1. n_0, the number of absorbing glasses;
2. d_0, the aperture of the cat's-eye;
3. f_0, the extension of the objective for focusing.

The same process is followed for the source to be studied, and the numbers n_1, d_1, f_1 are found.

k being the absorption coefficient of the tinted glasses, we have:

$$\frac{I}{1} = \left(\frac{1}{k}\right)^{(n_1 - n_0)} \cdot \left(\frac{d_0}{d_1}\right)^2 \cdot \left(\frac{f_1}{f_0}\right)^2.$$

For the glasses mentioned, the absorption coefficients are:

$k = \frac{1}{11}$, corresponding to $\lambda = 659$;
$k = \frac{1}{7}$, corresponding to $\lambda = 546$;
$k = \frac{1}{10}$, corresponding to $\lambda = 460$.

For very small objects which would have to be placed very near, a supplementary objective is put in front of the telescope; the object is placed in the principal focus of this new lens, the objective of the apparatus being focused for parallel rays. The absorptive power of this supplementary lens is reckoned as $\frac{1}{10}$.

Details of an Observation. — The first operation to make is the determination of the absorption coefficients of the absorbing glasses. For that, an object of suitable brightness is viewed once with the tinted glass before the cat's-eye and then without this glass. Let N be the aperture of the cat's-eye without tinted glass, and N' the aperture with such a glass. The coefficient k of absorption is

$$k = \left(\frac{N'}{N}\right)^2.$$

The following observations furnish data for the determination of the absorbing powers of different glasses employed in the course of studies relative to the radiations from incandescent mantles.

ABSORBING GLASS PLACED BEFORE THE SOURCE TO BE STUDIED.

Temperature.	Aperture of cat's-eye.		
	Red.	Green.	Blue.
1270° (+ 1 glass)	19.5	21.2	35
1270 (no glass)	5.5	7.9	11.1
	$k_r = 12.5$	$k_g = 7.2$	$k_b = 9.9$

ABSORBING GLASS PLACED BEFORE THE STANDARD LAMP.

1170° (− 1 glass)	2.9	5.95	10.2
1170 (no glass)	9.4	16.1	31.5
	$k_r = 10.5$	$k_g = 7.3$	$k_b = 9.5$

Emissive Power. — Before being able to establish the relation which exists between the intensity of radiation of incandescent bodies and their temperature, it is necessary to know the emissive powers of these bodies (see page 293). For this measurement use was made by Le Chatelier of the principle stated above, — that the interior of fissures in bodies may be considered as inclosed in an envelope at uniform temperature. The emissive power is thus, at the temperature considered, equal to the ratio of the luminous intensity of the surface to that of the bottom of deep fissures, with the condition, evidently, that the aperture of the fissures be sufficiently small.

The body to be studied was placed in the state of a paste, as dry as possible, on the end of a couple previously flattened so as to take the form of a disk of 2 or 3 mm. diameter. The drying was very slow, so as not to have any swelling of the mass, and one obtained in this way a coating possessing fissures; the conditions described above are then satisfied. The end of the couple thus prepared is heated either in a Bunsen flame or a blast lamp, and the temperature of the junction is noted, while, simultaneously, readings are taken with the optical pyrometer. In order to obtain a temperature as constant as possible, it is necessary to guard against currents of air and use a flame of small size.

Here are some results obtained:

I. COUPLE COVERED WITH A MIXTURE CONTAINING 99 PARTS OF THORIUM AND 1 OF CERIUM.

Temperatures.	Red. (1)	Red. (2)	Green. (1)	Green. (2)	Blue. (1)	Blue. (2)
950° (− 1 glass)	16.0	...	21.0	14.0	23.0	....
1170	15.5	9.0	11.0	9.0	12.0	12.0
1375	7.0	3.0	4.5	3.2	3.5	3.5
1525	3.2	2.0	2.0	2.0	1.9	1.9
1650 (+ 1 glass)	8.3	6.0	5.0	....	4.0	....

II. MAGNESIA.

Temperatures.	Red. (1)	Red. (2)	Green. (1)	Green. (2)	Blue. (1)	Blue. (2)
1340° (−1 glass)	12.2	4.0	18.5	6.7	19.0	9.0
1460 (−1 glass)	4.9	2.5	8.2	3.1	7.7	4.1
1540 (−1 glass)	2.4	1.3	3.1	1.8	3.2	2.1

The numbers give the divisions of the cat's-eye; those of column (1) refer to the surface, and those of column (2) to the bottom of the fissures. The indications (− 1 glass) and (+ 1 glass) mean that the absorbing glass is placed either before the standard lamp or before the source studied.

Measurements of Intensity.—The following table gives an idea of the order of magnitude of the intensities of different luminous sources, the measurements of brightness being made in the red. Unity is the brightness of the axial portion of stearine-candle flame.

Carbon beginning to glow (600°)	0.0001
Silver melting (960°)	0.015
Stearine candle; Gas flame; Acetate of amyl lamp	1.0
Pigeon lamp, with mineral oil	1.1
Argand burner, with chimney	1.9
Auer burner	2.05
Fe_3O melting (1350°)	2.25
Palladium melting	4.8
Platinum melting	15.0
Incandescent lamp	40
Crater of electric arc	10,000
Sun at midday	90,000

Calibration.—Le Chatelier made a first graduation of his optical pyrometer by measuring the brightness of iron oxide heated on the junction of a thermoelectric couple, and admitting that, for the red, the emissive power of this substance is equal

to unity. He found a law of variation of the intensity of the red radiations as function of the temperature, which is well represented by the formula

$$I = 10^{6.7} \cdot T^{-\frac{3210}{T}},$$

in which unit intensity corresponds to the most brilliant axial region of the flame of a candle. (T is absolute temperature.)

This formula has been shown by Rasch to be equivalent to (B), page 294, for red light, in which $\alpha = 13.02$. It is therefore a derivative from Wien's law (page 251).

The table below gives, for intervals of 100°, the intensities of red radiations emitted by bodies of an emissive power equal to unity. These numbers were calculated by means of the interpolation formula given above.

Intensities.	Temperatures.	Intensities.	Temperatures.
0.00008	600°	39	1800°
.00073	700	60	1900
.0046	800	93	2000
.020	900	1800	3000
.078	1000	9,700	4000
.24	1100	28,000	5000
.64	1200	56,000	6000
1.63	1300	100,000	7000
3.35	1400	150,000	8000
6.7	1500	224,000	9000
12.9	1600	305,000	10,000
22.4	1700		

These results are represented graphically in Fig. 111.

After having determined the value of the diaphragm opening d_0, which gives equality of brightness of the standard candle with that of the comparison lamp, and the absorbing power k of the tinted glasses, one may, as was said before, prepare a table which gives directly the temperature corresponding to each aperture of the cat's-eye.

With an apparatus for which

$$d_0 = 5.2, \quad k = \frac{1}{11},$$

the following table is obtained, in which the plus sign refers to tinted glasses placed before the objective, and the minus sign to those before the comparison lamp.

This graduation applies to all bodies placed in an inclosure at

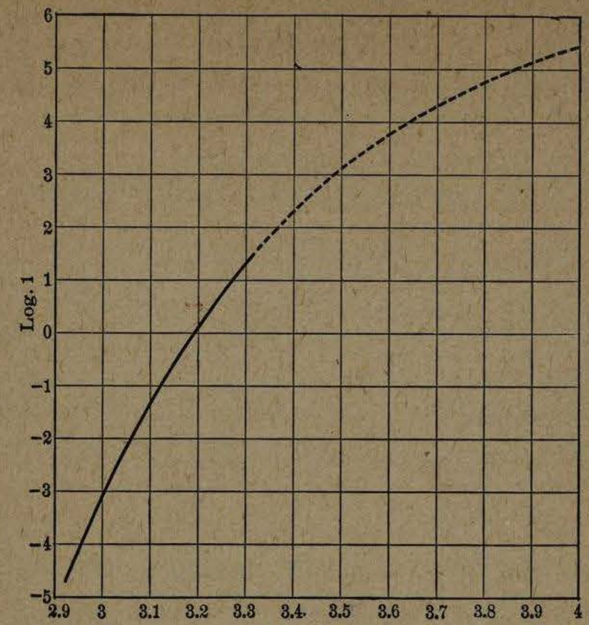

Fig. 111. Intensities in Terms of Temperatures.

the same temperature, — in the interior of furnaces for example, —and to black bodies whatever the temperature surrounding them; for example, it applies very closely for a piece of red-hot iron exposed to the free air. For bodies whose emissive power is inferior to unity, as platinum, magnesia, lime, it is necessary, when they are exposed to the air and not surrounded by an inclosure at the same temperature, to make a special calibration.

TYPICAL CALIBRATION TABLE FOR A LE CHATELIER OPTICAL PYROMETER.

Temperatures.	Aperture of the Cat's-eye. −2 glasses.	−1 glass.	0 glass.	+1 glass.	+2 glasses.
700°	17.3	....	...	....	....
800	6.9	23.0	...	....	....
900	....	11.0	....	....	....
1000	....	5.6	18.6	....	....
1100	....	....	10.5	....	....
1200	....	....	6.5	....	....
1300	....	....	4.0	13.6	....
1400	....	....	....	9.4	....
1500	....	....	....	6.6	....
1600	....	....	....	4.8	....
1700	....	....	....	3.6	12.0
1800	....	....	....	....	9.1
1900	....	....	....	....	7.3
2000	....	....	....	....	5.9

Le Chatelier and Boudouard made a series of measurements on radiations of different wave lengths. The junction of a thermoelectric couple was placed in a small platinum tube, to realize approximately an inclosed space. By taking as unity the brightness of melting platinum, the results obtained are the following for the red, green, and blue radiations:

TEMPERATURE VS. BRIGHTNESS (IN TERMS OF MELTING PLATINUM).

t	Log $(t+273)$	I_r	Log I_r	I_g	Log I_g	I_b	Log I_b
900°	3.0707	0.0009	$\bar{4}.95$	0.00018	$\bar{4}.25$	0.00002	$\bar{5}.3$
1180	3.161	.0024	$\bar{3}.88$	.0087	$\bar{3}.94$	.0015	$\bar{3}.17$
1275	3.190	.075	$\bar{2}.78$	.037	$\bar{2}.57$	.013	$\bar{2}.11$
1430	3.230	.23	$\bar{1}.36$	.16	$\bar{1}.67$	.058	$\bar{2}.76$
1565	3.265	.72	$\bar{1}.86$	.47	$\bar{1}.20$	.24	$\bar{1}.38$
1715	3.300	1.69	0.23	1.45	0.16	.9	0.95

Evaluation of Temperatures. — Finally, Le Chatelier has used his optical pyrometer to determine the very highest temperatures realized in some of the most important phenomena in nature and in the industries. These results, quite different from previous determinations, were at first regarded with considerable reserve; they are admitted to-day as reasonable, at least within the limits of precision, and in terms of the temperature scale used by him. Here are some of the figures obtained:

Siemens-Martin furnace	1490° to 1580° C.
Furnace of glass works	1375 to 1400
Furnace for hard porcelain	1370
Furnace for new porcelain	1250
Incandescent lamp	1800
Arc lamp	4100
Sun	7600

This determination of the temperature of the sun, generally believed to be low at the time it was found, was confirmed by the experiments of Wilson and Gray (page 271) by a totally different method. Later determinations of the sun's temperature, using the more recently established laws of radiation (Chapter VI), give values between 5500° and 6500°.

A series of measurements were made with the same apparatus in ironworks. Here are some results:

BLAST FURNACE SMELTING GRAY PIG.

Opening before the tuyère	1930° C.
Tapping the pig iron, beginning	1400
Tapping the pig iron, end	1520

BESSEMER CONVERTER.

Pouring the slag	1580°
Pouring the steel into the ladle	1640
Pouring the steel into the molds	1580
Reheating of the ingot	1200
End of the hammering	1080

SIEMENS-MARTIN FURNACE.

Flow of the steel into the ladle, beginning	1580°
Flow of the steel into the ladle, end	1420
Flow into the molds	1490

Calibration in Terms of Wien's Law. — As approximately monochromatic radiation is used, the Le Chatelier optical pyrometer may be calibrated in terms of Wien's law (III) (page 251) by sighting upon a black body (Fig. 86) whose temperature is given by means of a thermocouple. For this purpose Wien's law may be written:

$$\log I = K_1 - K_2 \frac{1}{T},$$

where I is the intensity of light, in terms of the center of the Hefner flame for example, and T is the absolute temperature. This method of graduation has the advantage that only two points are required to completely calibrate the instrument, for the relation between $\log I$ and $\frac{1}{T}$ is linear, so that these quantities being plotted give a straight line which may evidently be extended to lower and higher temperatures, since Wien's law has been shown (page 250) to hold over the widest temperature interval measurable, provided the light used is monochromatic and the bodies observed approximate blackness and are not luminescent, that is, their light not produced by chemical or electrical excitation. The value of I is given by the equation of page 300, and for a given absorption glass and focus is proportional to d^2.

Precision and Sources of Error. — We shall give in some detail a discussion of the factors which in the use of the Le Chatelier

optical pyrometer may influence the photometric settings and so affect the accuracy of temperature determinations, as results of such a discussion are illustrative of what may be expected from optical pyrometers in general. The results are taken from those of Waidner and Burgess, who have made an experimental comparison of all the available optical pyrometers.

The sources of error of this instrument may be those due to the standard Hefner amyl-acetate or other standard of constant photometric intensity or temperature placed before the cat's-eye when adjusting the pyrometer, the oil comparison lamp, the focusing system, the nature of the red glass used, and the coefficients of absorption of the tinted glasses. The first of these affects only comparative results with different instruments, while the others, if they exist, may be of considerable importance in work with a single instrument. We shall consider them in the order named.

As only the central portion of the amyl-acetate flame is used, variations in height and fluctuations in total intensity due to various causes such as moisture and carbonic acid in the atmosphere and changes due to differing samples of acetate become almost, if not quite, insignificant in this method of comparison; so that, when using only a small central area of the amyl-acetate flame, it is a very perfectly reproducible standard under the most varying conditions of burning. Again, the effects of any slight fluctuations in light intensity are further greatly reduced when transformed into temperature changes, as has been shown (page 238). Thus, the effect of varying the height of the Hefner flame by 1 mm., which amounts to 10 per cent of the total intensity when the whole flame is used, causes a change of less than 1 per cent in the intensity of light from the central area, which is equivalent to less than 0.5° C. change in temperature at 1000° C.

Although used intermittently as above indicated, the Hefner serves well enough as an ultimate standard by means of which the indications of all photometer pyrometers may be reduced to a common basis, yet the Hefner is not suited for use as comparison lamp in the pyrometer itself, as has been previously stated.

In a study of the constancy of the comparison lamp the following arrangement was adopted: In order to obtain a perfectly constant source of light with which to compare the flame, a 32-c.p. incandescent electric lamp was placed in a fixed position before the objective of the pyrometer and a glass diffusing screen inserted before the objective. The voltage across the lamp terminals was kept rigorously constant, thus giving an arbitrary but invariable standard of illumination.

The concordance of results obtained by different observers setting the gasoline flame and observing is shown below:

WITHOUT ABSORPTION GLASS.

Observer	1	2	3	4
Cat's-eye scale readings	7.4	7.8	7.6	7.3
	7.4	7.9	7.8	7.0
	7.2	7.7	7.6	8.0
	7.8	7.8	7.7	7.1
	7.7	7.7	7.8	8.3
	7.8	7.7	7.4	8.0
Means	7.55	7.73	7.65	7.60

Observers Nos. 2 and 4 had no previous experience in the use of the instrument.

WITH ABSORPTION GLASS.

Observer	1	3
Cat's-eye scale readings	25.7	25.8
	24.0	24.8
	23.6	26.0
	24.1	25.8
	25.4	24.8
	24.8	24.9
	24.8	25.3
Means	24.63	25.34

Here the greatest variation corresponds to less than 3 degrees in temperature at 1000° C.

To control accurately the flame height in the gasoline lamp, a sight was inserted consisting of a horizontal scratch 2 mm. above the window before the flame, and a very fine platinum wire in the same horizontal plane but in a collar behind the flame. With

this improvement an observer can set and control the flame height to 0.2 mm. Such provision, however, is not necessary except in the most refined work, for experiment showed that for most purposes changes of over 2 mm. may be made in the flame height with unimportant changes resulting in the temperature estimation.

Considering the time effect of burning upon the flame height and intensity due to local heating and change of depth of oil, it was found that the flame ceases creeping up after ten minutes and will then remain at constant height to within 0.5 mm. until the oil is used up, in three hours; and during all this period the brightness of the flame does not change by an amount corresponding to more than 5 degrees in temperature.

It might be expected that oils of different grades would give widely differing results, but an examination of this possible source of error showed that different samples of gasoline and gasolines mixed with several per cent of a heavy kerosene gave identical results. This is of great importance in the practical use of the instrument, as it shows that a calibration made with a given sample of gasoline remains good for any other gasoline.

From the above it is clear that variations in brightness of the comparison flame due to all possible causes need not produce errors in temperature measurement of over 5° C. at 1000° C., that is, within the experimental limits of making the photometric setting.

Considering now the sources of error due to focusing and sighting upon the object whose temperature is sought, it is first to be noticed that there is a minimum distance from the object at which the pyrometer can be focused, this distance being somewhat over a meter, depending, of course, upon the focal length of the objective and length of drawtube. There is also a minimum area which can be sighted upon and give an image of sufficient size to completely cover the desired photometric field; this minimum size of object is about 6 mm. on a side when the instrument is at its least distance; for greater distances a larger area must be viewed.