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BY

MAXIME BÔCHER

PROFESSOR OF MATHEMATICS IN HARVARD UNIVERSITY

PREPARED FOR PUBLICATION WITH THE COÖPERATION OF

E. P. R. DUVAL

INSTRUCTOR IN MATHEMATICS IN THE UNIVERSITY OF WISCONSIN

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PREFACE

AN American student approaching the higher parts of mathematics usually finds himself unfamiliar with most of the main facts of algebra, to say nothing of their proofs. Thus he has only a rudimentary knowledge of systems of linear equations, and he knows next to nothing about the subject of quadratic forms. Students in this condition, if they receive any algebraic instruction at all, are usually plunged into the detailed study of some special branch of algebra, such as the theory of equations or the theory of invariants, where their lack of real mastery of algebraic principles makes it almost inevitable that the work done should degenerate to the level of purely formal manipulations. It is the object of the present book to introduce the student to higher algebra in such a way that he shall, on the one hand, learn what is meant by a proof in algebra and acquaint himself with the proofs of the most fundamental facts, and, on the other, become familiar with many important results of algebra which are new to him.

The book being thus intended, not as a compendium, but really, as its title states, only as an *introduction* to higher algebra, the attempt has been made throughout to lay a sufficiently broad foundation to enable the reader to pursue his further studies intelligently, rather than to carry any single topic to logical completeness. No apology seems necessary for the omission of even such important subjects as Galois's Theory and a systematic treatment of invariants. A selection being necessary, those subjects have been chosen for treatment which have proved themselves of greatest importance in geometry and analysis, as well as in algebra, and the relations of the algebraic theories to geometry have been emphasized throughout. At the same time it must be borne in mind that the subject primarily treated is algebra, not analytic geometry, so that such geometric information as is given is necessarily of a fragmentary and somewhat accidental character.

No algebraic knowledge is presupposed beyond a familiarity with elementary algebra up to and including quadratic equations, and such a knowledge of determinants and the method of mathematical induction as may easily be acquired by a freshman in a week or two. Nevertheless, the book is not intended for wholly immature readers, but rather for students who have had two or three years' training in the elements of higher mathematics, particularly in analytic geometry and the calculus. In fact, a good elementary knowledge of analytic geometry is indispensable.

The exercises at the ends of the sections form an essential part of the book, not merely in giving the reader an opportunity to think for himself on the subjects treated, but also, in many cases, by supplying him with at least the outlines of important additional theories. As illustrations of this we may mention Sylvester's Law of Nullity (page 80), orthogonal transformations (page 154 and page 173), and the theory of the invariants of the biquadratic binary form (page 260).

On a first reading of Chapters I–VII, it may be found desirable to omit some or all of sections 10, 11, 18, 19, 20, 25, 27, 34, 35. The reader may then either take up the subject of quadratic forms (Chapters VIII–XIII), or, if he prefer, he may pass directly to the more general questions treated in Chapters XIV–XIX.

The chapters on Elementary Divisors (XX–XXII) form decidedly the most advanced and special portion of the book. A person wishing to read them without reading the rest of the book should first acquaint himself with the contents of sections 19 (omitting Theorem 1), 21–25, 36, 42, 43.

In a work of this kind, it has not seemed advisable to give many bibliographical references, nor would an acknowledgement at this point of the sources from which the material has been taken be feasible. The work of two mathematicians, however, Kronecker and Frobenius, has been of such decisive influence on the character of the book that it is fitting that their names receive special mention here. The author would also acknowledge his indebtedness to his colleague, Professor Osgood, for suggestions and criticisms relating to Chapters XIV–XVI.

This book has grown out of courses of lectures which have been delivered by the author at Harvard University during the last ten years. His thanks are due to Mr. Duval, one of his former pupils, without whose assistance the book would probably never have been written.

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