"*It matters not. They all subdivide decimally. Add, subtract, multiply, divide, equate, or proportionate, they are treated just the same as pure numbers.*"

To this statement we add for the beginner the relations of length to volume, and of the latter to weight, which indeed constitute the greatest beauty as well as simplicity of the system.

We shall now give a tabular statement of the system, accompanied by a few explanatory remarks.

Tables of ordinary concrete numbers are also given, and "conversion" factors from each system to the other.

Here we again warn the learner that he is not to perplex himself with these relations. *They are figures for reference.*

Under the head of problems in gas volumes are methods of transition from the metric system to the ordinary.

Verification of the problems under this head (metric system) will go far to render all the relations of the various tables of the system quite easy. Nothing, however, can make it half as easy as to actually do work with the linear, volumetric, and gravimetric units themselves.

METRIC SYSTEM, TABULATED

Length.—Unit, the METER (m.).

SUBDIVISIONS.		MULTIPLES.	
0.1 meter = 1 decimeter (dm.).	10 meters = 1 decameter.	10 meters = 1 decameter.	
0.01 meter = 1 centimeter (cm.).	100 meters = 1 hectometer.	100 meters = 1 hectometer.	
0.001 meter = 1 millimeter (mm.).	1000 meters = 1 kilometer.	1000 meters = 1 kilometer.	

The meter is properly the basis of the whole metric system. It may assist the learner to remember that it is very little over a yard in length.

In reading numbers in the system, remember that it is not usual, and certainly not necessary, to name each subdivision. In reading United States money we name only dollars and cents, e.g., \$315.28, three hundred and fifteen dollars and twenty-eight cents.

Suppose we are reckoning in meters. Take the number 6.139 m. We read it just as we would a number, probably "six (point), one, three, nine, meters." It is true that some unhappy children are still taught to say "six meters, one decimeter, etc." That, however, is merely a case of the illegal survival of infant torture.

Since the system is purely decimal, its divisions, as inculcated above, are treated in all respects, in calculation, like pure numbers.

Although in naming a compound quantity like the above we do not use the subdivisional names, yet the names centimeter and millimeter are familiar in the laboratory in naming lengths less than a meter. The name kilometer is becoming familiar in the United States. It is close to five-eighths of one mile.

Area.—The intended unit is the ARE or one hundred square meters. Being too large for laboratory use and too small for land measurement, it is not much heard of. In land measurement, 100 "ares" or one "hectare" are chiefly used. The "hectare" is about two and one-half acres.

As all areal measurements (metric or others) go upon squares for their bases, it is evident that as the linear dimensions are multiplied by 10, the areas are multiplied by 100.

100 square millimeters (smm.)	= 1 square centimeter (scm.).
100 square centimeters	= 1 square decimeter (sdm.).
100 square decimeters	= 1 square meter (sm.).
100 square meters	= 1 ARE.
100 ares	= 1 hectare.

We have very little to do with areas in the present work.

Volume.—All volumes are considered as cubes, whose edges are the standard length measures. Unit of volume, the LITER (the cube whose edge is one-tenth of a meter*).

1000 cubic millimeters (cmm.)	= 1 cubic centimeter (c.c.).
1000 cubic centimeters	= 1 LITER (cubic decimeter, edm.).
1000 liters	= 1 cubic meter ("Stere").

The liter happens to be very close to a quart. We shall use the common laboratory abbreviation, c.c., for cubic centimeter.

Weight.—Weights are based upon standard volumes of water at 4° Centigrade (maximum density). Unit, the GRAM. A gram is the weight of a cubic centimeter of water under above conditions. It is a convenient laboratory unit. Commercially, the KILOGRAM ("kilo") becomes the practical unit. The latter may be remembered as about two pounds avdp.

1000 milligrams	= 100 centigrams	= 10 decigrams	= 1 GRAM.
1000 grams	= 100 decagrams	= 10 hectograms	= 1 kilogram.

We have also, 1000 kilograms = 1 metric ton (tonne).

Considering the volumes above, we see that as the edge of a cube is multiplied by ten, its volume is multiplied by 1000 ($10^3 = 1000$).

As already mentioned in foot-note under volumes, the weight of a liter of water is one kilogram.

A few problems are annexed. But nearly every problem in the book involves the application of the system.

Questions given in grams, etc., might be solved in ounces provided we adhere throughout the problem to the same unit

*Strictly, volume occupied by 1 kilogram of water at maximum density. A cubic decimeter of water at 4° C. weighs 0.999972 kilos. It is too late to correct the unfortunate discrepancy, which, however, is too trivial to affect ordinary practice.

of comparison. Answers would then have to be given in decimals of an ounce.

No problems have been given in the comparison of one system with another, except in the one case of gas volumes. It is assumed that any reader having occasion to transform from metric to ordinary, or vice versa, can use the comparison factors, as given, for himself.

PROBLEMS IN THE METRIC SYSTEM.

(1) A rectangular tank is 4 meters long, 3 meters wide, and 2 meters deep. How many liters of water will it contain?

Ans. 24,000 liters.

(2) How many grams in a metric ton? *Ans.* One million.

(3) How many cubic millimeters in a cubic meter?

Ans. One thousand million.

(4) What does a cubic millimeter of water weigh?

(1 milligram) *Ans.* 0.001 gram.

(5) A rectangular tank is 4 meters long, 3 meters broad, and contains 39,000 liters of water. How deep is the water?

Ans. 3.25 meters deep.

(6) What is the weight of a cylinder of copper (sp. gr. = 8.9) whose diameter is 4 centimeters and whose height is 30 centimeters?

Ans. 3355.2288 grams.

(7) Cylindrical cistern is 4 meters deep and 3 meters diameter. How many liters will it hold?

Ans. 28,274 liters.

(8) A hollow globe has internal diameter of one-fourth meter. How many liters will it hold?

Ans. 8.1812 liters.

(9) What is the radius of a sphere, in millimeters, whose volume equals one liter?

Ans. 62.03 millimeters.

(10) A cubical vessel holds 15.625 liters, what is length of the edge of the cube? Answer in centimeters.

Ans. 25 centimeters.

(11) If the edge of a cube equals 1, what will be radius of a sphere of equal volume? (See problem 9 above.)

Ans. 0.6203.

(12) Cylinder of 6 centimeters internal diameter holds water to the height of 40 centimeters. What will the water weigh, assuming maximum density?

Ans. 1130.976 grams.

(13) Same as 12, but mercury (sp. gr. = 13.59) substituted for water. What will its weight be in grams?

Ans. 15369.9638 grams.

(14) Same dimensions, but hydrogen gas substituted, assumed weight per liter 0.09 gram. What weight? *Ans.* 0.1018 gram.

(15) We have one kilogram of each of the following metals, what volumes of each respectively? Answers in cubic centimeters.

Platinum.....sp. gr. = 21.5	Gold.....sp. gr. = 19.3
Silver.....sp. gr. = 10.5	Aluminum..sp. gr. = 2.67
Steel.....sp. gr. = 7.8	Lead.....sp. gr. = 11.4
Copper.....sp. gr. = 8.9	Mercury....sp. gr. = 13.6

Answers.

Platinum..... 46.51 c.c.	Gold..... 51.81 c.c.
Silver..... 95.24 c.c.	Aluminum.... 374.53 c.c.
Steel.....128.2 c.c.	Lead..... 87.72 c.c.
Copper.....112.36 c.c.	Mercury..... 73.53 c.c.

(16) A wire is 20 kilometers long, 2 mm. diameter. It is made of steel, sp. gr. 7.8. What is its weight in kilos?

Ans. 490.09 kilos.

(17) Sphere of silver (sp. gr. 10.5) weighs 43.9803 grams. What is radius of the sphere? *Ans.* 1 centimeter.

(18) How many cannon balls 2 decimeters in diameter can be made from 1 metric ton of iron (sp. gr. 7.8)?

Ans. 30 and a fraction.

ORDINARY SYSTEM.

Length.

12 inches = 1 foot.

3 feet = 1 yard = 36 inches.

63,360 inches = 5280 feet = 1760 yards = 1 mile.

(A "knot" is 1.152 statute miles = 6087 feet to nearest foot.)

Surface.

144 square inches = 1 square foot.

9 square feet = 1 square yard = 1296 square inches.

43,560 square feet = 1 acre = 4840 square yards.

640 acres = 1 square mile.

Volume or Capacity.

1728 cubic inches = 1 cubic foot.

231 cubic inches = 1 United States gallon.

(English or imperial gallon = 277.274 cubic inches, and taken in water at 62° Fahrenheit, weighs ten pounds or 70,000 grains.)

A cubic foot of water at 4° C. weighs 62.42 lbs. avdp.

A cubic foot of ice weighs 58 lbs., and has a sp. gr. of 0.929.

Liquid Measure.

1 gallon = 4 quarts = 8 pints = 231 cubic inches.

United States gallon taken in water, contains 58,318 grains.

Weight.

We have three systems; common unit is the GRAIN, same in all three.

AVOIRDUPOIS.

1 pound = 16 ounces = 7,000 grains.

(2000 lbs. = 1 ton.)

1 ounce = 437½ grains.

TROY.

1 pound = 12 ounces = 5,760 grains.

1 ounce = 20 dwt. = 480 grains.

1 dwt. (pennyweight) = 24 grains.

(Troy weight is still used for the precious metals; comparison with avdp. is given below.)

APOTHECARIES.

20 grains = 1 scruple.

3 scruples = 1 dram.

8 drams = 1 ounce.

12 ounces = 1 pound.

1 ounce = 480 grains.

1 pound = 5,760 grains.

Further comparisons of the ordinary weight system show that:

1 troy pound = 13.166 avoirdupois ounces.

1 avdp. pound = 14.583 troy ounces.

1 avdp. ton = 29,166 troy ounces.

The number of troy ounces in the ordinary ton is useful in assaying. We weigh ore by the (avdp.) ton of 2000 lbs., but we weigh the precious metals by troy weight.

So when we speak of "so many ounces gold or silver to the ton" we are speaking of troy ounces in an *avoirdupois* ton.

See under "Assay weights."

SOME DATA IN MENSURATION OCCASIONALLY
USED IN PHYSICAL CALCULATIONS.

Ratio of circumference to diameter of a circle is " π " = 3.1415926+. 3.1416 is a close approximation and is used in this work.

$$\frac{\pi}{2} = 1.5708. \quad \frac{\pi}{3} = 1.0472. \quad \frac{4\pi}{3} = 4.1888.$$

MULTIPLES OF " π ."

1.....	3.1416	6.....	18.8496
2.....	6.2832	7.....	21.9912
3.....	9.4248	8.....	25.1328
4.....	12.5664	9.....	28.2744
5.....	15.7080		

Area of a circle = πR^2 . (R = radius of the circle.)
Volume of a cylinder = $\pi R^2 \times H$. (R = radius of base, H = altitude.)

Surface of a sphere = $4\pi R^2$. (R = radius of the sphere.)

Volume of a sphere = $\frac{4\pi R^3}{3}$. (i.e., surface $\times \frac{R}{3}$.)

Examples.—(1) What is the volume of a cylinder 3 centimeters in diameter and 6 centimeters high?

R = 1.5. $R^2 = (1.5)^2 = 2.25$. $2.25 \times 6 = 13.50 =$ Answer in cubic cm.

(2) What is volume of a sphere whose diameter is 9 meters?

R = 4.5. $R^3 = 91.125$. $91.125 \times 4.1888 = 381.7044$ cubic meters.

(3) What is the radius of a cylinder, whose volume is 100 cubic centimeters, and whose altitude is 5 centimeters?

$$V = \pi R^2 \times H$$

That is, since

$$V = 100 \text{ and } H = 5 :$$

$$100 = 3.1416 R^2 \times 5. \quad R^2 = 6.3662 \text{ and } R = 2.5232 \text{ centimeters.}$$

(4) What is the radius of a sphere whose volume is 33.5104 c.c.?

$$V = \frac{4\pi R^3}{3}$$

That is:

$$33.5104 = 4.1888 R^3$$

Hence $R^3 = 8$ and $R = 2$ (centimeters).

(5) A cube has an edge of 10 centimeters.

A cylinder has diameter and altitude each 10 centimeters.

A sphere has diameter of 10 centimeters.

Find the volume of each. (Note that the cylinder is exactly inscribable in the cube, and the sphere in the cylinder.)

Answers. Volume of cube = $10^3 = 1000$ cubic centimeters.

Volume of cylinder = 785.4 cubic centimeters.

Volume of sphere = 523.6 cubic centimeters.

(6) What is the radius of a sphere whose volume is equal to that of a cylinder of 5 cm. radius and 4 cm. altitude?

Volume of cylinder = $\pi 25 \times 4$; that is, 100π , or 314.16.

This, by the terms of the problem, must equal $\frac{4}{3}\pi R^3$; that is, $4.1888 R^3$. R^3 , therefore, is 75 and $R = \sqrt[3]{75} = 4.2172$ centimeters.