Reduce to an integral or a mixed expression:

13. $\dfrac{x^2-9y^2+7}{x-3y}$.

14. $\dfrac{4a^3+20a^2b+27ab^2+9b^3}{2a+3b}$.

Reduce the following mixed expressions to fractions:

15. $x^2-xy+y^2-\dfrac{2y}{x+y}$.

16. $\dfrac{m^2+n^2}{m+n}-n-m$.

Simplify:

17. $\dfrac{s+t}{2rl}+\dfrac{s-t}{4rl}-\dfrac{s^2+t^2}{4rl(s+t)}$.

18. $\dfrac{a+x}{x+2}+\dfrac{a-x}{2-x}-\dfrac{2(x^2-2a)}{x^2-4}$.

19. $\left(\dfrac{x^3-y^3}{xy+y^2}\times\dfrac{x^2+xy}{x-y}\right)\div\left(\dfrac{x^2y+xy^2}{x^2+2xy+y^2}\times\dfrac{x+y}{y^2}\right)$.

20. $\left(\dfrac{a^2+2ab+b^2}{a^2-2ab+b^2}-\dfrac{a+b}{a-b}\right)\times\left(\dfrac{3a-3b}{2b}\div\dfrac{6a+6b}{2a-2b}\right)$.

21. What is meant by clearing an equation of fractions? State the axiom upon which it is based.

22. What precautions must be taken to secure correct results in solving equations that involve fractions?

Solve, and verify each result:

23. $\dfrac{5x}{2}-\dfrac{7x}{4}=6$.

24. $20x+\dfrac{10x}{3}=\dfrac{700}{6}$.

25. $\dfrac{x+7}{4}=\dfrac{x-3}{2}+1$.

26. $\dfrac{3x-2}{10}-\dfrac{7-2x}{5}=\dfrac{x-5}{15}$.

27. $\dfrac{x+3}{x-3}-\dfrac{x}{4}=\dfrac{12}{24}-\dfrac{x-3}{4}$.

28. $\dfrac{x+7}{x+2}+\dfrac{x+8}{3}=\dfrac{3x-15}{9}-3$.

29. $2.04x-3.1-2.95x=8.12-5x+1.05$.

SIMPLE EQUATIONS

ONE UNKNOWN NUMBER

207. The student already knows what an equation is; he has solved several different kinds; and he knows some of the kinds by name. In this chapter and the next he will meet some of the same kinds with the treatment extended to a few new forms and some additional methods of solution.

208. An equation that does not involve an unknown number in any denominator is called an **integral equation.**

$x+5=8$ and $\dfrac{2x}{3}+5=8$ are integral equations. Though the second equation contains a fraction, the unknown number x does not appear in the denominator.

209. An equation that involves an unknown number in any denominator is called a **fractional equation.**

$x+5=\dfrac{8}{x}$ and $\dfrac{2x}{x-1}=7$ are fractional equations.

210. Any number that satisfies an equation is called a **root** of the equation.

2 is a root of the equation $3x+4=10$.

211. Finding the roots of an equation is called **solving the equation.**

212. Two equations that have the same roots, each equation having all the roots of the other, are called **equivalent equations.**

$x+3=7$ and $2x=8$ are equivalent equations, each being satisfied for $x=4$ and for no other value of x.

Numerical Equations

213. By applying axioms to the solution of equations, the endeavor is made to change to *equivalent* equations, each simpler than the preceding, until an equation is obtained having the unknown number in one member and the known numbers in the other.

Solve, and verify each result:

1. $8v=24.$
2. $9r=54.$
3. $\frac{1}{4}r=1.5.$
4. $\frac{1}{2}x=2.5.$
5. $11+x=15.$
6. $20+x=30.$
7. $7y-5=2.$
8. $2z+3=9.$
9. $4h+3=7.$
10. $6r-7=5.$
11. $\frac{1}{2}b+3=8.$
12. $\frac{1}{5}x+2=6.$
13. $8x-7=3+6x.$
14. $7x+6=6x+8.$
15. $5x-10=2x+20.$
16. $4r-18=20+\frac{1}{5}r.$
17. $5n-(2n+3)=12.$
18. $17t+5(2-3t)=18.$
19. $5x-(4-\overline{6x-3})=26.$
20. $(2w-1)^2=4(w-3)^2.$
21. $21x+(x-4)^2=(5+x)^2.$
22. $(12x+6)\div 3=9-3x.$
23. $(x+1)(x+2)=11+x^2.$
24. $\frac{1}{4}x-4+\frac{3}{4}x=16+\frac{1}{4}x-10.$
25. $x(x+5)-6=x(x-1)+12.$
26. $3(2-x)-2(x+3)=6-2x.$
27. $x-(2+4x)=13-5(x+5).$
28. $2\{x-\overline{2x-2}\}=3\{x-(3x-3)\}.$
29. $6x-13-9x+x=4x-12+3x-6x-13.$
30. $36+5x-22-(7x-16)=5x+17-(2x+22).$
31. $2(r-5)(r-4)=(r-4)(r-3)+(r-2)(r-5).$
32. $12x-(6x-17x-15-x)=15-(2-17x+6x-4-12x).$

33. $3x-\frac{x}{5}=14.$
34. $\frac{2x}{3}-\frac{5}{6}=\frac{x}{4}.$
35. $\frac{2x}{3}-\frac{7x}{8}+\frac{5x}{18}+\frac{x}{24}=\frac{4}{9}.$
36. $\frac{3t-5}{4}-\frac{7t-13}{6}=3-\frac{t+3}{2}.$
37. $\frac{r}{2}(2-r)-\frac{r}{4}(3-2r)=\frac{r+10}{6}.$
38. $\frac{6r+3}{15}-\frac{3r-1}{5r-25}=\frac{2r-9}{5}.$
39. $\frac{s+1}{s+2}+\frac{s+6}{s+7}=\frac{s+2}{s+3}+\frac{s+5}{s+6}.$
40. $\frac{x^3+1}{x-1}-\frac{x^3-1}{x+1}=\frac{8}{x^2-1}+2x.$
41. $\frac{5x+2}{3}-\left(x-\frac{3x-1}{2}\right)=\frac{3x+19}{2}-\left(\frac{x+1}{6}+5\right).$

Literal Equations

214. 1. Solve the equation $\frac{x-b^2}{a}=\frac{x-a^2}{b}$ for x.

SOLUTION

$$\frac{x-b^2}{a}=\frac{x-a^2}{b}.$$

Clearing of fractions, $bx-b^3=ax-a^3.$

Transposing, etc., $ax-bx=a^3-b^3.$

$(a-b)x=a^3-b^3.$

Dividing by $(a-b)$, $x=a^2+ab+b^2.$

VERIFICATION. — Since a and b may have any numerical value, let $a=2$ and $b=1$; then $x=a^2+ab+b^2=4+2+1=7$, and the given equation becomes $\frac{7-1}{2}=\frac{7-4}{1}$, or $3=3$; consequently, the equation is satisfied for $x=a^2+ab+b^2$.

Solve for x, and verify each result:

2. $\frac{c^2-x}{nx}+\frac{n^2}{cx}=\frac{1}{c}$.

3. $1-\frac{ab}{x}=\frac{7}{ab}-\frac{49}{abx}$.

4. $rx+s^2=r^2-sx$.

5. $a^2x-b^4=b^2x-a^4$.

6. $\frac{a^2+b^2}{2\,bx}-\frac{a-b}{2\,bx^2}=\frac{b}{x}$.

7. $\frac{2\,x-a}{x-a}-\frac{x-a}{x+a}=1$.

8. $\frac{x}{b}-\frac{x+2\,b}{a}=\frac{a}{b}-3$.

9. $\frac{x-a}{b}+\frac{2\,x}{a}=5+\frac{6\,b}{a}$.

10. $6+l-2\,x=l(x-2)$.

11. $c^5x+d^{10}=c^{10}+d^5x$.

12. $\frac{x-2\,a}{a}+\frac{x}{b}=\frac{a^2+b^2}{ab}$.

13. $\frac{a^2}{bx}+\frac{b^2}{ax}=\frac{a+b}{ab}-\frac{3(a+b)}{x}$.

14. $6\,x+18(1-\frac{1}{2}\,a)=a(x-a)$.

15. $a^2(a-x)=abx+b^2(b+x)$.

16. $b(2\,x-9\,c-14\,b)=c(c-x)$.

17. $a(x-a-2\,b)+b(x-b)+c(x+c)=0$.

18. $(a-x)(x-b)+(a+x)(x-b)=(a-b)^2$.

19. $(m+x)^2+(m+x)(n-x)=(m+n)^2$.

20. $(a-b)(x-c)-(b-c)(x-a)=(c-a)(x-b)$.

21. $\frac{a-b+c}{x+a}=\frac{b-a+c}{x-a}$.

22. $\frac{x-1}{a-1}-\frac{a-1}{x-1}=\frac{x^2-a^2}{(a-1)(x-1)}$.

23. $\frac{1}{m+n}-\frac{2\,mn}{(m+n)^3}-\frac{m}{(m+n)^2}=\frac{x-n}{(m+n)^2}$.

24. $\frac{a+x}{a}-\frac{2\,x}{a+x}+\frac{x^2(x-a)}{a(a^2-x^2)}=\frac{1}{3}$.

Suggestion. — Simplify as much as possible before clearing the equation of fractions.

25. $\frac{x^2(b-x)}{b^2(b^2-x^2)}+\frac{5\,(b-x)}{b(x-b)}-\frac{x-b}{b^2}=0$.

Problems

215. Review the general directions for solving problems given on page 45.

1. What is the weight of a turtle, from which $6\frac{3}{4}$ pounds of tortoise shell is taken, if this is $\frac{1}{20}$ of the turtle's whole weight?

2. The powder and the shell used in a twelve-inch gun weigh 1265 pounds. The powder weighs 15 pounds more than $\frac{1}{4}$ as much as the shell. Find the weight of each.

3. One day three lace makers earned 80 cents. The beginner earned $\frac{1}{4}$ as much as the expert maker, and the average worker earned 3 times as much as the beginner. How much did each earn?

4. One ton of coal will make 8.7 tons of steam. If the *Lusitania* requires 1200 tons of coal a day for this purpose, how many tons of steam are required an hour?

5. A grocer paid \$8.50 for a molasses pump and 5 feet of tubing. He paid 12 times as much for the pump as for each foot of tubing. How much did the pump cost? the tubing?

6. In lighting a hall a certain number of 16-candle power electric lamps and twice as many 20-candle power lamps were used. The total illumination amounted to 224 candle power. Find the number of lamps of each kind used.

7. At the waterworks 2 large pumps and 4 small ones delivered 4800 gallons of water per minute. Each of the large pumps delivered 4 times as much water as each small pump. How many gallons per minute did each pump deliver?

8. The crew of a United States battleship in target practice made 11 hits in less than a minute. If $\frac{3}{4}$ of the number of shots fired was 9 times the number of misses, how many shots were fired?

9. The courtyard of a palace is 101 feet longer than it is wide. If its width were decreased 25 feet, its length would be twice its width. Find the dimensions of the courtyard.

10. In making 5000 pounds of brass there were used $8\frac{1}{2}$ times as much copper as tin, and twice as much tin as zinc. How many pounds of each metal were used?

11. A merchant bought 62 barrels of flour, part at \$$4\frac{3}{4}$ per barrel, the rest at \$$5\frac{1}{2}$ per barrel. If he paid \$320 for the flour, how many barrels of each grade did he buy?

12. A dealer paid \$185 for 25 boxes of candles. If he paid \$9 a box for part of them and \$6.50 a box for the rest, how many did he buy at each price?

13. A merchant purchased an assortment of bath robes for \$480. By selling $\frac{1}{4}$ of them at \$6 each, $\frac{1}{6}$ of them at \$7 each, $\frac{1}{3}$ of them at \$5 each, and the rest, or $\frac{1}{4}$ of them, at \$8 each, he gained \$128. How many did he sell at each price?

14. In a certain balloon race, the sum of the distances covered by the *Lotus II* and the *United States* was 1025 miles. The distance covered by the former was 50 miles more than $\frac{1}{2}$ of that covered by the latter. How far did each travel?

15. A newspaper reporter saved $\frac{1}{5}$ of his weekly salary, or \$1 more than was saved by an artist on the same paper, whose salary was \$5 greater but who saved only $\frac{1}{7}$ of it. How much did the reporter earn per week? the artist?

16. During a year of 365 days one locality had 6 days less of 'clear' weather than of 'cloudy' weather, and 4 days more of 'clear' than of 'partly cloudy' weather. Find the number of days of each kind of weather during the year.

17. The bark from a cork tree lost $\frac{1}{5}$ of its weight by being boiled. The boiled cork was then scraped, its weight thus being reduced $\frac{1}{4}$. How much did the cork weigh before and after these two operations, if the entire loss was 16 pounds?

18. At a certain depth a diver saw the sun as a reddish disk. At a depth 25 feet more than twice this depth it could still be faintly seen. If darkness occurred on descending 100 feet more, or at a total depth of 325 feet, at what depth did the sun appear as a reddish disk?

19. Find a fraction whose value is $\frac{4}{7}$ and whose denominator is 15 greater than its numerator.

20. Find a fraction whose value is $\frac{2}{3}$ and whose numerator is 3 greater than half of its denominator.

21. The numerator of a certain fraction is 8 less than the denominator. If each term of the fraction is decreased by 5, the resulting fraction equals $\frac{1}{3}$. What is the fraction?

22. An acre of wheat yielded 2000 pounds more of straw than of grain. The weight of the grain was .3 of the total weight of grain and straw. How many 60-pound bushels of wheat were produced?

23. The total diameter of a large wooden fly wheel is 30 feet. The number of inches in the thickness of the rim is 2 less than the number of feet from the center to the rim. How thick is the rim?

24. A shipment of 83,000 postal cards in two sizes weighed 472 pounds. The smaller cards weighed 5 pounds per 1000 and the larger ones weighed 6 pounds 3 ounces per 1000. Find the number of cards of each size in the shipment.

25. I paid 18¢ more for a screen door, 7 feet by 3 feet, than for 3 window screens, each $2\frac{2}{3}$ feet by 3 feet. Find the price per square foot in each case, if it was 3¢ less for window screens.

26. A grocer bought a box of soap containing 72 cakes for \$4.50. Some of the soap he sold at 3 cakes for 25¢, and the rest at 10¢ a cake. This gave a profit of \$1.90. How many cakes did he sell at each price?

27. It costs 3.6¢ less to travel 100 miles on the Swiss railroads under public management than it did to travel 90 miles when they were under private management. The average fare per mile under private management was 1.9¢ more than it is under public management. Find the rate per mile under each.

28. Two orange pickers together earned \$4.50 a day, and one of them picked 20 boxes more than the other. If the slower one had picked twice as many as he did, they would have earned \$6.50. How much did each receive a box?

29. A can do a piece of work in 8 days. If B can do it in 10 days, in how many days can both working together do it?

SOLUTION

Let $x =$ the required number of days.

Then, $\frac{1}{x} =$ the part of the work both can do in 1 day,

$\frac{1}{8} =$ the part of the work A can do in 1 day,

$\frac{1}{10} =$ the part of the work B can do in 1 day;

$\therefore \frac{1}{x} = \frac{1}{8} + \frac{1}{10}.$

Solving, $x = 4\frac{4}{9}$, the required number of days.

30. A can do a piece of work in 10 days, B in 12 days, and C in 8 days. In how many days can all together do it?

31. It takes a man 6 days to make a Panama hat, and a boy 7 days. How long would it take them, if they could work together?

32. The average amount of coal blasted out by a keg of powder can be mined by one man in 2 days and by another in 3 days. How long would it take them to mine it if they worked together?

33. A and B can dig a ditch in 10 days, B and C can dig it in 6 days, and A and C in $7\frac{1}{2}$ days. In what time can each man do the work?

SUGGESTION. — Since A and B can dig $\frac{1}{10}$ of the ditch in 1 day, B and C $\frac{1}{6}$ of it in 1 day, and A and C $\frac{2}{15}$ of it in 1 day, $\frac{1}{10} + \frac{1}{6} + \frac{2}{15}$ is twice the part they can all dig in 1 day.

34. A and B can load a car in $1\frac{7}{8}$ hours, B and C in $2\frac{1}{2}$ hours, and A and C in $2\frac{1}{3}$ hours. How long will it take each alone to load it?

35. In a certain year, New York State furnished 153.9 million pens, or 54 % of all that were made in the United States. How many pens were made in the United States?

36. The per cent of copper contained in an ancient die found in Egypt was $2\frac{1}{2}$ % more than 3 times the per cent of tin. If these metals formed $92\frac{1}{2}$ % of the die, what per cent of each did it contain?

37. Of the population of Mexico at one time the per cent of whites was $\frac{1}{2}$ that of Indians. Mixed races formed 5 % more than the per cent of Indians. Find the per cent of each.

38. Crude oil when refined produces $2\frac{1}{4}$ times as much kerosene as it does gasoline, and the remainder, which is 65 %, is fuel oil. If a certain refinery produces 2250 barrels of kerosene a day, what is its daily capacity of crude oil?

39. The units' digit of a two-digit number exceeds the tens' digit by 5. If the number increased by 63 is divided by the sum of its digits, the quotient is 10. Find the number.

SOLUTION

Let $x =$ the digit in tens' place.

Then, $x + 5 =$ the digit in units' place,

and $10x + (x + 5) =$ the number;

$\therefore \frac{10x + (x+5) + 63}{2x + 5} = 10;$

whence, $x = 2,$

and $x + 5 = 7.$

Therefore, the number is 27.

40. The tens' digit of a two-digit number is 3 times the units' digit. If the number diminished by 33 is divided by the difference of the digits, the quotient is 10. Find the number.

41. The tens' digit of a two-digit number is $\frac{1}{2}$ of the units' digit. If the number increased by 27 is divided by the sum of its digits, the quotient is $6\frac{1}{4}$. Find the number.

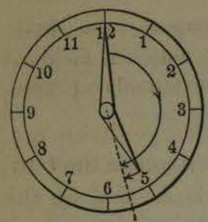
42. An officer, attempting to arrange his men in a solid square, found that with a certain number of men on a side he had 34 men over, but with one man more on a side he needed 35 men to complete the square. How many men had he?

SUGGESTION. — With x men on a side, the square contained x^2 men; with $(x + 1)$ men on a side, there were places for $(x + 1)^2$ men.

43. A regiment drawn up in the form of a solid square was reënforced by 240 men. When the regiment was formed again in a solid square, there were four more men on a side. How many men were there in the regiment at first?

44. At what time between 5 and 6 o'clock will the hands of a clock be together?

Solution



Starting with the hands in the position shown, at 5 o'clock, let x represent the number of minute spaces passed over by the minute hand after 5 o'clock until the hands come together. In the same time the hour hand will pass over $\frac{1}{12}$ of x minute spaces.

Since they are 25 minute spaces apart at 5 o'clock,

$$x - \frac{x}{12} = 25;$$

$\therefore x = 27\frac{3}{11}$, the number of minutes after 5 o'clock.

45. At what time between 1 and 2 o'clock will the hands of a clock be together?

46. At what time between 10 and 11 o'clock will the hands of a clock point in opposite directions?

47. At what two different times between 4 and 5 o'clock will the hands of a clock be 15 minute spaces apart?

48. Mr. Reynolds invested \$800, a part at 6 %, the rest at 5 %. The total annual interest was \$45. Find how much money he invested at each rate.

Suggestion. — Let x = the number of dollars invested at 6 %.
Then, $800 - x$ = the number of dollars invested at 5 %;

$$\therefore \tfrac{6}{100}x + \tfrac{5}{100}(800 - x) = 45.$$

49. A man put out \$4330 in two investments. On one of them he gained 12 %, and on the other he lost 5 %. If his net gain was \$251, what was the amount of each investment?

50. Mr. Bailey loaned some money at 4 % interest, but received \$48 less interest on it annually than Mr. Day, who had loaned $\frac{4}{5}$ as much at 6 %. How much did each man loan?

51. A man paid \$80 for insuring two houses for \$6000 and \$4000, respectively. The rate for the second house was $\frac{1}{8}$ % greater than that for the first. What were the two rates?

52. United States silver coins are $\frac{9}{10}$ pure silver, or '$\frac{9}{10}$ fine.' How much pure silver must be melted with 250 ounces of silver $\frac{4}{5}$ fine to render it of the standard fineness for coinage?

Suggestion. — Let x = the number of ounces of pure silver to be added.
Then, $\frac{4}{5}(250) + x$ = the number of ounces of pure silver after the addition.
Also, $\frac{9}{10}(250 + x)$ = the number of ounces of pure silver after the addition.

53. In an alloy of 90 ounces of silver and copper there are 6 ounces of silver. How much copper must be added that 10 ounces of the new alloy may contain $\frac{3}{5}$ of an ounce of silver?

54. If 80 pounds of sea water contain 4 pounds of salt, how much fresh water must be added that 49 pounds of the new solution may contain $1\frac{3}{4}$ pounds of salt?

55. Four gallons of alcohol 90 % pure is to be made 50 % pure. What quantity of water must be added?

56. Of 24 pounds of salt water, 12 % is salt. In order to have a solution that shall contain 4 % salt, how many pounds of pure water should be added?

57. A man rows downstream at the rate of 6 miles an hour and returns at the rate of 3 miles an hour. How far downstream can he go and return within 9 hours?

58. An airship traveled 11 miles with the wind in the same time as 1 mile against it. If it traveled 55 miles and returned in 12 hours, what was its rate against the wind? with the wind?

59. A train went 905.4 miles in a certain length of time. Another train with a speed 3 miles greater per hour covered 54 miles more in the same length of time. What was the speed of each train?

60. An express train whose rate is 40 miles an hour starts 1 hour and 4 minutes after a freight train and overtakes it in 1 hour and 36 minutes. How many miles does the freight train run per hour?

Solution of Formulæ

216. A **formula** expresses a principle or a rule in symbols. The solution of problems in commercial life, and in mensuration, mechanics, heat, light, sound, electricity, etc., often depends upon the ability to solve formulæ.

EXERCISES

217. 1. The circumference of a circle is equal to π (= 3.1416) times the diameter, or

$$C = \pi D.$$

Solve the formula for D and find, to the nearest inch, the diameter of the wheel of a locomotive, if the circumference of the wheel is 194.78 inches.

SOLUTION

From $C = \pi D$, $\pi D = C$.

$$\therefore D = \frac{C}{\pi} = \frac{194.78}{3.1416} = 62.0+.$$

Hence, to the nearest inch, the diameter is 62 inches.

2. Area of a triangle = $\frac{1}{2}$ (base × altitude), or

$$A = \tfrac{1}{2}bh.$$

Solve for b, then find the base of a triangle whose area is 600 square feet and altitude 40 feet.

3. The area of a trapezoid is equal to the product of the altitude and half the sum of the bases; that is,

$$A = h \cdot \tfrac{1}{2}(b + b').$$

The bases are b and b'. b' is read 'b-prime.'

Solve for h, then find the altitude of a trapezoid whose area is 96 square inches and whose bases are 14 inches and 10 inches, respectively.

4. The volume of a pyramid = $\frac{1}{3}$ (base × altitude), or

$$V = \tfrac{1}{3}Bh.$$

Solve for B, then find the area of the base of a pyramid whose volume is 252 cubic feet and altitude 9 feet.

5. The charge (c) for a telegram from New York to Chicago, 40¢ for 10 words and 3¢ for each additional word, may be found by the formula,

$$c = 40 + 3(n - 10),$$

in which n stands for the number of words.

Find the cost of a 16-word message.

Solve for n, then find how many words can be sent for \$1.

6. In the formula, $i = p \cdot \frac{r}{100} \cdot t$,

i denotes the interest on a principal of p dollars at simple interest at r% for t years.

Solve for t, then find the time \$300 must be on interest at 5% to yield \$60 interest.

Solve for r. At what rate of interest will \$4500 yield \$900 interest in 5 years?

Solve for p. What principal at $3\frac{1}{2}$% will yield \$210 annually?

7. The formula for the space (s) passed over by a body that moves with uniform velocity (v) during a given time (t) is

$$s = vt.$$

Solve for v, then find the velocity of sound when the conditions are such that it travels 8640 feet in 8 seconds.

8. The formula for converting a temperature of F degrees Fahrenheit into its equivalent temperature of C degrees Centigrade is

$$C = \tfrac{5}{9}(F - 32).$$

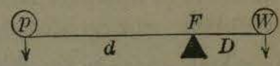
Solve for F and express 25° Centigrade (the mean annual temperature in Havana) in degrees Fahrenheit.

Solve:

9. $s = \frac{1}{2}at^2$, for a.
10. $F = Ma$, for a.
11. $W = Fs$, for s.
12. $P = I^2R$, for R.
13. $Mv_1 = mv_2$, for m.
14. $E = \frac{1}{2}Mv^2$, for M.
15. $s = v_0t + \frac{1}{2}at^2$, for a.
16. $s = \frac{1}{2}a(2t - 1)$, for t.

17. Any sort of a bar resting on a fixed point or edge is called a **lever**; the point or edge is called the **fulcrum**.

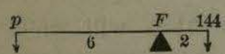
A lever will just balance when the numerical product of the power (p) and its distance (d) from the fulcrum (F) is equal to the numerical product of the weight (W) and its distance (D) from the fulcrum; that is, when

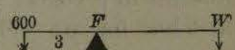


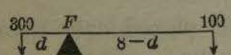
$$pd = WD.$$

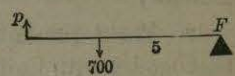
Solve for W and find what weight a power of 150 (pounds) will support by means of the lever shown, if $d = 7$ (feet) and $D = 3$ (feet).

Find for what values of p, d, W, or D the following levers will balance, each lever being 8 feet long:

18. 

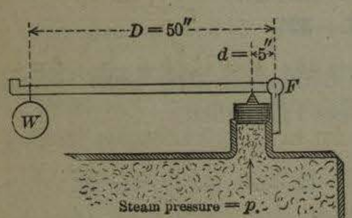
20. 

19. 

21. 

22. Philip, who weighs 114 pounds, and William, who weighs 102 pounds, are balanced on the ends of a 9-foot plank. Neglecting the weight of the plank, how far is Philip from the fulcrum?

23. The figure illustrates the lever of a safety valve, the power being the steam pressure (p) acting on the end of the piston above. The area of the end of the piston is 16 square inches. What weight (W) must be hung on the end of the lever so that when the steam pressure rises to 100 pounds per square inch the piston will rise and allow steam to escape?



24. The number of pounds pressure (P) on A square feet of surface of any body submerged to a depth of h feet in a liquid that weighs w pounds per cubic foot is given by the formula

$$P = wAh.$$

Fresh water weighs about $62\frac{1}{2}$ pounds per cubic foot, and ordinary sea water about 64 pounds per cubic foot.

Find the pressure on 1 square foot of surface at the bottom of a standpipe in which the water is 30 feet high; at the bottom of the ocean at a depth of 3000 feet.

25. Solve $P = wAh$ for h and find the value of h when $P = 5000$, $w = 62\frac{1}{2}$, and $A = 8$.

26. At what depth in fresh water will the pressure on an object having a total area of 4 square feet be 2000 pounds?

27. How deep in the ocean can a diver go, without danger, in a suit of armor that can sustain safely a pressure of 140 pounds per square inch (20,160 pounds per square foot)?

28. If the pressure per square foot on the bottom of a tank holding 18 feet of petroleum is 990 pounds, what is the weight of the petroleum per cubic foot?

29. The side of a chest lying in 25 feet of water was 5 square feet in area and sustained a pressure of 8000 pounds. Was the chest submerged in fresh water or in salt water?

Solve:

30. $\frac{E}{E'} = \frac{R}{R'}$, for R.

31. $I = \frac{E}{R + r}$, for r.

32. $a = \frac{v_1 - v_0}{t}$, for v_1.

33. $V = V_0\left(1 + \frac{t}{273}\right)$, for t.

34. Solve $\frac{E}{e} = \frac{R + r}{r}$, for R; for r.

35. Solve $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ for f_1; for f_2.

36. Solve $\frac{1}{8}Wl = \frac{SI}{c}$, for W; for S; for $\frac{I}{c}$.

SIMULTANEOUS SIMPLE EQUATIONS

TWO UNKNOWN NUMBERS

218. In the equation $x + y = 12$,
x and y may have an unlimited number of pairs of values,

as $\quad x = 1$ and $y = 11$;

or $\quad x = 2$ and $y = 10$; etc.

For since $\quad y = 12 - x$,
if any value is assigned to x, a corresponding value of y may be obtained.

An equation that is satisfied by an unlimited number of sets of values of its unknown numbers is called an **indeterminate equation.**

219. PRINCIPLE. — *Any single equation involving two or more unknown numbers is indeterminate.*

220. The equations
$$\left.\begin{aligned}2x + 2y &= 10\\ 3x + 3y &= 15\end{aligned}\right\}$$
and

express but one relation between x and y; namely, that their sum is 5. In fact, the equations are *equivalent* to

$$x + y = 5$$

and to each other. Such equations are often called **dependent equations**, for either may be *derived* from the other.

221. The equations
$$\left.\begin{aligned}x + y &= 5\\ x - y &= 1\end{aligned}\right\}$$
express two distinct relations between x and y, namely, that

their sum is 5 and their difference is 1. The equations cannot be reduced to the same equation; that is, they are not *equivalent*.

Equations that express different relations between the unknown numbers involved, and so cannot be reduced to the same equation, are called **independent equations.**

222. Each of the equations
$$\left.\begin{aligned}x + y &= 5\\ x - y &= 1\end{aligned}\right\}$$
is satisfied separately by an unlimited number of sets of values of x and y, but these letters have only one set of values in both equations, namely,

$$x = 3 \text{ and } y = 2.$$

Two or more equations that are satisfied by the same set or sets of values of the unknown numbers form a **system of simultaneous, or consistent, equations.**

223. The equations
$$\left.\begin{aligned}x + y &= 5\\ x + y &= 7\end{aligned}\right\}$$
have no set of values of x and y in common

Such equations are called **inconsistent equations.**

224. The student is familiar with the methods of solving simple equations involving *one* unknown number. The general method of solving a system of two independent simultaneous simple equations in *two* unknown numbers, as
$$\left.\begin{aligned}x + y &= 5\\ x - y &= 3\end{aligned}\right\}$$
is to combine the equations, using axioms 1–5 (§§ 68, 74) in such a way as to obtain an equation involving x alone, and another involving y alone, which may be solved separately by previous methods.

The process of deriving from a system of simultaneous equations another system involving fewer unknown numbers is called **elimination.**

Elimination by Addition or Subtraction

225. Elimination by addition or subtraction has been discussed and applied to the solution of simultaneous equations in §§ 99–103.

EXERCISES

226. 1. Solve the equations $2x + 3y = 7$ and $3x + 4y = 10$.

SOLUTION

$$\begin{cases} 2x + 3y = 7, & (1) \\ 3x + 4y = 10. & (2) \end{cases}$$

(1) × 4, $8x + 12y = 28.$ (3)

(2) × 3, $9x + 12y = 30.$ (4)

(4) − (3), $x = 2.$ (5)

Substituting (5) in (1), $4 + 3y = 7.$

$\therefore y = 1.$

To *verify*, substitute 2 for x and 1 for y in the given equations.

RULE. — *If necessary, multiply or divide the equations by such numbers as will make the coefficients of the quantity to be eliminated numerically equal.*

Eliminate by addition if the resulting coefficients have unlike signs, or by subtraction if they have like signs.

Solve by addition or subtraction, and verify results:

2. $\begin{cases} 7x - 5y = 52, \\ 2x + 5y = 47. \end{cases}$

3. $\begin{cases} 3x + 2y = 23, \\ x + y = 8. \end{cases}$

4. $\begin{cases} 3x - 4y = 7, \\ x + 10y = 25. \end{cases}$

5. $\begin{cases} 2x - 10y = 15, \\ 2x - 4y = 18. \end{cases}$

6. $\begin{cases} 3d + 4y = 25, \\ 4d + 3y = 31. \end{cases}$

7. $\begin{cases} 5p + 6q = 32, \\ 7p - 3q = 22. \end{cases}$

8. $\begin{cases} 3a + 6z = 39, \\ 9a - 4z = 51. \end{cases}$

9. $\begin{cases} 6x - 5y = 33, \\ 4x + 4y = 44. \end{cases}$

10. $\begin{cases} 2a + 3b = 17, \\ 3a + 2b = 18. \end{cases}$

11. $\begin{cases} 3m + 11n = 67, \\ 5m - 3n = 5. \end{cases}$

Elimination by Comparison

227. If $x = 8 - y,$ (1)

and also $x = 2 + y,$ (2)

by axiom 5, the two expressions for x must be equal.

$$\therefore 8 - y = 2 + y.$$

By *comparing* the values of x in the given equations, (1) and (2), we have eliminated x and obtained an equation involving y alone.

This method is called **elimination by comparison.**

EXERCISES

228. 1. Solve the equations $2x - 3y = 10$ and $5x + 2y = 6$.

SOLUTION

$$\begin{cases} 2x - 3y = 10, & (1) \\ 5x + 2y = 6. & (2) \end{cases}$$

From (1), $x = \frac{10 + 3y}{2}.$ (3)

From (2), $x = \frac{6 - 2y}{5}.$ (4)

Comparing the values of x in (3) and (4),

$$\frac{10 + 3y}{2} = \frac{6 - 2y}{5}.$$

Solving, $y = -2.$

Substituting -2 for y in either (3) or (4),

$x = 2.$

To *verify*, substitute 2 for x and -2 for y in the given equations.

RULE. — *Find an expression for the value of the same unknown number in each equation, equate the two expressions, and solve the equation thus formed.*

Solve by comparison, and verify results:

2. $\begin{cases} 3x - 2y = 10, \\ x + y = 70. \end{cases}$

3. $\begin{cases} 5x + y = 22, \\ x + 5y = 14. \end{cases}$

4. $\begin{cases} 2x + 3y = 24, \\ 5x - 3y = 18. \end{cases}$

5. $\begin{cases} 3x + 5y = 14, \\ 2x - 3y = 3. \end{cases}$

6. $\begin{cases} 3v + 2y = 36, \\ 5v - 9y = 23. \end{cases}$

7. $\begin{cases} 2y + 3x = 22, \\ 7x - 3y = 13. \end{cases}$

8. $\begin{cases} 2s + 7t = 8, \\ 3s + 9t = 9. \end{cases}$

9. $\begin{cases} 4u + 6v = 19, \\ 3u - 2v = \frac{9}{2}. \end{cases}$

10. $\begin{cases} 4v + 3w = 34, \\ 11v + 5w = 87. \end{cases}$

11. $\begin{cases} 4x - 13y = 5, \\ 3x + 11y = -17. \end{cases}$

12. $\begin{cases} 4x + 3y = 10, \\ 12x - 11y = -10. \end{cases}$

13. $\begin{cases} 18x - 3y = 4y, \\ 1 - 4x + 3y = 27. \end{cases}$

Elimination by Substitution

229. Given $3x + 2y = 27,$ (1)

and $x - y = 4.$ (2)

On solving (2) for x, its value is found to be $x = 4 + y$.

If $4 + y$ is *substituted* for x in (1), $3x$ will become $3(4 + y)$, and the resulting equation

$$3(4 + y) + 2y = 27 \quad (3)$$

will involve y only, x having been eliminated.

Solving (3), $y = 3$.

Substituting 3 for y in (2), $x = 7$.

This method is called **elimination by substitution**.

RULE. — *Find an expression for the value of either of the unknown numbers in one of the equations.*

Substitute this value for that unknown number in the other equation, and solve the resulting equation.

EXERCISES

230. Solve by substitution, and verify results:

1. $\begin{cases} x - y = 4, \\ 4y - x = 14. \end{cases}$

2. $\begin{cases} x + y = 10, \\ 6x - 7y = 34. \end{cases}$

3. $\begin{cases} 3x - 4y = 26, \\ x - 8y = 22. \end{cases}$

4. $\begin{cases} 6y - 10x = 14, \\ y - x = 3. \end{cases}$

5. $\begin{cases} y + 1 = 3x, \\ 5x + 9 = 3y. \end{cases}$

6. $\begin{cases} 17 = 3x + z, \\ 7 = 3z - 2x. \end{cases}$

7. $\begin{cases} 4y = 10 - x, \\ y - x = 5. \end{cases}$

8. $\begin{cases} 7z - 3x = 18, \\ 2z - 5x = 1. \end{cases}$

9. $\begin{cases} 3 - 15y = -x, \\ 3 + 15y = 4x. \end{cases}$

10. $\begin{cases} 1 - x = 3y, \\ 3(1 - x) = 40 - y. \end{cases}$

231. Three standard methods of elimination have been given. Though each is applicable under all circumstances, in special cases each has its peculiar advantages. The student should endeavor to select the method best adapted or to invent a method of his own.

EXERCISES

232. Solve by any method, verifying all results:

1. $\begin{cases} x + z = 13, \\ x - z = 5. \end{cases}$

2. $\begin{cases} 3x + y = 10, \\ x + 3y = 6. \end{cases}$

3. $\begin{cases} 4x + 5y = -2, \\ 5x + 4y = 2. \end{cases}$

4. $\begin{cases} 5x - y = 28, \\ 3x + 5y = 28. \end{cases}$

5. $\begin{cases} x + 3 = y - 3, \\ 2(x + 3) = 6 - y. \end{cases}$

6. $\begin{cases} 5x - y = 12, \\ x + 3y = 12. \end{cases}$

7. $\begin{cases} 4(2 - x) = 3y, \\ 2(2 - x) = 2(y - 2). \end{cases}$

8. $\begin{cases} (x + 1) + (y - 2) = 7, \\ (x + 1) - (y - 2) = 5. \end{cases}$

Eliminate before or after clearing of fractions, as may be more advantageous:

9. $\begin{cases} x+\dfrac{y}{3}=11, \\ \dfrac{x}{3}+3y=21. \end{cases}$

11. $\begin{cases} \dfrac{x}{2}-\dfrac{2y}{3}=-2, \\ \dfrac{5x}{2}+\dfrac{y}{3}=12. \end{cases}$

10. $\begin{cases} \dfrac{3x}{4}+\dfrac{4y}{5}=21, \\ \dfrac{2x}{3}+\dfrac{3y}{5}=17. \end{cases}$

12. $\begin{cases} \dfrac{x+y}{2}-\dfrac{x-y}{3}=8, \\ \dfrac{x+y}{3}+\dfrac{x-y}{4}=11. \end{cases}$

13. $\begin{cases} x+\frac{1}{2}(3x-y-1)=\frac{1}{4}+\frac{3}{4}(y-1), \\ \frac{1}{5}(4x+3y)=\frac{1}{10}(7y+24). \end{cases}$

Equations of the form $\dfrac{a}{x}+\dfrac{b}{y}=c$, though not simple equations, may be solved as simple equations for some of their roots by first regarding $\dfrac{1}{x}$ and $\dfrac{1}{y}$ as the unknown numbers.

14. Solve the equations $\begin{cases} \dfrac{4}{x}-\dfrac{3}{y}=\dfrac{14}{5}, & (1) \\ \dfrac{4}{x}+\dfrac{10}{y}=\dfrac{50}{3}. & (2) \end{cases}$

SOLUTION. (2) — (1), $\dfrac{13}{y}=\dfrac{208}{15}.$

$\therefore \dfrac{1}{y}=\dfrac{16}{15}.$ (3)

Substituting (3) in (1), $\dfrac{4}{x}-\dfrac{48}{15}=\dfrac{14}{5}.$

$\therefore \dfrac{1}{x}=\dfrac{3}{2}.$ (4)

From (4) and (3), $x=\dfrac{2}{3}$ and $y=\dfrac{15}{16}.$

Solve, and verify results:

15. $\begin{cases} \dfrac{5}{x}-\dfrac{3}{y}=-2, \\ \dfrac{25}{x}+\dfrac{1}{y}=6. \end{cases}$

18. $\begin{cases} \dfrac{2}{x}-\dfrac{3}{y}=5, \\ \dfrac{5}{x}-\dfrac{2}{y}=7. \end{cases}$

16. $\begin{cases} \dfrac{7}{x}-\dfrac{8}{y}=-1, \\ \dfrac{1}{x}+\dfrac{3}{y}=\dfrac{25}{28}. \end{cases}$

19. $\begin{cases} \dfrac{4}{x}+\dfrac{3}{y}=\dfrac{9}{8}, \\ \dfrac{3}{x}+\dfrac{4}{y}=\dfrac{11}{12}. \end{cases}$

17. $\begin{cases} \dfrac{3}{2x}-\dfrac{1}{y}=-3, \\ \dfrac{5}{2x}+\dfrac{3}{y}=23. \end{cases}$

20. $\begin{cases} \dfrac{5}{3x}+\dfrac{4}{y}=3, \\ \dfrac{7}{y}-\dfrac{1}{6x}=2\frac{1}{6}. \end{cases}$

Literal Simultaneous Equations

233. 1. Solve the equations $\begin{cases} ax+by=m, \\ cx+dy=n. \end{cases}$

SOLUTION

$ax + by = m$ (1)

$cx + dy = n$ (2)

(1) × d, $adx + bdy = dm$ (3)

(2) × b, $bcx + bdy = bn$ (4)

(3) − (4), $(ad - bc)x = dm - bn$

$\therefore x = \dfrac{dm - bn}{ad - bc}.$ (5)

(1) × c, $acx + bcy = cm$ (6)

(2) × a, $acx + ady = an$ (7)

(7) − (6), $(ad - bc)y = an - cm$

$\therefore y = \dfrac{an - cm}{ad - bc}.$ (8)

In solving literal simultaneous equations, elimination is usually performed most easily by addition or subtraction.

13. A grocer bought 1416 oranges of two sizes. Of one kind it took 360 oranges to fill a box and of the other 48. If there were 10 boxes in all, find the number of boxes of each kind.

14. The cost of firing 20 shots from a Japanese battleship was \$4040. The shots from the large cannon cost \$400 each and every shot from the small cannon cost \$70. How many shots of each kind were fired?

15. In Berlin, Germany, a mason received 80¢ more in 5 days than a painter received in 6 days, each working 10 hours a day. The former earned the same in 9 days as the latter did in 12 days. What was the hourly wage of each man?

16. The champion National League baseball team one year won 62 games more than it lost. The team that came second played 154 games, winning 16 less than the first and losing 18 more than the first. How many games did each team win and how many did each lose?

17. During a rate war between rival steamship lines the passage for 2 immigrants from Bremen to New York cost \$7.14 less than the normal rate for 1, and the passage for 7 immigrants \$4.76 less than the normal cost for 3. Find the normal rate and the reduced rate.

18. A man noticed that a 15-word message by telegraph cost him 40¢ and a 22-word message 54¢, between the same two cities. Find the charge for the first 10 words and the charge for each additional word.

19. The United States imported 38 million bunches of bananas one year. The cost, at 30¢ each for the larger bunches and 20¢ each for the smaller ones, was 8.55 million dollars How many bunches of each size were imported?

20. A steam pipe was inclosed in a wooden case. The diameter of the pipe was $\frac{2}{7}$ of the diameter of the case. The radius of the case was 2 inches less than the diameter of the pipe. What was the diameter of each?

21. A man invested \$4000, a part at 5% and the rest at 4%. If the annual income from both investments was \$175, what was the amount of each investment?

22. At a factory where 1000 men and women were employed, the average daily wage was \$2.50 for a man and \$1.50 for a woman. If labor cost \$2340 per day, how many men were employed? how many women?

23. It required 60 inches of tape to bind the four edges of a card on which a photograph was mounted. The length of the card was 6 inches greater than the width. How many inches long was the card? how many inches wide?

24. The German railroads carried 153 million first-class and second-class passengers one year. The number would have been 180 million if 4 times as many had traveled first class. How many traveled first class? second class?

25. In one hour 1375 vehicles passed a merchant's door on Broadway, New York City. The horse-drawn vehicles would have equaled the automobiles in number, had there been 50 more of the former and 25 less of the latter. How many of each passed his door?

26. Probably the highest dock in the world is on the Victoria Nyanza. Its height above sea level is 50 feet less than 15 times its length, and the sum of its height and length is 3950 feet. Find its height above sea level.

27. The American and British tourists to Japan during a recent year numbered 2794. If there had been twice as many Americans and 3 times as many British, the number would have been 6682. How many tourists were there from each country?

28. The number of students attending the University of Berlin at one time was 9277 more than the number attending at Munich, and $\frac{1}{2}$ the number at Berlin plus $\frac{1}{3}$ the number at Munich equaled 9591. How many attended each university?

29. Under the present contract, it costs \$24.15 less a year per lamp to maintain electric lights in a certain city than it did under the previous one, and the expense of 6 lamps then was \$46.35 more than that of 7 lamps now. Find the present yearly expense per lamp.

30. On the same day two seats in the New York Stock Exchange sold for \$192,500. If one of them had sold for \$1500 less, and the other for \$1000 more, the prices of the two would have been equal. Find the price of each.

31. A man had 10 fox skins, some of which were silver fox, worth \$300 a pelt, and some black fox, worth \$750 a pelt. If he had had 3 less of the former and 3 more of the latter, the total value would have been \$6150. How many had he of each?

32. The quantity of peanuts raised in the United States in a year is 135 million pounds more than the quantity of all nuts imported, and $\frac{1}{5}$ of the former equals $\frac{1}{2}$ of the latter. Find the number of pounds of nuts imported and of peanuts raised in the United States.

33. The receipts from a football game were \$700. Admission tickets to the grounds were sold for 50¢, and to the grand stand for 25¢ in addition. If twice as many persons had purchased tickets for the grand stand, the receipts would have been \$800. How many tickets of each kind were sold?

34. A train of 25 cars loaded with iron ore was run out on a dock and the ore emptied into pockets beneath the tracks. The ore filled 7 pockets and $\frac{1}{2}$ of another. To fill this last pocket, then, required 16 tons less than 2 extra car loads. What was the capacity of a car? of a pocket?

35. If 100 pounds of soft coal in burning can evaporate 50 pounds more water than 6 gallons of oil, and if 60 pounds of coal can evaporate 10 pounds less water than 4 gallons of oil, how many pounds of water can 1 pound of coal evaporate? 1 gallon of oil?

36. A proposed tunnel under Bering Strait would be in three sections, each of which would be $\frac{1}{4}$ of a mile longer, and two of which together would be $12\frac{3}{4}$ miles longer than the Simplon tunnel. Find the length of the proposed tunnel; of the Simplon tunnel.

37. If 1 is added to the numerator of a certain fraction, the value of the fraction becomes $\frac{3}{4}$; if 2 is added to the denominator, the value of the fraction becomes $\frac{1}{2}$. What is the fraction?

SUGGESTION.—Let x = the numerator and y = the denominator.

38. If the numerator of a certain fraction is decreased by 2, the value of the fraction is decreased by $\frac{1}{2}$; but if the denominator is increased by 4, the value of the fraction is decreased by $\frac{5}{8}$. What is the fraction?

39. A certain number expressed by two digits is equal to 7 times the sum of its digits; if 27 is subtracted from the number, the difference will be expressed by reversing the order of the digits. What is the number?

SUGGESTION. — The sum of x tens and y units is $(10x + y)$ units; of y tens and x units, $(10y + x)$ units.

40. Find a number that is 3 greater than 6 times the sum of its two digits, if the units' digit is 2 less than the tens' digit.

41. A crew can row 8 miles downstream and back, or 12 miles downstream and halfway back in $1\frac{1}{2}$ hours. What is their rate of rowing in still water and the velocity of the stream?

42. A man rows 12 miles downstream and back in 11 hours. The current is such that he can row 8 miles downstream in the same time as 3 miles upstream. What is his rate of rowing in still water, and what is the velocity of the stream?

43. A quantity of wheat could be thrashed by two machines in 6 days, but the larger machine worked alone for 8 days and was then replaced by the smaller, which finished in 3 days. How long would it have taken the larger machine to thrash all of the wheat? the smaller machine?

THREE UNKNOWN NUMBERS

235. The student has been solving systems of *two* independent simultaneous equations involving *two* unknown numbers. In general,

PRINCIPLE. — *Every system of independent simultaneous simple equations, involving the same number of unknown numbers as there are equations, can be solved, and is satisfied by one and only one set of values of its unknown numbers.*

EXERCISES

236. 1. Solve $\begin{cases} x+2y+3z=14, & (1) \\ 2x+y+2z=10, & (2) \\ 3x+4y-3z=2. & (3) \end{cases}$

SOLUTION. — Eliminating z by combining (1) and (3),

$(1)+(3),\qquad 4x+6y=16. \qquad (4)$

Eliminating z by combining (2) and (3),

$(2)\times 3,\qquad 6x+3y+6z=30$

$(3)\times 2,\qquad 6x+8y-6z=4$

Adding, $\qquad 12x+11y\quad =34 \qquad (5)$

Eliminating x by combining (5) and (4),

$(4)\times 3,\qquad 12x+18y\quad =48 \qquad (6)$

$(6)-(5),\qquad 7y\quad =14;\ \therefore y=2.$

Substituting the value of y in (4), $4x+12=16;\ \therefore x=1.$

Substituting the values of x and y in (1),

$$1+4+3z=14;\ \therefore z=3.$$

VERIFICATION. — Substituting $x=1$, $y=2$, and $z=3$ in the given equations,

(1) becomes $1+4+9=14$, or $14=14$;

(2) becomes $2+2+6=10$, or $10=10$;

and (3) becomes $3+8-9=2$, or $2=2$;

that is, the given equations are satisfied for $x=1$, $y=2$, and $z=3$.

Solve, and verify all results:

2. $\begin{cases} x+3y-z=10, \\ 2x+5y+4z=57, \\ 3x-y+2z=15. \end{cases}$

3. $\begin{cases} x+y+z=53, \\ x+2y+3z=105, \\ x+3y+4z=134. \end{cases}$

4. $\begin{cases} x-y+z=30, \\ 3y-x-z=12, \\ 7z-y+2x=141. \end{cases}$

5. $\begin{cases} 8x-5y+2z=53, \\ x+y-z=9, \\ 13x-9y+3z=71. \end{cases}$

6. $\begin{cases} x+3y+4z=83, \\ x+y+z=29, \\ 6x+8y+3z=156. \end{cases}$

7. $\begin{cases} 3x-2y+z=2, \\ 2x+5y+2z=27, \\ x+3y+3z=25. \end{cases}$

8. $\begin{cases} x+\frac{1}{2}y+\frac{1}{3}z=32, \\ \frac{1}{3}x+\frac{1}{4}y+\frac{1}{5}z=15, \\ \frac{1}{4}x+\frac{1}{5}y+\frac{1}{6}z=12. \end{cases}$

9. $\begin{cases} \frac{1}{6}x-\frac{1}{5}y+\frac{1}{4}z=3, \\ \frac{1}{5}x-\frac{1}{4}y+\frac{1}{3}z=1, \\ \frac{1}{4}x-\frac{1}{3}y+\frac{1}{2}z=5. \end{cases}$

10. There are three numbers such that the sum of $\frac{1}{2}$ of the first, $\frac{1}{3}$ of the second, and $\frac{1}{4}$ of the third is 12; of $\frac{1}{3}$ of the first, $\frac{1}{4}$ of the second, and $\frac{1}{5}$ of the third is 9; and the sum of the numbers is 38. What are the numbers?

11. A, B, and C have certain sums of money. If A gives B \$100, they will have the same amount; if A gives C \$100, C will have twice as much as A; and if B gives C \$100, C will have 4 times as much as B. What sum has each?

12. A quantity of water sufficient to fill three jars of different sizes will fill the smallest jar 4 times; the largest jar twice with 4 gallons to spare; or the second jar 3 times with 2 gallons to spare. What is the capacity of each jar?

13. A contractor used 3 scows to convey sand from his dredge to the dumping ground. He was credited by the inspector for:

April 20, scows $a, b, c, a, b, c, a,$ and b, 8 loads, 3230 cu. yd.
April 21, scows $c, a, b, c, a, b,$ and c, 7 loads, 2820 cu. yd.
April 22, scows $a, b, c, a, b, c,$ and a, 7 loads, 2870 cu. yd.

Find the capacity of each scow.