CHAPTER VI.

THE LAWS OF RADIATION.

General Principles. — The temperature of bodies may be estimated from the radiant energy they send out, either in the form of visible light radiation or of the longer infra-red waves that are studied by their thermal effects. For the estimation of temperature in this way use is made of the laws of radiation.

Temperature and Intensity of Radiation. — When we consider the enormous increase in the intensity of radiation with rise in temperature, this method appears especially well adapted to the measurement of high temperatures. Thus, for example, if the intensity of the red light ($\lambda = 0.65\ \mu$) emitted by a body at 1000° C. is called 1, at 1500° C. the intensity will be over 130 times as great, and at 2000° C. over 2100 times as great.

The rapid increase of the photometric intensity of the light in comparison with that of the temperatures is shown by the following table, from Lummer and Kurlbaum, for light emitted by incandescent platinum. If I_1 and I_2 are the intensities of the total light emitted at the absolute temperatures T_1 and T_2 (not differing many degrees from one another), then if we write with Lummer and Pringsheim

$$\text{(A)} \qquad \frac{I_1}{I_2} = \left(\frac{T_1}{T_2}\right)^x.$$

the values of x at various absolute temperatures ($T°$ C. + 273°) are as follows:

$T°$ abs.	x.
900°	30
1000	25
1100	21
1200	19
1400	18
1600	15
1900	14

From this table it will at once be seen that at 1000° absolute (727° C.) the intensity of the light increases twenty-five times as rapidly as the temperature; at 1900° absolute (1627° C.) fourteen times as rapidly. The product $Tx = 25{,}000$ as shown by Rasch seems to express the relation between T and the exponent x.

Emissive Powers. — It would therefore appear that a system of optical pyrometry based on the intensity of the light emitted by incandescent bodies would be an ideal one, inasmuch as a comparatively rough measurement of the photometric intensity would measure the temperature quite accurately. This, however, is only partly true; it is limited somewhat by the fact that different bodies, although at the same temperature, emit vastly different amounts of light. Thus the intensity of the radiation from incandescent iron or carbon at 1000° C., for example, is many times greater than that emitted by such substances as magnesia, polished platinum, etc., at the same temperature. Consequently, if any conclusions were drawn as to the temperatures of these bodies from the light that they emit, it might lead to large errors. Thus at 1500° C. this difference in the intensity of the light emitted by carbon and by polished platinum would lead to a difference in the estimated temperature of these bodies of about 100° C., and less at lower temperatures.

The "Black Body." — Kirchhoff in one of the most important contributions to the theory of radiation was led to the important conception of what he termed a "black body," which he defined as one which would absorb all radiations falling on it, and would neither reflect nor transmit any. He further pointed out clearly the important fact that the radiation from such a black body was a function of the temperature alone, and was identical with the radiation inside an inclosure all parts of which have the same temperature. Various expressions are in use for the "black body," such as "integral radiator," "full radiator," etc.

Experimental Realization. — The first experimental realization of a black body as a practical laboratory apparatus was made by Lummer and Wien, by heating the walls of a hollow opaque inclosure as uniformly as possible and observing the radiation coming from the inside through a very small opening in the walls of the inclosure. No substance is known, however, whose surface radiation is exactly that of a black body. The radiations from such substances as carbon and iron oxide approximate fairly near to black-body radiation, while such bodies as polished platinum and magnesia, etc., depart very far from it. Black-body radiations, corresponding to temperatures from that of liquid air or lower, up to 2500° C. or higher (if suitable materials are chosen), are now available in the laboratory. For temperatures up to 600° or thereabouts, this is realized by immersing a metallic or other vessel in a constant-temperature bath (liquid, gas, vapor, or fused salt) and observing the radiation from the interior through a small opening in the walls. At higher temperatures it is very difficult to heat the walls of the inclosure uniformly, especially with gas flames. Lummer and Kurlbaum have very satisfactorily overcome this difficulty in their electrically heated black body, which is shown in section in Fig. 86.

The central porcelain tube is wound over with thin platinum foil through which an electric current is sent which can be adjusted to maintain any desired temperature up to 1500° C. This tube is provided with a number of diaphragms to minimize the disturbing effects of air currents. To protect this inner tube from external influences and to diminish unnecessary heat losses, it is surrounded by several porcelain tubes and air spaces, as shown in the figure. The radiation from the uniformly heated region near the center, and which passes out through the end of the tube at *O*, is a very close approximation of the ideal black-body radiation of Kirchhoff. The temperature of this central region is measured by means of one or more carefully calibrated thermocouples. By adding supplementary heating coils at the ends the temperature distribution may be improved. Waidner and Burgess were able to obtain a constancy of 1° C. throughout the greater part of the length of such an apparatus. The calibration of optical and radiation pyrometers is carried out by means of such a black body. For higher temperatures special furnaces are used, which we shall describe later.

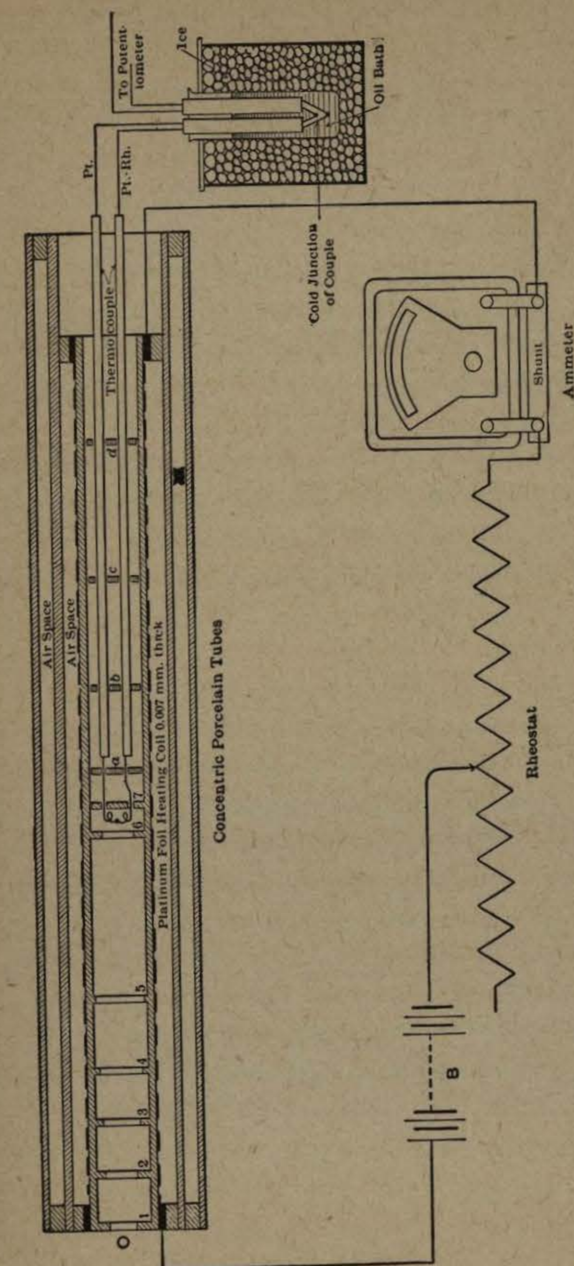

Fig. 86. Experimental Black Body.

As has already been stated, if magnesia, porcelain, platinum, iron, etc., are heated to the same temperature, they will emit vastly different amounts of light. If, however, these bodies* are heated inside a black body, they will all emit the same radiation, and on looking into the small opening all details of their contour will be lost, the whole region being of uniform brightness. Thus, in the black body described above, before the heating has become uniform, the platinum wires of the thermocouple can be seen as dark lines against the brighter background, but when the heating current has been maintained constant for some time, so that the heating has become uniform in the inner central chamber, the wires of the couple almost completely disappear, notwithstanding that, of all substances, platinum and the black oxide of the radiating walls differ most widely in their radiating powers (emissivities).

Realization in Practice. — Fortunately, in pyrometric practice it is often easy to realize very nearly the conditions of a black or totally absorbing body. Thus the interior of most furnaces, kilns, and ovens approximates this condition, or the bottom of a closed tube of any material thrust into any space heated to incandescence. Again, iron and coal observed in the open are not far removed in their optical properties from the black body.

Black-body Temperature. — The term "black-body temperature" has come into quite extensive use and is of great convenience in the discussion of pyrometric problems. The temperatures indicated by a radiation pyrometer that has been calibrated against a black body are known as black-body temperatures. Thus, were a piece of iron and a piece of porcelain both at 1200°, the optical pyrometer, which used the red light emitted by these bodies, would give, as the temperature of these bodies, 1140° and 1100° respectively. This means that iron and porcelain at 1200° emit red light of the same intensity as is emitted by a black body at 1140° and 1100° C. respectively. The "black-body temperature" of these materials for green light might differ quite appreciably from that for red light. It is at once evident that if the "black-body temperatures" of different bodies, e.g., carbon and platinum, are equal, their actual temperatures may differ considerably (180° C., or so, at 1500° C.). This violates our ordinary conception of equal temperatures, which is based on thermal equilibrium between the bodies if brought into contact. The term "equivalent temperature," suggested by Guillaume, is also used for "black-body temperature." Waidner and Burgess have suggested the notation: 1500° $K_{.65}$, meaning 1500° absolute centigrade as viewed with light of wave length 0.65μ.

* It is here assumed that the radiation is purely thermal and that no part is due to luminescence, as the laws of radiation are only directly applicable where such is the case.

The temperature of any body, therefore, as measured by an optical or by a radiation pyrometer, will always be lower than its true temperature by an amount depending on the departure of its radiation from that of a black body. There is another source of error, however, that may act in the direction of making the pyrometer read too high, due to light reflected by the body whose temperature is being measured. This source of error may very often be eliminated, where the accessibility of the work permits, by running a tube down to the incandescent surface, which will cut off stray radiation from the surrounding flames. The magnitude of the error that may arise from light reflected from surrounding hotter objects may be quite considerable (several hundred degrees), depending on the temperature, area, and position of the surrounding hot objects and the reflecting power of the surface whose temperature is under observation.

Kirchhoff's Law. — If we consider an opaque object and let radiation fall upon it, the relation between the proportions reflected (r) and absorbed (a) is:

$$r + a = 1.$$

For such objects, therefore, eliminating the effects of polarized light and angle of incidence and assuming we are dealing with matt surfaces and thermal radiation only, the determination of either the absorbing or reflecting power gives also the other.

The quantity a depends on the nature of the substance and is a function of the wave length and temperature only, or

$$a = f(\lambda, T).$$

For a radiating body the emissive power e is a similar function of λ and T.

By definition, a black body absorbs all the radiation incident upon it, therefore in this case $a = 1$ and $r = 0$ for all values of λ and T. The emissive power ϵ of a radiating black body is evidently fundamental to the theory of radiation, and the function $\epsilon = F(\lambda, T)$ forms the basis of several of the radiation laws.

Kirchhoff's law as applied to monochromatic radiation may be stated in the following way: $e = a\epsilon$, or more completely:

$$\frac{e}{a} = \ldots \frac{e_n}{a_n} = \epsilon \equiv F(\lambda, T).$$

The ratio of the emission to the absorption is for all bodies the same function of wave length and temperature, and is equal to the emission of a black body.

There are a number of corollaries to Kirchhoff's law, some of which we may emphasize as being of interest in temperature measurements, bearing in mind that we are here dealing only with radiation due to thermal causes.

The emissive power of a black body is greater than that of any other body at the same temperature. Every body absorbs the same rays that it emits at a given temperature. It may also absorb other rays, but they will be among those that a black body does not emit at the given temperature.

In general, the ratio $e:a$, which is the same for all bodies for given values of λ and T, does not depend on the degree or kind of polarization of the radiation.

The energy curves $e = f(\lambda)$ for each value of T lie wholly within the corresponding black-body curves $\epsilon = f(\lambda)$.

In the case of composite radiations, that is, of spectral bands having for limiting case the whole spectrum, Kirchhoff's law applies only under special conditions. Thus Kirchhoff's law holds for any composite radiations, taken between the limits λ_1 and λ_2, if the total absorption is referred to the radiation from a black body at the same temperature as the bodies to be compared.

Again, Kirchhoff's law holds for composite radiations, when the two given bodies are at the same temperature and when each of them serves as source to the radiation which measures the total absorption of the other.

A corollary of considerable practical importance is the following: If two surfaces of any substances whatever at the same temperature radiate only on each other, the radiation from each is equivalent to the emission of a black body; from which it follows that, within an inclosed space at constant temperature, all bodies emit radiation identical to that of a black body. And finally, the radiation from a small opening in such an inclosure at constant temperature is black-body radiation, and depends only on the temperature.

It is worthy of remark that in the measurement of radiation from such a black body the receiver should also be a black body, or at least its coefficient of absorption should be known for the kind of radiation to be studied, if the radiation laws as applied to a black body are assumed to hold, as is often the case.

Stefan's Law. — Naturally the first numerical relation sought between intensity of radiation and temperature was one for the total energy of radiation sent out by a body, as it required less delicate instruments for measurement than the study of the spectral distribution of energy. Numerous attempts to express such a relation were made by Newton, Dulong and Petit, Rosetti, and others. These attempts, however, merely resulted in empirical expressions that held only through narrow ranges of temperature. The first important step was made by Stefan, who examined some of the experimental data of Tyndall on the radiation of incandescent platinum wire in the interval 525° C. to 1200° C., and was led to the conclusion that the energy radiated was proportional to the fourth power of the absolute temperatures. This relation seemed to be further supported by the best experimental data of other observers, at least to within the limit of accuracy of their observations, being strictly true, however, only for the energy of total radiation from a black body. This relation received independent confirmation from Boltzmann, who deduced it from thermodynamic reasoning. The conditions imposed by Boltzmann in his discussion on the nature of the radiation were such as are fulfilled by the radiation from a black body. This relation, which has now come to be generally known as the *Stefan-Boltzmann radiation law*, may then be stated as follows:

The total energy radiated by a black body is proportional to the fourth power of the absolute temperature, or

(B) $$E = \sigma (T^4 - T_0^4) = \int_0^\infty \epsilon (\lambda, T)\, d\lambda,$$

when E is the total energy radiated by the body at absolute temperature $T°$ to the body at absolute temperature $T_0°$, and σ is a constant depending on the units used. Usually T_0 is small compared with T, so that practically we may write

(B′) $$E = \sigma T^4.$$

This law has received abundant experimental support from the researches of Lummer, Kurlbaum, Pringsheim, Paschen, and others, throughout the widest range within which temperature measurements can be made.

An illustration of the experimental evidence in support of this law is given in the table taken from the experiments of Lummer and Kurlbaum.

Absolute temperature.		$\sigma = \frac{E}{T^4 - T_0^4}$.		
T	T_0	Black body.	Polished platinum.	Iron oxide.
372.8	290.5	108.9	.....	....
492	290	109.0	2.28	33.1
654	290	108.4	6.56	33.1
795	290	109.9	8.14	36.6
1108	290	109.0	12.18	46.9
1481	290	110.7	16.69	65.3
1761	290	.....	19.64	.....

It will also be seen from this table that while the intensity of the total radiation of iron oxide is 4 or 5 times that of polished platinum, it is still considerably less than that emitted by a black body. The total radiation from objects other than a black body increases more rapidly than the fourth power of the absolute temperature, so that as the temperature is raised the radiation of all bodies appears to approach that of the black body. Whether or not there is a maximum limiting value of radiation due to purely thermal causes, is still an unsettled question, however.

The numerical value of the constant σ for a black body is of interest in absolute measurements and in checking the constants of radiation instruments. For the radiation per degree C. from 1 $\overline{\text{cm.}}^2$, expressed in gram-calories per second, the values found for σ range from less than $1.0 \cdot 10^{-12}$ by Bottomly and King to $1.52 \cdot 10^{-12}$ by Féry. The following observers, however, have obtained results agreeing more closely:

Kurlbaum,	$1.277 \cdot 10^{-12}$
Valentiner,	1.286
Bauer and Moulin,	1.275

$$\text{Mean} = 1.279 \cdot 10^{-12} \simeq 5.34 \cdot 10^{-12} \frac{\text{watts}}{\overline{\text{cm.}}^2}.$$

The last two series of measurements were carried out over very extended temperature intervals — in the case of Valentiner to nearly 1600° C. His results correspond to the temperature scale, established by Holborn and Valentiner. It should be noted, however, that if compensating errors are present in the energy measurements or in the non-blackness of the radiator or receiver, it is possible to obtain a correct value of σ from an incorrect temperature scale, as is evident from the logarithmic form of the Stefan-Boltzmann equation: $\log E = \log \sigma + n \log T$.

Laws of Energy Distribution. — Among the first facts to be noticed about the nature of the radiations sent out by bodies were, that at low temperatures these radiations consisted of waves too long to affect the human eye. As the temperature was raised, shorter and shorter waves were added, which could finally be detected by the eye; the first of the visible radiations producing the sensation called red, then orange, etc., until the violet waves were reached, which were the shortest waves that the eye could detect.

Soon after Langley brought out the bolometer, which was so admirably adapted to the measurement of the minute energy of radiations, a great mass of valuable experimental data was obtained, bearing on the spectral distribution of the energy of the radiation emitted by various bodies. Among the most important of these contributions must be mentioned the researches of Paschen, who examined the distribution of energy in the emission and absorption spectra of various substances. Among the experimental facts established by these researches were, that by far the largest portion of the energy in the spectrum was found in the infra-red region, that the position of the wave length having the maximum energy depended on the temperature of the body, and that, as the temperature was raised, the energy of all the waves emitted increased, but the shorter waves more rapidly than the longer, so that the position (wave length) of maximum energy in the spectrum shifted toward shorter wave lengths. These facts are well illustrated by the curves shown in Fig. 85, taken from a paper by Lummer and Pringsheim, in which the ordinates are proportional to the intensity of radiation emitted by a black body, and the abscissas are wave lengths (in thousandths of a millimeter). Such curves, as are here shown, where the temperature is constant and the energy is measured corresponding to radiations of different wave lengths emitted by a body, are called *energy curves*, i.e., the relation determined is $I = f(\lambda)$ for T = constant, where I = energy corresponding to wave length λ, strictly the energy comprised in the region of the spectrum between λ and $\lambda + d\lambda$, and T is the absolute temperature of the radiating source. It is also interesting to study the change in the intensity of some particular wave length as the temperature of the radiating source is changed, i.e., to find $I = F(T)$ for λ = constant. This can of course be done by exposing the bolometer strip in a fixed part of the spectrum and

observing the galvanometer deflections as the temperature is changed. The curves obtained in this way for $I = F(T)$ are called *isochromatic curves*.

Wien's Laws. — Wien was led from theoretical considerations to state that "when the temperature increases, the wave length

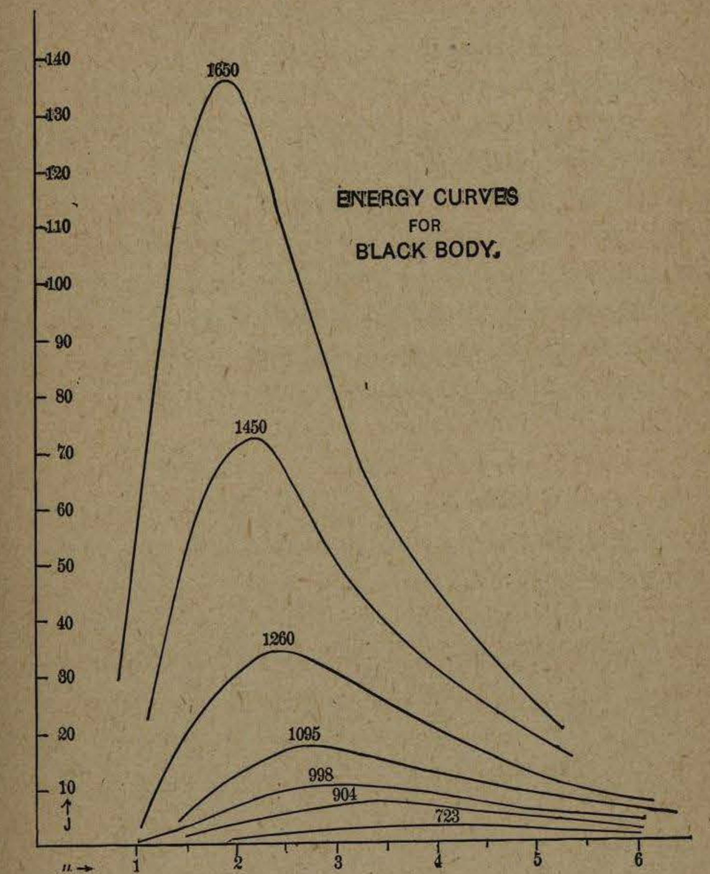

Fig. 87. Energy Curves.

of every monochromatic radiation diminishes in such a way that the product of the temperature and the wave length is a constant,"

$$\lambda T = \lambda_0 T_0.$$

Hence for the wave length of the maximum energy, λ_m, we have

$$\lambda_m T = \text{const.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(I)}$$

This is known as the " Wien displacement law " and is simply a mathematical statement of the fact that as the temperature of the radiating source is changed the wave length having maximum energy in the spectrum will be changed in such a way that the product of this wave length and the corresponding absolute temperature of the source, T, is equal to a constant. Wien then combined the above relation with the Stefan-Boltzmann law and was led to the relation that

$$I_{\max.} T^{-5} = \text{const.} = B, \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(II)}$$

in which $I_{\max.}$ indicates the energy corresponding to the wave length of the maximum energy and T is the absolute temperature of the radiating source (black body). Both of these generalizations of Wien for the radiations emitted by a black body have received the most convincing experimental verification throughout the widest ranges of measurable temperatures that are at present available to the experimentalist.

As an illustration of the experimental evidence in support of these two laws of radiation, the following table has been added, taken from a paper by Lummer and Pringsheim on the radiation from a black body:

λ_m	I_m	$A = \lambda_m T$	$B = I_m T^{-5}$	Absolute temperature.	$T = \sqrt[5]{\frac{I_m}{B_{\text{mean}}}}$	Diff.
4.53	2.026	2814	2190.10^{-17}	621.2°	621.3	+0.1°
4.08	4.28	2950	2166	723.0	721.5	−1.5
3.28	13.66	2980	2208	908.5	910.1	+1.6
2.96	21.50	2956	2166	998.5	996.5	−2.0
2.71	34.0	2966	2164	1094.5	1092.3	−2.2
2.35	68.8	2959	2176	1259.0	1257.5	−1.5
2.04	145.0	2979	2184	1460.4	1460.0	−0.4
1.78	270.8	2928	2246	1646.0	1653.5	+7.5
Mean........		2940	2188.10^{-17}			

As will be seen, these results of experiment are in most satisfactory agreement with these laws, when one considers the experimental difficulties that are involved in the measurements. In the value for B the temperature enters to the fifth power, so that a small error in the temperature produces a very marked effect on the value of B. Paschen later obtained $\lambda_m T = 2920$.

Wien also published the result of a further theoretical investigation on the spectral distribution of energy in the radiation of a black body, in which he was led to the conclusion that the energy I corresponding to any wave length was represented by

$$I = c_1 \lambda^{-5} \epsilon^{-\frac{c_2}{\lambda T}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(III)}$$

where I is the energy corresponding to wave length λ, T is the absolute temperature of the radiating black body, ϵ is the base of the natural system of logarithms, and c_1 and c_2 are constants. We have further the relation: $c_2 = 5\,\lambda_m T$, to which we may assign the value $c_2 = 14{,}500$.

The subsequent experimental work of Beckman, Rubens, and others has shown that Wien's distribution law does not hold for long wave lengths, although it amply suffices throughout the whole visible spectrum, and may be applied in all cases where $\lambda T < 3000$.

Planck has deduced an expression analogous to Wien's which applies with exactness for all wave lengths and temperatures. His law, which reduces to Wien's for small values of λ, may be written

$$I = c_1 \lambda^{-5} \left(\epsilon^{\frac{c_2}{\lambda T}} - 1 \right)^{-1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IV)}$$

in which $c_2 = 4.965\,\lambda_m T$, a more exact solution than that given by Wien's law III.

Other radiation laws have also been suggested, but Planck's seems to best satisfy both experiment and theory.

For the radiation from all substances that have been examined experimentally, it has been found generally that the "displacement law,"

$$\lambda_{m_1} T = \text{const.} = A_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(Ia)}$$

still holds true, although the radiation may depart far from that for a black body. In this case, however, the value of the constant is different from that for a black body. Thus for polished platinum Lummer and Pringsheim found $A_1 = 2626$.