

For the radiation from other than a black body the law of maximum energy applies only in the modified form

$$I_m T^{-\alpha} = \text{const.} = B_1, \dots \dots \dots \text{(IIa)}$$

where α cannot be less than 5 and is not probably ever greater than 6, the value found by Lummer and Pringsheim for polished platinum. The general form of Wien's law (III) takes the form

$$I = c_1 \lambda^{-\alpha} e^{-\frac{c_2}{\lambda T}}, \dots \dots \dots \text{(III')}$$

where $6 > \alpha > 5$.

The corresponding form assumed by Stefan's law for non-black bodies is

$$E = \sigma' T^{\alpha-1}, \dots \dots \dots \text{(B')}$$

where α is the same as in the preceding equation.

Lummer and Pringsheim found the following limits of temperature as given by the Wien relation (Ia):

	λ_m	$T_{\text{max.}}$	$T_{\text{min.}}$
Electric arc	0.7 μ	4200 abs.	3750 abs.
Nernst lamp	1.2	2450	2200
Auer burner	1.2	2450	2200
Incandescent lamp	1.4	2100	1875
Candle	1.5	1960	1750
Argand burner	1.55	1900	1700

Lummer and Pringsheim also heated a carbon tube electrically to about 2000° C. and observed the temperature inside simultaneously with instruments making use of the several radiation laws:

Method.	T absolute.
Photometric.....	{ 2310
	{ 2320
	{ 2330
Total radiation.....	{ 2330
	{ 2345
	{ 2325
Energy maximum.....	{ 2330
	{ 2320

This complete concordance at such a high temperature between the different radiation methods gives further confidence in the

legitimacy of their indefinite extrapolation for nonluminescent bodies. Waidner and Burgess have also found that this accord probably exists at the temperature of the electric arc, 3600° C.

In spite of the excellent agreement among the above-mentioned experiments in confirming the validity for temperature measurements of the several radiation laws, there is, nevertheless, too great an outstanding uncertainty in the numerical values of the characteristic constants of the equations representing these laws, such, for instance, as the σ in Stefan's law (B), page 246, and the c_2 in the Wien equation (III), page 251, for black-body radiation, as well as in the corresponding quantities, and particularly the value of the exponent α (equation (II), etc.), for other substances.

We have seen (page 80) that there is a discrepancy of considerable magnitude, of at least 25° C., at the melting point of palladium, on the scale of the nitrogen-gas thermometer as given by Holborn and Valentiner and by Day and Sosman. The measurements of the former, made simultaneously with the gas-thermometer determinations, lead to a value of $c_2 = 14,200$ in Wien's equation (III), and in the hands of Valentiner to a constant value of 5.34 watts for σ in Stefan's equation (B). The gas scale of Day and Sosman, however, corresponds more nearly to a value of $c_2 = 14,500$ as deduced from the optical measurements of Nernst and Wartenberg and of Waidner and Burgess. It is very important that this outstanding discrepancy be settled, and further work is in progress in several laboratories on these and allied subjects.

Warburg and Leithauser, in a preliminary announcement of new determinations of c_2 made at the Reichsanstalt, find values ranging between 14,200 and 14,600, depending on the experimental conditions. Their best value so far announced is 14,570.

Coblentz at the Bureau of Standards has also published preliminary results on c_2 approximating 14,600.

Applications to Pyrometry. — It is evident that theoretically any of these laws and their various consequences might be used as a basis of pyrometry, but practically it is not convenient

to make use of all of them. The displacement law ($\lambda_m T = A$) and the maximum-energy law ($I_m T^{-5} = B$) of Wien are well-established relations, but in practice it is exceedingly difficult to construct instruments of sufficient sensibility to give any considerable precision, and any industrial pyrometer using these principles is out of the question at the present time. The reason of the lack of sensibility with the relation $\lambda_m T = A$ is due to the fact that the exact position of the wave length possessing the maximum of energy is very difficult to locate, especially at relatively low temperatures (see Fig. 87). The value of the maximum energy could perhaps be measured more readily, but as this quantity varies as the fifth power of the temperature, there would hardly be any preference for this over the former method.

There have been, however, several most convenient, simple, and very accurate instruments devised which are based either on the use of Stefan's law, $E = \sigma (T^4 - T_0^4)$, or Wien's distribution law, $I = c_1 \lambda^{-5} \epsilon^{-\frac{c_2}{\lambda T}}$, either directly or indirectly, and in the two following chapters we shall treat of these at some length.

The last equation is conveniently put into the logarithmic form for computation:

$$\log I = C - \frac{c_2 \log \epsilon}{\lambda} \cdot \frac{1}{T},$$

or, when comparing two intensities, as in temperature measurements, the determination of an absorption or reflection coefficient, or when comparing one type of radiation with another, we have:

$$\log \frac{I}{I_1} = \frac{c_2 \log \epsilon}{\lambda} \left(\frac{1}{T_1} - \frac{1}{T} \right) \dots \dots \dots \text{(IIIa)}$$

By expressing numerical results graphically, it is evident that $\log I$ vs $\frac{1}{T}$ is a straight line, which greatly simplifies the reduction of observations. It will be seen also that the constant c_2 must be known exactly if (IIIa) is to be used as a basis of correct temperature measurement. Unfortunately there is still a considerable lack of certainty in our knowledge of this constant, as shown

above, although its value has been shown by several observers to remain sufficiently constant throughout the visible spectrum. We shall use the value $c_2 = 14,500$ in what follows.

It has been shown by Henning, v. Wartenberg, and others that the absorption coefficient a , for the various wave lengths of the visible spectrum, does not change appreciably with temperature, at least for several of the metals; and this fact may be taken advantage of in temperature measurements when sighting on an object for which we know the value of the absorption coefficient a , the reflection coefficient $r = 1 - a$, or the emissivity $e = a$ when that of a black body is taken as unity. If S_λ is the black-body temperature absolute ($= s + 273$), that is, the apparent temperature of the substance as given by an optical pyrometer, using light of wave length λ , and $T (= t + 273)$ the corresponding true temperature of the substance, then Wien's equation gives:

$$\frac{1}{T} - \frac{1}{S_\lambda} = \frac{\lambda}{c_2 \log \epsilon} \cdot \log a. \dots \dots \dots \text{(IIIb)}$$

This form of Wien's equation is of the greatest importance in the practical application of optical pyrometry to the estimation of the temperature of incandescent metals and other substances for which the absorption coefficients are known.

It is also possible to estimate a temperature by Wien's law from the measurement of the light intensity at two wave lengths. The general expression is:

$$\log \frac{I_1}{I_2} = 5 \log \frac{\lambda_2}{\lambda_1} + \frac{c_2 \log \epsilon}{T} \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right) + \log \frac{a_1}{a_2} \dots \dots \dots \text{(V)}$$

where a_1 and a_2 are the absorption coefficients for the wave lengths λ_1 and λ_2 , and where I_1 and I_2 are the corresponding intensities corrected for the sensibility of the eye. In the case of a black body, the last term is zero. The eye-sensibility correction is different for each individual and the visibility curve usually has the general form shown in Fig. 88 as given by Nutting for several observers and reduced to a common maximum.

In view of their general utility in the application of optical

pyrometers to the measurement of the temperatures of exposed surfaces, we give in the Appendix, Table X, the values of the emissivities or absorption coefficients ($a = e/\epsilon$), where $\epsilon = 1$, for certain opaque substances as determined by various observers; and if the assumption is made in all cases that a has no, or a very small, temperature coefficient, the above equations may be used to compute the true temperatures of such substances when their absorbing power a is known, as well as their apparent tempera-

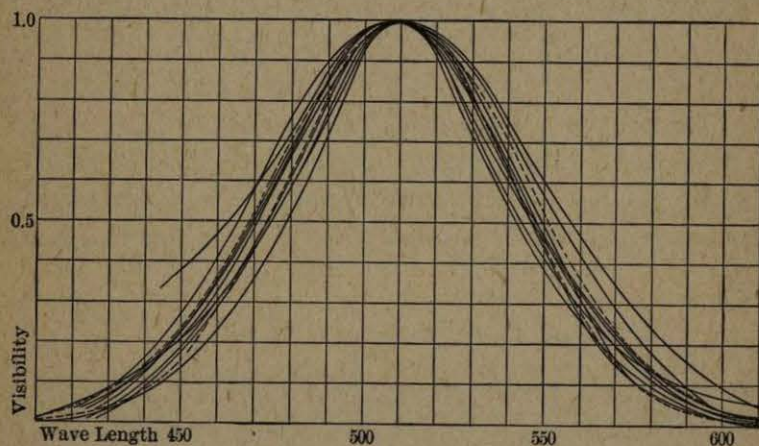


Fig. 88. Visibility Curve of Eye.

tures as given by an optical or radiation pyrometer, remembering that the above formulæ apply only when expressed in degrees absolute ($t + 273^\circ \text{C.}$).

We may illustrate by a numerical example the use of Table X for the estimation of the true temperature of a metal from observations with an optical pyrometer, using red light of wave length $\lambda = 0.65 \mu$. Let us take the case of the pouring of a stream of iron whose surface is clear, and suppose the optical pyrometer reading to be 1427°C. when the pyrometer is sighted on the clear stream. From Table X, the value of the absorption coefficient a for iron at $\lambda = 0.65 \mu$ is 0.415. In equation (IIIb), we have to solve for T as follows:

$$\frac{1}{T} - \frac{1}{1427 + 273} = \frac{0.65 \cdot \log 0.415}{14,500 \cdot 0.4343}$$

whence $t = T - 273 = 1549^\circ \text{C.}$, the true temperature of the stream of metal.

Equation (IIIb) may also be solved graphically as shown in Fig. 89, for pyrometers using red light, in which each of the

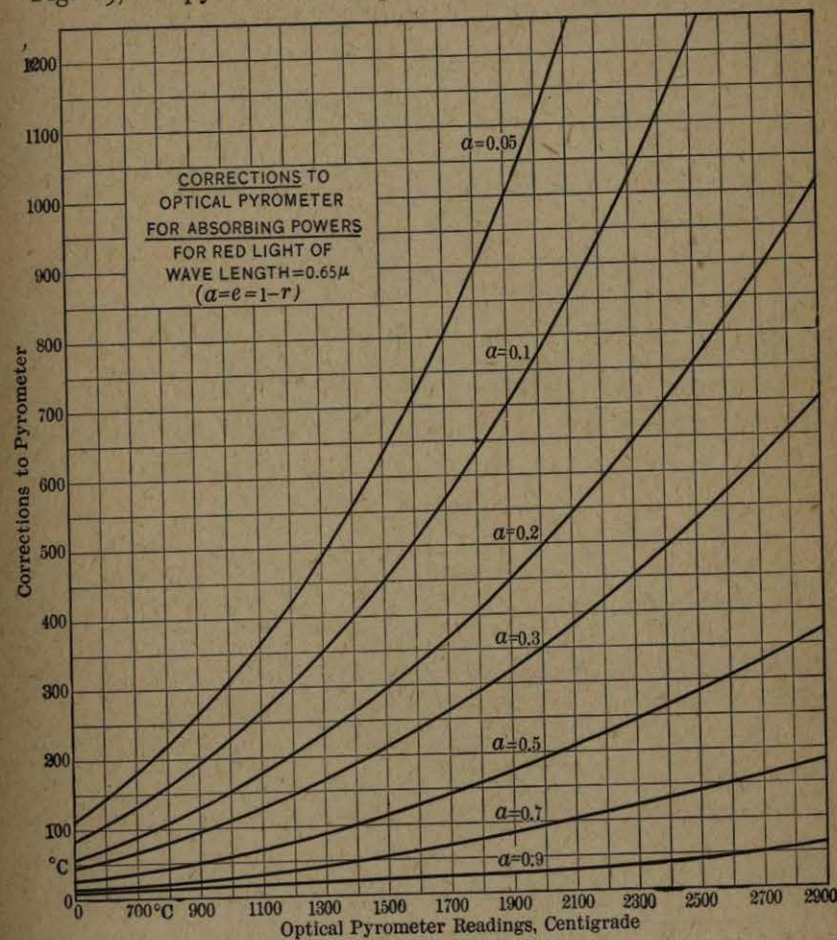


Fig. 89. Corrections to Optical Pyrometer.

curves represents a given absorbing, emissive, or reflecting power ($a = e = 1 - r$), the abscissæ are pyrometer readings in centigrade, and the ordinates the corresponding corrections to be added to the pyrometer readings for substances of known emissive powers. Similar graphs could be constructed for pyrometers using other than red light. Methods of graphical reduction have

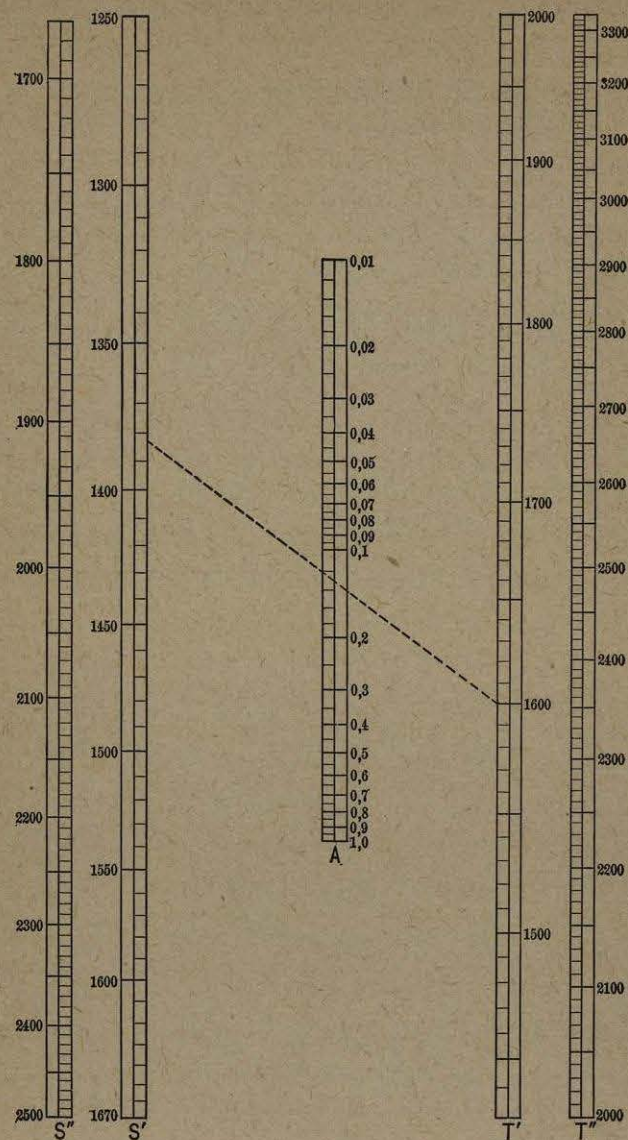


Fig. 90. Pirani's Method.

also been described by Wartenberg, who plots lines of temperature readings with absorption coefficients as ordinates and temperature corrections as abscissæ; and by Pirani, who has devised a protractor or vernier method for the solution of Wien's equation, reproduced in Fig. 90, in which a line drawn from any observed temperature S or S_1 through the value a of the absorption coefficient for the substance observed intersects the lines T or T_1 at the point corresponding to the true temperature. This figure is constructed for $\lambda = 0.65 \mu$ and $c_2 = 14,200$. To transform the value of a to the basis of any other value of c_2 , as 14,500, use may be made of the equation $a \log a = \log a'$, where a' is the ratio of the c 's. (See also Table IX in the Appendix.)

It should be noted, in making use of the figures given in Table X, that they apply, with some exactness, in the case of the metals, to bright or polished surfaces only of either the solid or liquid. When rough surfaces are met with, the values of the absorbing factors should in general be increased considerably over those given in the table. Also, these last are uncertain for a single metal, in some cases by as much as 10 per cent, as determined by different observers, due in large part, apparently, to the varying degree of polish of the samples used. Also, it should be noted that the surfaces of many substances undergo changes of emissivity on continued heating.

The expression for the emissivity E_x of total radiation, to use with radiation pyrometers based on Stefan's law, is:

$$\log E_x = 4 (\log T - \log S). \dots \dots (Ba)$$

There is little data on the total emissivity E for either the metals or other substances of interest in the measurement of temperatures by means of total-radiation pyrometers. A rough idea of the order of the value of the total emissivity E (equation (Ba)) is given by an examination of both the visible and infra-red values of the absorbing powers. The values at 2, 5, and 8 μ are given in Table X. Thus for iron, the average value of a from the table is 0.28, and the observed value of E is 0.29. The infra-red values of a as well as those of E , in general,

vary with the temperature, which fact renders their exact determination for high temperatures a difficult operation. Determinations of E at high temperatures have been made for liquid copper, 0.14; iron, 0.29; cuprous oxide, 0.60.

There is room for a great deal of experimental work in determining satisfactorily the emissive properties at high temperatures of those substances met with in pyrometric practice.

CHAPTER VII.

RADIATION PYROMETER.

Principle. — The quantity of heat a body receives by radiation from another body depends on certain conditions relative to each of the two bodies, which are:

1. Temperature;
2. Area of surface;
3. Distance apart;
4. Emissive and absorbing power.

In order to utilize heat radiation for the determination of temperatures, one measures a heat change produced on the object used as an instrument by the object to be studied; this heat change is either a rise of temperature or a resulting phenomenon, such as a change of electrical resistance, thermoelectromotive force, expansion, etc.

The quantity of heat given off is proportional to the area of the radiating surface S , and varies inversely as the square of the distance l .

$$q = k \frac{S}{l^2} = k' \frac{d^2}{l^2} = k'' E \cdot \frac{d^2}{l^2},$$

d being the diameter of the radiating surface and E its emissive power.

Now, $\frac{d}{l}$ is the apparent diameter of the object; the quantity of heat radiated depends, then, upon the solid angle under which the object is seen. Any instrument making use of the intensity of radiation must, therefore, have a receiving device of sufficiently small area so that it may be completely covered by the desired radiation.

The emissive power E is very variable from one substance to another, as we have seen, and for the same substance variable