such a knowledge of determinants and the method of mathematical induction as may easily be acquired by a freshman in a week or two. Nevertheless, the book is not intended for wholly immature readers, but rather for students who have had two or three years' training in the elements of higher mathematics, particularly in analytic geometry and the calculus. In fact, a good elementary knowledge of analytic geometry is indispensable.

The exercises at the ends of the sections form an essential part of the book, not merely in giving the reader an opportunity to think for himself on the subjects treated, but also, in many cases, by supplying him with at least the outlines of important additional theories. As illustrations of this we may mention Sylvester's Law of Nullity (page 80), orthogonal transformations (page 154 and page 173), and the theory of the invariants of the biquadratic binary form (page 260).

On a first reading of Chapters I–VII, it may be found desirable to omit some or all of sections 10, 11, 18, 19, 20, 25, 27, 34, 35. The reader may then either take up the subject of quadratic forms (Chapters VIII–XIII), or, if he prefer, he may pass directly to the more general questions treated in Chapters XIV–XIX.

The chapters on Elementary Divisors (XX–XXII) form decidedly the most advanced and special portion of the book. A person wishing to read them without reading the rest of the book should first acquaint himself with the contents of sections 19 (omitting Theorem 1), 21–25, 36, 42, 43.

In a work of this kind, it has not seemed advisable to give many bibliographical references, nor would an acknowledgement at this point of the sources from which the material has been taken be feasible. The work of two mathematicians, however, Kronecker and Frobenius, has been of such decisive influence on the character of the book that it is fitting that their names receive special mention here. The author would also acknowledge his indebtedness to his colleague, Professor Osgood, for suggestions and criticisms relating to Chapters XIV–XVI.

This book has grown out of courses of lectures which have been delivered by the author at Harvard University during the last ten years. His thanks are due to Mr. Duval, one of his former pupils, without whose assistance the book would probably never have been written.

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