6. A farmer keeps his chickens in a rectangular lot that is 4 times as long as it is wide. If its area is 2500 square feet, find its length and width.

7. At a luncheon with Menelik of Abyssinia there was a pile of bread containing 448 cubic feet. Its height was twice its width and its length was 14 feet. Find its height and width.

8. A large rectangular freight station in Atlanta, Georgia, covers an area of 41,750 square feet. If the length is 16.7 times the width, what is the width of the station?

9. An automobilist paid $3.60 for gasoline. If the number of cents he paid per gallon was 2 less than the number of gallons he bought, find how many gallons he bought and the price per gallon.

10. St. Louis has the largest steam whistle in the world. The number of times it is blown each day is 3 more than the number of dollars it costs to blow it once. How many times is it blown a day, if the cost for 12 days is $48?

11. In the Panama Canal Zone a washerwoman washed as many dozen pieces as she received dollars a dozen for her labor. If she had washed 2 dozen more, she would have received $15. How much did she receive a dozen?

12. The number of pounds of duck feathers that a man bought was the same as the number of cents that he paid a pound for them. If he had bought 10 pounds more, they would have cost $20. How many pounds did he buy?

13. The area of one of the largest photographic prints ever made is 180 square feet. Its dimensions are 18 times those of the picture from which it was enlarged. Find the dimensions of the picture, if its length is 2 inches greater than its width.

14. The cages holding canaries imported into this country are arranged in rows in crates. The number of rows in a crate is 2 less than 5 times the number of cages in a row. If there are 231 cages in a crate, how many rows are there?

# FRACTIONS

**167.** In algebra, an indicated division is called a **fraction**.

The fraction $\frac{a}{b}$ means $a \div b$ and is read '$a$ divided by $b$.'

**168.** The *dividend*, written above a line, is the **numerator**; the *divisor*, written below the line, is the **denominator**; the numerator and denominator are called the **terms** of the fraction.

An arithmetical fraction also indicates division, but the arithmetical notion is that a fraction is one or more of the *equal* parts of a unit; that is, in arithmetic, the terms of a fraction are *positive integers*, while in algebra they may be *any numbers whatever*.

**169.** The student will find no difficulty with algebraic fractions, if he will bear in mind that they are essentially the same as the fractions he has met in arithmetic. He will have occasion to change fractions to higher or lower terms; to write integral and mixed expressions in fractional form; to change fractions to integers or mixed numbers; to add, subtract, multiply, and divide with algebraic fractions just as he has learned to do with arithmetical fractions, except that *signs* must be considered in dealing with positive and negative numbers.

### Signs in Fractions

**170.** There are *three* signs to be considered in connection with a fraction; namely, the sign of the numerator, the sign of the denominator, and the sign written before the dividing line, called the **sign of the fraction**.

In $-\frac{x}{3z}$ the sign of the fraction is $-$, while the signs of its terms are $+$.

**171.** An expression like $\frac{-a}{-b}$ indicates a process in division, in which the quotient is to be found by dividing $a$ by $b$ and prefixing the sign according to the law of signs in division; that is,

$$\frac{-a}{-b}=+\frac{a}{b},\qquad \frac{+a}{+b}=+\frac{a}{b},$$

$$\frac{-a}{+b}=-\frac{a}{b},\qquad \frac{+a}{-b}=-\frac{a}{b}.$$

By observing the above fractions and their values the following principles may be deduced:

**172.** PRINCIPLES. — 1. *The signs of both the numerator and the denominator of a fraction may be changed without changing the sign of the fraction.*

2. *The sign of either the numerator or the denominator of a fraction may be changed, provided the sign of the fraction is changed.*

When either the numerator or the denominator is a polynomial, its sign is changed by changing the sign of each of its terms. Thus, the sign of $a-b$ is changed by writing it $-a+b$, or $b-a$.

EXERCISES

**173.** Reduce to fractions having positive numbers in both terms:

1. $\frac{-3}{-4}$.
2. $\frac{2}{-5}$.
3. $\frac{-a-x}{2x}$.
4. $\frac{-4c}{-b-y}$.
5. $-\frac{-a-b}{c+d}$.
6. $-\frac{-2}{-a-y}$.
7. $-\frac{-2-m}{2+n}$.
8. $\frac{-4(a+b)}{5(-x-y)}$.

**174.** In accordance with § 80,

PRINCIPLES. — 3. *The sign of either term of a fraction is changed by changing the signs of an odd number of its factors.*

4. *The sign of either term of a fraction is not changed by changing the signs of an even number of its factors.*



EXERCISES

**175.** 1. Show that $\frac{(a-b)(d-c)}{(c-a)(b-c)}=\frac{(a-b)(c-d)}{(a-c)(b-c)}$.

SOLUTION OR PROOF

Changing $(d-c)$ to $(c-d)$ changes the sign of *one* factor of the numerator and therefore changes the sign of the numerator (Prin. 3).

Similarly, changing $(c-a)$ to $(a-c)$ changes the sign of the denominator (Prin. 3).

We have changed the signs of both terms of the fraction. Therefore, the sign of the fraction is not affected (Prin. 1).

2. Show that $\frac{(b-a)(d-c)}{(c-b)(a-c)}=-\frac{(a-b)(c-d)}{(b-c)(a-c)}$.

SOLUTION OR PROOF

Changing the signs of *two* factors of the numerator does not change the sign of the numerator (Prin. 4).

Changing the sign of *one* factor of the denominator changes the sign of the denominator (Prin. 3).

Since we have changed the sign of only *one* term of the fraction, we must change the sign of the fraction (Prin. 2).

3. Show that $\frac{-b}{b-a}$ may be properly changed to $\frac{b}{a-b}$.

4. From $\frac{-a}{b-a+c}$ derive $\frac{a}{a-b-c}$ by proper steps.

5. Prove that $\frac{3}{1-x}=-\frac{3}{x-1}$; that $-\frac{2}{4-x^2}=\frac{2}{x^2-4}$.

6. Prove that $\frac{1}{(b-a)(c-b)}=\frac{1}{(a-b)(b-c)}$.

7. Prove that $\frac{(m-n)(m+n)}{(a-c)(b-a)}=\frac{-m^2+n^2}{(a-c)(a-b)}$.

8. Prove that $\frac{(a-b)(b-a+c)}{(y-x)(z-y)(z-x)}=\frac{(a-b)(a-b-c)}{(x-y)(y-z)(x-z)}$.

## REDUCTION OF FRACTIONS

**176.** The process of changing the form of an expression without changing its value is called **reduction.**

**177.** An expression, some of whose terms are integral and some fractional, is called a **mixed number,** or a **mixed expression.**

Thus, $a-\frac{a-b}{c}$, $\frac{x^2}{a^2}-2+\frac{a^2}{x^2}$, and $a-b+\frac{1}{ab}$ are mixed expressions.

**178. To reduce a fraction to an integer or a mixed expression.**

This change in form is made in algebra in the same manner as in arithmetic.

### EXERCISES

**179.** 1. Reduce to a mixed number: $\frac{13}{4}$; $\frac{ax+b}{x}$.

PROCESS

$$\frac{13}{4}=13\div 4=3+\frac{1}{4}=3\tfrac{1}{4}.$$

PROCESS

$$\frac{ax+b}{x}=(ax+b)\div x=a+\frac{b}{x}.$$

Explanation. — Since a fraction is an indicated division, by performing the division indicated the fraction is changed into the form of a mixed number.

Reduce to an integral or a mixed expression:

2. $\frac{27a}{9}$.

3. $\frac{26b}{13b}$.

4. $\frac{45x^2y^3z}{15xy}$.

5. $\frac{36ac+9c}{9c}$.

6. $\frac{a^2+c^2}{a}$.

7. $\frac{12x^2-6x}{6x}$.

8. Reduce $\frac{a^3-3a^2-a+1}{a^2}$ to a mixed number.

Solution. $\frac{a^3-3a^2-a+1}{a^2}=a-3+\frac{-a+1}{a^2}=a-3-\frac{a-1}{a^2}$.

Note. — It is not necessary to write the step $(a^3-3a^2-a+1)\div a^2$. The division should be continued until the undivided part of the numerator no longer contains the denominator.



Reduce to an integral or a mixed expression:

9. $\frac{4x^3-8x^2+2x-1}{2x}$.

10. $\frac{ab-bc-cd+d^2}{b}$.

11. $\frac{a^2x^3-ax^2-x-1}{ax}$.

12. $\frac{x^2-x-12}{x-4}$.

13. $\frac{x^3-2xy-y^3}{x-y}$.

14. $\frac{x^2-6xy+4y^2}{2xy}$.

15. $\frac{x^3-6x^2+14x-9}{x-2}$.

16. $\frac{x^4-3x^2+5x-1}{x-3}$.

17. $\frac{x^3+5x^2+3x-6}{x+2}$.

18. $\frac{x^4+2x^3-x^2+5}{x^3+x^2}$.

19. $\frac{a^3+9a^2+24a+18}{a+3}$.

20. $\frac{4a^5+12a^3-a^2+34}{2a^3+5}$.

21. $\frac{a^2+3ab-5b^2+bc}{a-2b}$.

22. $\frac{x^3-7x^2-4x+40}{x^2-3}$.

23. $\frac{a^3+3a^2b-ab^2+ab}{a^2+b}$.

24. $\frac{x^2-xy-3y^2-z}{x+y}$.

25. $\frac{x^{3n}-3x^{2n}+5x^n-3}{x^n-1}$.

26. $\frac{l^3-l^2t-7lt^2+3t^3}{l-3t}$.

27. $\frac{x^2+x^2y-y^2-xy^2-3}{x-y}$.

28. $\frac{3x^2y^2-12x^2+2y^2-3}{3x^2+2}$.

**180. To reduce a fraction to its lowest terms.**

Just as $\frac{3}{4}=\frac{9}{12}$ or $\frac{9}{12}=\frac{3}{4}$,

so $\frac{a}{b}=\frac{am}{bm}$ or $\frac{am}{bm}=\frac{a}{b}$. That is,

**181.** Principle. — *Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.*

**182.** A fraction is in its **lowest terms** when its terms have no *common* factor.

**183.** The product of all the common prime factors of two or more numbers is called their **highest common factor** (h. c. f.).

Thus, to find the h.c.f. of two expressions, as $9ax^2y + 3bx^2y$ and $6x^4y^2 - 6x^2y^4$, we first factor them. Since $9ax^2y + 3bx^2y = 3 \cdot x^2 \cdot y(3a + b)$ and $6x^4y^2 - 6x^2y^4 = 2 \cdot 3 \cdot x^2 \cdot y^2(x + y)(x - y)$, their *common* factors are 3, $x$, $x$, and $y$; hence, their h.c.f. $= 3x^2y$. (See also §§ 407–411.)

NOTE. — The number of *literal* factors in a term determines its **degree**, and the term of an expression that has the greatest number of literal factors determines the *degree of the expression*. Thus, $2a^2b$ is of the *third degree* and is *higher* than $5ab$, which is of the *second degree;* the expression $2a^2b + 5ab$, then, is of the *third degree.*

### EXERCISES

**184.** 1. Reduce to lowest terms: $\frac{20}{24}$; $\frac{21a^2x^2y}{30a^3xz}$.

PROCESS

$$\frac{20}{24} = \frac{2 \cdot 2 \cdot 5}{2 \cdot 2 \cdot 6} = \frac{5}{6}$$

PROCESS

$$\frac{21a^2x^2y}{30a^3xz} = \frac{3 \cdot 7a^2x^2y}{3 \cdot 10a^3xz} = \frac{7xy}{10az}$$

EXPLANATION. — Since a fraction is in its lowest terms when its terms have no common factor, a fraction may be reduced to its lowest terms by removing in succession all common factors of its numerator and denominator; or by dividing the terms by their *highest common factor.*

Reduce to lowest terms:

2. $\frac{a^2xy^3}{a^3xy}$.

3. $\frac{m^3n^3}{am^2n^4}$.

4. $\frac{a^2b^2x^2}{b^3xy^2}$.

5. $\frac{16m^2nx^2z^2}{40am^3yz^3}$.

6. $\frac{210bc^2d}{750ab^2c}$.

7. $\frac{-25x^2y^5z^2}{-100x^4y^3}$.

8. $\frac{-7a^2bcd^3}{42ab^2cd^4}$.

9. $\frac{tr^2 + ts^2}{3t}$.

10. $\frac{2x}{4x^2 - 6ax}$.

11. Reduce $\frac{bx - ax}{a^2 - b^2}$ to its lowest terms.

SOLUTION. $\frac{bx - ax}{a^2 - b^2} = \frac{x(b - a)}{(a + b)(a - b)}$

§ 174, Prin. 4, $= \frac{-x(a - b)}{(a + b)(a - b)} = \frac{-x}{a + b} = -\frac{x}{a + b}$.



Reduce to lowest terms:

12. $\frac{a^2 - b^2}{a^2 + 2ab + b^2}$.

13. $\frac{a^2 - 2ab + b^2}{b^2 - a^2}$.

14. $\frac{4a^2 - 9x^2}{8a^3 + 27x^3}$.

15. $\frac{6xy - 3x^2y}{x^4y - 8xy}$.

16. $\frac{3a^2b - 3b^3}{2b^4 - 2a^3b}$.

17. $\frac{x^4y + x^2y^3 + y^5}{x^6 - y^6}$.

18. $\frac{x^4y - x^2y^3 + y^5}{x^6 + y^6}$.

19. $\frac{x^3 - 6x^2 + 5x}{x^3 + 2x^2 - 35x}$.

20. $\frac{7x - 2x^2 - 3}{4 - 7x - 2x^2}$.

21. $\frac{x^2 - 2x^4 + x^5}{x^6 - x^2}$.

22. $\frac{a^3 + 2a^2b + ab^2}{a^5 - 2a^3b^2 + ab^4}$.

23. $\frac{x^3 + 5x^2 - 6x}{2x^2 - 2}$.

24. $\frac{x^3 - 7x + 6}{x^4 - 10x^2 + 9}$.

25. $\frac{20 - 21x + x^3}{x^4 - 26x^2 + 25}$.

26. $\frac{3a^2 + 4ax - 4x^2}{9a^2 - 12ax + 4x^2}$.

27. $\frac{x^3 + 1 + 3x^2 + 3x}{4 + 4x - x^2 - x^3}$.

28. $\frac{m - m^2 - n + mn}{m - mn + n^2 - n}$.

29. $\frac{am - an - m + n}{am - an + m - n}$.

30. $\frac{a^3 - b^3 - 3a^2b + 3ab^2}{3ab^2 - 3a^2b}$.

31. $\frac{x^3 + 5x^2 - 9x - 45}{x^3 + 3x^2 - 25x - 75}$.

32. $\frac{2ax - ay - 4bx + 2by}{4ax - 2ay - 2bx + by}$.

33. $\frac{x^3 + 2x^2 - 23x - 60}{x^3 - 11x^2 - 10x + 200}$.

**185.** **To reduce a fraction to an equal fraction having a given denominator or a given numerator.**

In order to change $\frac{3}{4}$ to a fraction whose denominator is 12, both terms must be multiplied by $12 \div 4$, or 3; similarly, to change $\frac{x}{z}$ to a fraction whose denominator is $nz^2$, both terms must be multiplied by $nz^2 \div z$, or $nz$.

**EXERCISES**

**186.** 1. Reduce $\frac{a}{a+b}$ to a fraction whose denominator is $a^2-b^2$.

PROCESS

$$(a^2-b^2)\div(a+b)=a-b.$$

Then, $$\frac{a}{a+b}=\frac{a(a-b)}{(a+b)(a-b)}=\frac{a^2-ab}{a^2-b^2}.$$

EXPLANATION. — Since the required denominator is $(a-b)$ times the given denominator, in order that the value of the fraction shall not be changed (§ 181) both terms of the fraction must be multiplied by $(a-b)$.

2. Reduce $\frac{5a}{6}$ to a fraction whose denominator is 42.

3. Reduce $\frac{3x}{11b}$ to a fraction whose denominator is $55b$.

4. Reduce $\frac{3a}{14x}$ to a fraction whose denominator is $84xy$.

5. Reduce $\frac{4a^2}{5y}$ to a fraction whose denominator is $20y^3$.

6. Reduce $\frac{x-3}{x-1}$ to a fraction whose denominator is $(x-1)^2$.

7. Reduce $\frac{2x-5}{2x+5}$ to a fraction whose denominator is $(2x+5)^2$.

8. Reduce $\frac{a}{3-a}$ to a fraction whose numerator is $3a+a^2$.

9. Reduce $\frac{x-y}{2x+y}$ to a fraction whose numerator is $x^2-y^2$.

10. Reduce $\frac{-1}{x-2}$ to a fraction whose denominator is $4-x^2$.

11. Reduce $\frac{ab}{3-b}$ to a fraction whose denominator is $b^2-9$.

12. Reduce $x-5$ to a fraction whose denominator is $x+5$.

13. Reduce $3t+2l$ to a fraction whose denominator is $2l-3t$.



**187. Reduction to lowest common denominator.**

In algebra, as in arithmetic, it is frequently desirable to reduce fractions that have different denominators to respectively equal fractions that have a *common* denominator.

**188.** In algebra, **lowest common denominator** corresponds to *least common denominator* in arithmetic.

The word 'lowest' has reference to the *degree* of the denominator.

**189.** It is not always easy to discover by inspection the *lowest common denominator* (l. c. d.), that is, the *lowest common multiple* (l. c. m.) of the given denominators. However, it may be found, as in arithmetic, by factoring the denominators, for it is the product of all their *different* prime factors, each factor used the greatest number of times that it occurs in any denominator. (See also §§ 412–414.)

Thus, if the given denominators are $ax-bx$, $a^2-b^2$, and $a^2-2ab+b^2$, on factoring we find: $ax-bx=x(a-b)$; $a^2-b^2=(a+b)(a-b)$; and $a^2-2ab+b^2=(a-b)(a-b)$.

Then, the factors of the l. c. d. are $x$, $a+b$, $a-b$, and $a-b$.

Hence, the l. c. d. $=x(a+b)(a-b)^2$.

**EXERCISES**

**190.** 1. Reduce to fractions having their lowest common denominator: $\frac{5}{6}$ and $\frac{3}{8}$; $\frac{a}{3bc}$ and $\frac{c}{6ab}$.

PROCESS

l. c. d. $=24$

$$\frac{5}{6}=\frac{5\times4}{6\times4}=\frac{20}{24}$$

$$\frac{3}{8}=\frac{3\times3}{8\times3}=\frac{9}{24}$$

PROCESS

l. c. d. $=6abc$

$$\frac{a}{3bc}=\frac{a\times2a}{3bc\times2a}=\frac{2a^2}{6abc}$$

$$\frac{c}{6ab}=\frac{c\times c}{6ab\times c}=\frac{c^2}{6abc}$$

EXPLANATION. — The l. c. m. of the given denominators is found for the l. c. d. in accordance with § 189. Then, each fraction is reduced to an equal fraction having this denominator, as in § 185.

NOTE. — All fractions should first be reduced to lowest terms.

2. Reduce $2m$ and $\frac{m+n}{m-n}$ to fractions having their lowest common denominator.

SUGGESTION. — First write $2m$ as a fraction with the denominator 1.

Reduce to fractions having their lowest common denominator:

3. $\frac{x}{2}$ and $\frac{3y}{5}$.

4. $\frac{2a}{5b}$ and $3x$.

5. $\frac{a^2b}{c^2d}$ and $\frac{ab^2}{cd^2}$.

6. $\frac{x^2}{2xy}$ and $\frac{3}{4ay}$.

7. $\frac{3xy}{cx}$ and $\frac{2ay}{3by}$.

8. $\frac{3}{x^2y^4}, \frac{-6}{x^3y^3}, \frac{3}{x^4y^2}$.

9. $\frac{3ab}{8a^2c}, \frac{7a^2}{4b^2c}, \frac{5a^3}{a^3bc^2}$.

10. $\frac{m-n}{a}, 2, \frac{a}{m+n}$.

11. $\frac{x+y}{2}, \frac{x-y}{4}, \frac{x^2-y^2}{6}$.

12. $\frac{x^2}{x^2-1}, \frac{x}{x+1}, \frac{x}{x-1}$.

13. $\frac{a^3}{a^4-16}, \frac{a}{a^2+4}, \frac{2a}{4-a^2}$.

SUGGESTION. — By § 172, Prin. 1, $\frac{2a}{4-a^2}=\frac{-2a}{a^2-4}$.

14. $\frac{4a}{b-a}, \frac{3b}{a+b}, \frac{1}{a^2-b^2}$.

15. $\frac{a}{1-ax}, \frac{x}{1+ax}, \frac{-ax}{ax-1}$.

16. $\frac{1}{x^2+7x+10}, \frac{1}{x^2+x-2}, \frac{1}{x^2+4x-5}$.

17. $\frac{3x}{x^2-3x+2}, \frac{x-1}{x^2-5x+6}, \frac{x+3}{x^2-4x+3}$.

18. $\frac{a+5}{a^2-4a+3}, \frac{a-2}{a^2-8a+15}, \frac{a+1}{a^2-6a+5}$.



## ADDITION AND SUBTRACTION OF FRACTIONS

**191**. The method of adding and subtracting fractions is the same in algebra as in arithmetic. In algebra, however, subtraction of fractions practically reduces to addition of fractions, for every fraction to be subtracted is really added with its sign changed (§ 64, Prin.).

Just as $\frac{1}{3}+\frac{1}{4}=\frac{4}{12}+\frac{3}{12}=\frac{4+3}{12}$,

so $\frac{a}{b}+\frac{c}{d}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$.

Also as $\frac{1}{3}-\frac{1}{4}=\frac{4}{12}-\frac{3}{12}=\frac{4-3}{12}$,

so $\frac{a}{b}-\frac{c}{d}=\frac{ad}{bd}-\frac{bc}{bd}=\frac{ad-bc}{bd}$.

### EXERCISES

**192**. 1. Add $\frac{3x}{4}, \frac{7x}{10}$, and $\frac{5z}{12}$.

SOLUTION. — Since the fractions have unlike denominators, they must be reduced to fractions having a common denominator. By § 189, the l. c. d. = 60.

$$\frac{3x}{4}+\frac{7x}{10}+\frac{5z}{12}=\frac{45x}{60}+\frac{42x}{60}+\frac{25z}{60}$$
$$=\frac{45x+42x+25z}{60}=\frac{87x+25z}{60}.$$

2. Subtract $\frac{x-2}{7}$ from $\frac{5x-1}{8}+\frac{x}{4}$.

SOLUTION

$$\frac{5x-1}{8}+\frac{x}{4}-\frac{x-2}{7}=\frac{35x-7}{56}+\frac{14x}{56}-\frac{8x-16}{56}$$
$$=\frac{35x-7+14x-(8x-16)}{56}$$
$$=\frac{35x-7+14x-8x+16}{56}=\frac{41x+9}{56}.$$

SUGGESTION. — When a fraction is preceded by the sign −, it is well for the beginner to inclose the numerator in a parenthesis, if it is a polynomial, as shown above.

Rule. — *Reduce the fractions to similar fractions having their lowest common denominator.*

*Change the signs of all the terms of the numerators of fractions preceded by the sign −, then find the sum of all the numerators, and write it over the common denominator.*

Reduce the resulting fraction to its lowest terms, if necessary.

Add:

3. $\frac{2x}{5}$ and $\frac{3x}{2}$.

4. $\frac{4a}{3}$ and $\frac{6b}{5}$.

5. $\frac{2a}{3b}$ and $\frac{3a}{2b}$.

6. $\frac{-5}{7x}$ and $\frac{-2}{3x}$.

Subtract:

7. $\frac{5m}{6}$ from $\frac{4m}{3}$.

8. $\frac{4x}{9}$ from $-\frac{x}{2}$.

9. $-\frac{y}{3}$ from $\frac{3}{y}$.

10. $\frac{a+b}{3}$ from $\frac{a-b}{2}$.

Simplify:

11. $\frac{2x+1}{3}+\frac{x-2}{4}-\frac{x-3}{6}+\frac{5-x}{2}$.

12. $\frac{x-2}{6}-\frac{x-4}{9}+\frac{2-3x}{4}-\frac{2x+1}{12}$.

13. $\frac{x-1}{3}-\frac{x-2}{18}-\frac{4x-3}{27}+\frac{1-x}{6}$.

14. $\frac{2-6x}{5}+\frac{4x-1}{2}-\frac{5x-3}{6}-\frac{1-x}{3}$.

15. $\frac{x+3}{4}-\frac{x-2}{5}+\frac{x-4}{10}-\frac{x+3}{6}$.

16. $\frac{1-2a}{5}+\frac{2a-1}{4}-\frac{2a-a^2+1}{8}$.

17. $\frac{3+x-x^2}{4}-\frac{1-x+x^2}{6}-\frac{1-2x-2x^2}{3}$.



18. Reduce $\frac{5a^2+b^2}{a^2-b^2}-2$ to a fraction.

Solution

$$\frac{5a^2+b^2}{a^2-b^2}-\frac{2}{1}=\frac{5a^2+b^2-2(a^2-b^2)}{a^2-b^2}$$
$$=\frac{5a^2+b^2-2a^2+2b^2}{a^2-b^2}$$
$$=\frac{3(a^2+b^2)}{a^2-b^2}.$$

Reduce the following **mixed expressions to fractions**:

19. $a+\frac{b}{2}$.

20. $x-\frac{y}{2}$.

21. $\frac{a^2-c^2}{c}+5c$.

22. $\frac{1-x}{3}-4x$.

23. $a-\frac{a^2-ab}{b}$.

24. $a-\frac{a-b-c}{2}$.

25. $a+x-\frac{x^2}{a-x}$.

26. $a^2-ab+b^2-\frac{b^3}{a+b}$.

Perform the additions and subtractions indicated:

27. $\frac{a-b}{ab}+\frac{b-c}{bc}$.

28. $\frac{a+b}{a-b}-\frac{a-b}{a+b}$.

29. $x+y-\frac{x^2+y^2}{x-y}$.

30. $\frac{x}{x-2}-\frac{x-2}{x+2}$.

31. $x+1+\frac{x^3-3}{x-1}$.

32. $m-\frac{m^2+n^2}{m-n}+n$.

33. $\frac{1}{x}+1+\frac{2x}{1+x}-2$.

34. $3a-2x-\frac{8a^2-4x^2}{3a+2x}$.

35. $\frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{x^2}$.

36. $\frac{1}{a+b}-\frac{1}{a-b}+\frac{2a}{a^2-b^2}$.

37. $\frac{a+x}{a-x}+\frac{a-x}{a+x}+\frac{4ax}{a^2-x^2}$.

38. $3x+\frac{5}{ax}-\left(2x+\frac{3}{ax}\right)$.

39. $\frac{a}{a-2}-\frac{a-2}{a+2}+\frac{3}{4-a^2}$.

SUGGESTION. — By § 172, Prin. 1, $\frac{3}{4-a^2}=\frac{-3}{a^2-4}$.

40. $\frac{a+1}{a-1}+\frac{2}{a+1}+\frac{4a}{1-a^2}$.

41. $\frac{5x+2}{x^2-4}+\frac{2}{x-2}-\frac{3}{2-x}$.

42. $\frac{x(a+x)}{a-x}-\frac{3ax-x^2}{x-a}+4a$.

43. $\frac{a}{a-b}-\frac{a}{a+b}-\frac{2ab}{a^2+b^2}-\frac{4ab^3}{a^4+b^4}$.

SUGGESTION. — Combine the first two fractions, then the result and the third fraction, then this result and the fourth fraction.

44. $\frac{a+b}{a-b}-\frac{a-b}{a+b}-\frac{4ab}{a^2+b^2}+\frac{8ab^3}{a^4+b^4}$.

45. $\frac{1}{a-b}-\frac{1}{a+b}-\frac{2b}{a^2+b^2}+\frac{2b^3}{a^4+b^4}$.

46. $\frac{x+y}{(y-z)(z-x)}-\frac{y+z}{(x-z)(x-y)}+\frac{z+x}{(y-x)(z-y)}$.

SOLUTION

$$\text{Sum}=\frac{x+y}{(y-z)(z-x)}+\frac{y+z}{(z-x)(x-y)}+\frac{z+x}{(x-y)(y-z)}$$
$$=\frac{(x^2-y^2)+(y^2-z^2)+(z^2-x^2)}{(x-y)(y-z)(z-x)}$$
$$=\frac{0}{(x-y)(y-z)(z-x)}=0.$$

47. $\frac{1}{(b-c)(a-c)}+\frac{1}{(c-a)(a-b)}+\frac{1}{(b-a)(b-c)}$.

48. $\frac{a+1}{(a-b)(a-c)}+\frac{b+1}{(b-c)(b-a)}+\frac{c+1}{(a-c)(b-c)}$.



## MULTIPLICATION OF FRACTIONS

**193.** Fractions are multiplied in algebra just as they are in arithmetic.

Thus, $\frac{3}{4}\times\frac{5}{2}=\frac{3\times 5}{4\times 2}$.

In general, $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$. That is,

PRINCIPLE. — *The product of two or more fractions is equal to the product of their numerators divided by the product of their denominators*

### EXERCISES

**194.** 1. Multiply $\frac{x-5}{x+5}$ by $x^2-25$.

SOLUTION

$$\frac{x-5}{\cancel{x+5}}\cdot\frac{\overset{x-5}{\cancel{x^2-25}}}{1}=(x-5)^2=x^2-10x+25.$$

2. Multiply $\frac{x+3}{x+2}$ by $1+\frac{1}{x+1}$.

SOLUTION

$$\left(\frac{x+3}{x+2}\right)\left(1+\frac{1}{x+1}\right)=\frac{x+3}{x+2}\left(\frac{x+1}{x+1}+\frac{1}{x+1}\right)$$
$$=\frac{x+3}{\cancel{x+2}}\cdot\frac{\cancel{x+2}}{x+1}$$
$$=\frac{x+3}{x+1}.$$

GENERAL SUGGESTIONS. — 1. Any integer may be written with the denominator 1.

2. After finding the product of the numerators and the product of the denominators the resulting fraction may be reduced to lowest terms, in many cases, by canceling common factors from numerator and denominator. It is, however, more convenient to remove the common factors before performing the multiplications.

3. Generally, mixed numbers should be reduced to fractions.

39. $\frac{a}{a-2}-\frac{a-2}{a+2}+\frac{3}{4-a^2}.$

SUGGESTION. — By § 172, Prin. 1, $\frac{3}{4-a^2}=\frac{-3}{a^2-4}.$

40. $\frac{a+1}{a-1}+\frac{2}{a+1}+\frac{4a}{1-a^2}.$

41. $\frac{5x+2}{x^2-4}+\frac{2}{x-2}-\frac{3}{2-x}.$

42. $\frac{x(a+x)}{a-x}-\frac{3ax-x^2}{x-a}+4a.$

43. $\frac{a}{a-b}-\frac{a}{a+b}-\frac{2ab}{a^2+b^2}-\frac{4ab^3}{a^4+b^4}.$

SUGGESTION. — Combine the first two fractions, then the result and the third fraction, then this result and the fourth fraction.

44. $\frac{a+b}{a-b}-\frac{a-b}{a+b}-\frac{4ab}{a^2+b^2}+\frac{8ab^3}{a^4+b^4}.$

45. $\frac{1}{a-b}-\frac{1}{a+b}-\frac{2b}{a^2+b^2}+\frac{2b^3}{a^4+b^4}.$

46. $\frac{x+y}{(y-z)(z-x)}-\frac{y+z}{(x-z)(x-y)}+\frac{z+x}{(y-x)(z-y)}.$

SOLUTION

$$\text{Sum}=\frac{x+y}{(y-z)(z-x)}+\frac{y+z}{(z-x)(x-y)}+\frac{z+x}{(x-y)(y-z)}$$
$$=\frac{(x^2-y^2)+(y^2-z^2)+(z^2-x^2)}{(x-y)(y-z)(z-x)}$$
$$=\frac{0}{(x-y)(y-z)(z-x)}=0.$$

47. $\frac{1}{(b-c)(a-c)}+\frac{1}{(c-a)(a-b)}+\frac{1}{(b-a)(b-c)}.$

48. $\frac{a+1}{(a-b)(a-c)}+\frac{b+1}{(b-c)(b-a)}+\frac{c+1}{(a-c)(b-c)}.$



## MULTIPLICATION OF FRACTIONS

**193.** Fractions are multiplied in algebra just as they are in arithmetic.

Thus, $\frac{3}{4}\times\frac{5}{2}=\frac{3\times 5}{4\times 2}.$

In general, $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}.$ That is,

PRINCIPLE. — *The product of two or more fractions is equal to the product of their numerators divided by the product of their denominators*

### EXERCISES

**194.** 1. Multiply $\frac{x-5}{x+5}$ by $x^2-25$.

SOLUTION

$$\frac{x-5}{\cancel{x+5}}\cdot\frac{\overset{x-5}{\cancel{x^2-25}}}{1}=(x-5)^2=x^2-10x+25.$$

2. Multiply $\frac{x+3}{x+2}$ by $1+\frac{1}{x+1}$.

SOLUTION

$$\left(\frac{x+3}{x+2}\right)\left(1+\frac{1}{x+1}\right)=\frac{x+3}{x+2}\left(\frac{x+1}{x+1}+\frac{1}{x+1}\right)$$
$$=\frac{x+3}{\cancel{x+2}}\cdot\frac{\cancel{x+2}}{x+1}$$
$$=\frac{x+3}{x+1}.$$

GENERAL SUGGESTIONS. — 1. Any integer may be written with the denominator 1.

2. After finding the product of the numerators and the product of the denominators the resulting fraction may be reduced to lowest terms, in many cases, by canceling common factors from numerator and denominator. It is, however, more convenient to remove the common factors before performing the multiplications.

3. Generally, mixed numbers should be reduced to fractions.

Multiply:

3. $\frac{2}{3}$ by $\frac{5}{6}$.

4. $\frac{3}{4}$ by $\frac{4}{15}$.

5. $\frac{3ab}{4xy}$ by $\frac{2y}{3a^2}$.

6. $\frac{5xy}{2ac}$ by $\frac{3ax}{10y^2}$.

7. $\frac{4ab}{10c^2}$ by $\frac{3bc}{a^3}$.

8. $\frac{4mn}{3xy}$ by $-\frac{15bx}{16m^2}$.

9. $\frac{2ax}{12by}$ by $-\frac{10b^2}{x^2}$.

10. $\frac{a}{a+b}$ by $\frac{b}{a-b}$.

11. $\frac{xy^2}{20-8x}$ by $\frac{25-10x}{x^2y}$.

12. $\frac{1-6x+5x^2}{x^2-3x+2}$ by $\frac{2-x}{1-x}$.

Simplify each of the following:

13. $\frac{(a-b)^2}{a+b} \times \frac{b}{a^2-ab} \times \frac{(a+b)^2}{a^2-b^2}$.

14. $\frac{a^4-x^4}{a^3+x^3} \times \frac{a+x}{a^2-x^2} \times \frac{a^2-ax+x^2}{(a+x)^2}$.

15. $\frac{4a-b}{2x+y} \times \frac{2a}{4a^2-ab} \times \frac{4x^2-y^2}{4}$.

16. $\frac{p+2}{x-3} \times \frac{3x^2-27}{2p^2-8} \times \frac{4}{px+3p}$.

17. $\frac{p^4-q^4}{(p-q)^2} \times \frac{p-q}{p^2+pq} \times \frac{p^2}{p^2+q^2}$.

18. $\frac{a^3+8}{a^3-8} \times \frac{a^2+2a+4}{a^2-2a+4}$.

19. $\frac{a^4+a^2x^2+x^4}{a^4-ax^3} \times \frac{x}{a^2-ax+x^2}$.

20. $\frac{a^4+4}{a^4+a^2+1} \times \frac{a^2+a+1}{a^2+2a+2}$.

21. $\frac{4r^2-4s^2}{5r^2+10rs+5s^2} \times \frac{10r+10s}{8r^3-8s^3}$.



22. $\left(1-\frac{x-1}{x^2+6x+5}\right)\left(1-\frac{2}{x^2+7x+12}\right)$.

23. $\left(1+\frac{7x+11}{x^2-4x-21}\right)\left(1-\frac{17x-11}{x^2+7x+10}\right)$.

24. $\frac{4ax^2-4ay^2}{3ax^2+3axy+bxy+by^2} \cdot \frac{3ax^2-3axy+bxy-by^2}{5ax^2-10axy+5ay^2}$.

25. $\frac{x^3-5x^2+8x-4}{x^3-8x^2+19x-12} \cdot \frac{x^3-10x^2+33x-36}{x^3-6x^2+11x-6}$.

26. $\frac{x^4-3x^3-23x^2+75x-50}{x^4-5x^3-21x^2+125x-100} \cdot \frac{x^3-10x^2+29x-20}{x^3-12x^2+45x-50}$.

27. $\frac{a^2+ab+ac+bc}{ax-ay-x^2+xy} \cdot \frac{a^2-ax+ay-xy}{a^2+ac+ax+cx} \cdot \frac{x^2-x(y-a)-ay}{a^2-a(y-b)-by}$.

## DIVISION OF FRACTIONS

**195.** The **reciprocal** of a fraction is the fraction *inverted*, or 1 divided by the fraction.

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$; of $\frac{x}{z}$ is $\frac{z}{x}$; of $m$, or $\frac{m}{1}$, is $\frac{1}{m}$.

**196.** The **reciprocal** of a number is 1 divided by the number.

**197.** Just as $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{3 \times 3}{4 \times 2}$,

so $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$.

PRINCIPLE. — *Dividing by a fraction is equivalent to multiplying by its reciprocal.*

### EXERCISES

**198.** Write the reciprocal of:

1. $\frac{a}{b}$.
2. $\frac{3m}{p}$.
3. $rs$.
4. $\frac{2a^2}{bc}$.
5. $3tu$.
6. $\frac{1}{3m}$.
7. $\frac{4}{ab}$.
8. $\frac{a-x}{b-y}$.

9. Divide $\frac{x^2-4}{x^2-1}$ by $\frac{x+2}{x-1}$.

SOLUTION

$$\frac{x^2-4}{x^2-1}\div\frac{x+2}{x-1}=\frac{(x+2)(x-2)}{(x+1)(x-1)}\times\frac{x-1}{x+2}=\frac{x-2}{x+1}.$$

Simplify:

10. $\frac{5mn}{6bx}\div\frac{10m^2n}{3ax^2}$.

11. $\frac{3abm}{7}\div abx$.

12. $\frac{my-y^2}{(m+y)^2}\div\frac{y^2}{m^2-y^2}$.

13. $\frac{(a-b)^2}{a+b}\div\frac{a^2-ab}{b}$.

14. $(4a+2)\div\frac{2a+1}{5a}$.

15. $\frac{a^4-b^4}{a^2-2ab+b^2}\div\frac{a^2+b^2}{a^2-ab}$.

16. $\frac{x^3+y^3}{x^2-y^2}\div\frac{x^2+xy+y^2}{x-y}$.

17. $\left(x\div\frac{1}{y}\right)\div\left(y^2\div\frac{1}{x^2}\right)$.

18. $\left(\frac{a^3}{b}\div b^2\right)\div\left(\frac{a^2}{b^2}\times ab\right)$.

19. $(a+c)\div\left(\frac{a^2-c^2}{1+x}\div\frac{a-c}{1-x^2}\right)$.

20. $\left(y-x+\frac{x^2}{y}\right)\div\left(\frac{x}{y^2}+\frac{y}{x^2}\right)$.

SUGGESTION. — Reduce the dividend to a fraction.

21. $\left(x^2-\frac{1}{x^2}\right)\div\left(x-\frac{1}{x}\right)$.

22. $\left(1-\frac{y^2}{x^2}\right)\div\left(1-\frac{2x}{y}+\frac{x^2}{y^2}\right)$.

23. $\left(1+\frac{1}{y^2}+\frac{1}{y^4}\right)\div\left(1+\frac{1}{y}+\frac{1}{y^2}\right)$.

24. $\frac{a^2+b^2-c^2+2ab}{a^2-b^2-c^2+2bc}\div\frac{a^2-b^2+c^2-2ac}{a^2-b^2+c^2+2ac}$.

25. $\frac{x^3-6x^2+11x-6}{x^3+2x^2-19x-20}\div\frac{x^3-13x+12}{x^3+10x^2+29x+20}$.



26. $\left(x-4+\frac{9}{x+2}\right)\div\left(1-\frac{4x-7}{x^2-4}\right)$.

27. $\left(x+\frac{3x+6}{x^2-1}+2\right)\div\left(x+3+\frac{1}{x+1}\right)$.

## Complex Fractions

**199.** A fraction one or both of whose terms contains a fraction is called a **complex fraction**.

### EXERCISES

**200.** 1. Simplify the expression $\frac{\frac{a}{b}}{\frac{x}{y}}$.

SOLUTION. $\frac{\frac{a}{b}}{\frac{x}{y}}=\frac{a}{b}\div\frac{x}{y}=\frac{a}{b}\times\frac{y}{x}=\frac{ay}{bx}$.

Simplify:

2. $\frac{\frac{x+y}{ab}}{\frac{x^2-y^2}{ab^2}}$.

3. $\frac{a+\frac{b}{c}}{b+\frac{c}{a}}$.

4. $\frac{2+\frac{3a}{4b}}{a+\frac{8b}{3}}$.

5. $\frac{1-\frac{y^2}{x^2}}{1+\frac{y^2}{x^2}}$.

6. $\frac{b-\frac{c}{2}}{\frac{b}{2}-c}$.

7. $\frac{ax-\frac{x^2}{2}}{\frac{a^2}{2}-ax}$.

8. Simplify the expression $\frac{\frac{x^2}{y^2}-\frac{x}{y}+1}{\frac{x^2}{y^2}+\frac{x}{y}+1}$.

SOLUTION. — On multiplying the numerator and denominator of the fraction by $y^2$, which is the l. c. d. of the fractional parts of the numerator and denominator, the expression becomes $\frac{x^2-xy+y^2}{x^2+xy+y^2}$.

Simplify:

9. $\dfrac{\dfrac{x^2-1}{x}}{\dfrac{x+1}{x^2}}$.

11. $\dfrac{\dfrac{x^3+y^3}{xy}}{\dfrac{x^2-xy+y^2}{xy}}$.

13. $\dfrac{\dfrac{x^2+y^2}{2y}-x}{\dfrac{x}{y}-\dfrac{y}{x}}$.

10. $\dfrac{\dfrac{1}{x}+\dfrac{1}{y+z}}{\dfrac{1}{x}-\dfrac{1}{y+z}}$.

12. $\dfrac{\dfrac{1}{a+1}}{1-\dfrac{1}{a+1}}$.

14. $-\dfrac{\dfrac{1}{1-a}}{\dfrac{a}{a-1}}$.

15. $\dfrac{\dfrac{1}{x+1}}{1-\dfrac{1}{1+x}}+\dfrac{\dfrac{1}{x+1}}{\dfrac{x}{1-x}}+\dfrac{\dfrac{1}{1-x}}{\dfrac{x}{1+x}}$.

16. $\dfrac{\dfrac{1}{a}+\dfrac{1}{b+c}}{\dfrac{1}{a}-\dfrac{1}{b+c}}\div\dfrac{1}{1+\dfrac{b^2+c^2-a^2}{2bc}}$.

17. Simplify the expression $\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{x}}}$.

Solution. — By successive reductions and divisions,

$$\frac{1}{1+\dfrac{1}{1+\dfrac{1}{x}}}=\frac{1}{1+\dfrac{1}{\dfrac{x+1}{x}}}=\frac{1}{1+\dfrac{x}{x+1}}=\frac{x+1}{x+1+x}=\frac{x+1}{2x+1}.$$

Simplify:

18. $\dfrac{1}{x+\dfrac{1}{1+\dfrac{x+1}{3-x}}}$.

20. $\dfrac{2}{2-\dfrac{2}{2-\dfrac{2}{2-x}}}$.

19. $\dfrac{1}{a+\dfrac{1}{a+\dfrac{1}{a}}}$.

21. $\dfrac{x-2}{x-2-\dfrac{x}{x-\dfrac{x-1}{x-2}}}$.



## EQUATIONS AND PROBLEMS

**201.** Since the student has learned how to perform operations when fractions are involved, he is now prepared to solve certain equations that heretofore he could solve only by a roundabout method, and others that he could not solve at all.

### Clearing Equations of Fractions

**202.** The process of changing an equation containing fractions to an equation without fractions is called **clearing the equation of fractions.**

#### EXERCISES

**203.** 1. Solve the equation $\dfrac{x}{2}=10-\dfrac{x}{3}$.

Solution

$$\frac{x}{2}=10-\frac{x}{3}.$$

Since the first fraction will become an integer if the members of the equation are multiplied by 2 or some number of times 2, and since the second fraction will become an integer if the members are multiplied by 3 or some multiple of 3, the equation may be *cleared of fractions* in a single operation by multiplying both members by some *common multiple* of 2 and 3, as 6, or 12, or 18, etc.

It is best to multiply by the l. c. m. of the denominators, that is, by the l. c. d. of the fractions, which in this case is 6.

Multiplying by 6, Ax. 3, $3x=60-2x$.

Transposing, etc., § 71, $5x=60$.

Hence, Ax. 4, $x=12$.

Verification. — When 12 is substituted for $x$, the given equation becomes $6=6$; that is, the equation is satisfied for $x=12$.

2. Solve the equation $\dfrac{x-1}{2}-\dfrac{x-2}{3}=\dfrac{2}{3}-\dfrac{x-3}{4}$.

Suggestion. — Multiplying both members of the equation by the l. c. d., which in this case is 12, we obtain

$$6(x-1)-4(x-2)=8-3(x-3).$$

To clear an equation of fractions:

RULE. — *Multiply both members of the equation by the lowest common denominator of the fractions.*

CAUTIONS. — 1. To insure correct results in *solving* equations:
*Before clearing, reduce all fractions to lowest terms, and unite fractions that have like denominators.*
*Test results and reject such as do not satisfy the equation.*

2. If a fraction is negative, the sign of each term of the numerator must be changed when the denominator is removed.

Solve, and verify each result:

3. $2x+\frac{x}{3}=\frac{35}{3}$.

4. $\frac{x}{4}+10=13$.

5. $\frac{x}{2}+\frac{x}{6}=\frac{10}{3}$.

6. $7\frac{1}{2}-\frac{3x}{14}=\frac{x}{7}$.

7. $\frac{x}{2}+\frac{x}{3}-\frac{x}{4}+\frac{3x}{10}-\frac{5x}{12}=7$.

8. $\frac{25x}{18}-\frac{5x}{9}+\frac{2x}{3}-\frac{5x}{6}=2$.

9. $\frac{7z+2}{6}-\frac{12-z}{4}+\frac{z+2}{2}=6$.

10. $\frac{u-3}{7}+\frac{u+5}{3}-\frac{u+2}{6}=4$.

11. $\frac{y-1}{2}+\frac{y-2}{3}+\frac{y-3}{4}=\frac{5y-1}{6}$.

12. $\frac{x-5}{3}+\frac{2x+2}{8}-\frac{x-1}{4}=\frac{x+4}{6}$.

13. $1.07x+.32=.15x+10.12+.675x$.

SUGGESTION. — Clear of decimal fractions by multiplying by 1000.

14. $.604x-3.16-.7854x+7.695=0$.

15. $\frac{.2x}{7}-\frac{.1x}{4}-\frac{.1x}{2}+\frac{.4x}{7}=\frac{.3}{14}$.



16. Solve $\frac{9x+5}{14}+\frac{8x-7}{6x+2}=\frac{36x+15}{56}+\frac{10\frac{1}{4}}{14}$.

SUGGESTION. — The equation may be written,

§ 36, $\frac{9x}{14}+\frac{5}{14}+\frac{8x-7}{6x+2}=\frac{36x}{56}+\frac{15}{56}+\frac{41}{56}$, or $\frac{8x-7}{6x+2}=\frac{9}{14}$.

17. $\frac{3x-2}{2x-5}+\frac{3x-21}{5}=\frac{6x-22}{10}$.

18. $\frac{4x+3}{9}=\frac{8x+19}{18}-\frac{7x-29}{5x-12}$.

19. $\frac{6p^2+p}{15p}-\frac{2p-4}{7p-13}=\frac{2p-1}{5}$.

20. Solve the equation $\frac{x-1}{x-2}+\frac{x-6}{x-7}=\frac{x-5}{x-6}+\frac{x-2}{x-3}$.

SOLUTION. — It will be observed that if the fractions in each member were connected by the sign $-$, and if the terms of each member were united, the numerators of the resulting fractions would be simple. The fractions can be made to meet this condition by transposing one fraction in each member before clearing of fractions.

Transposing, $\frac{x-1}{x-2}-\frac{x-2}{x-3}=\frac{x-5}{x-6}-\frac{x-6}{x-7}$.

Uniting terms, $\frac{-1}{x^2-5x+6}=\frac{-1}{x^2-13x+42}$.

Since the fractions are equal and their numerators are equal, their denominators must be equal.

Then, $x^2-5x+6=x^2-13x+42$.

$\therefore x=4\frac{1}{2}$.

21. $\frac{x-1}{x-2}+\frac{x-7}{x-8}=\frac{x-5}{x-6}+\frac{x-3}{x-4}$.

22. $\frac{x-3}{x-4}+\frac{x-7}{x-8}=\frac{x-6}{x-7}+\frac{x-4}{x-5}$.

23. $\frac{v+2}{v+1}-\frac{v+3}{v+2}=\frac{v+5}{v+4}-\frac{v+6}{v+5}$.

### Algebraic Representation

**204.** 1. What part of $m-n$ is $p$?

2. Indicate the sum of $l$ and $m$ divided by 2, and that result multiplied by $n$.

3. Indicate the product of $s$ and $(r-1)$ divided by the $n$th power of the sum of $t$ and $v$.

4. A boy who had $m$ marbles lost $\frac{1}{a}$ of them. How many marbles had he left?

5. By what number must $x$ be multiplied that the product shall be $z$?

6. Indicate the result when the sum of $a$, $b$, and $-c$ is to be divided by the square of the sum of $a$ and $b$.

7. It is $t$ miles from Albany to Utica. The Empire State Express runs $s$ miles an hour. How long does it take this train to go from Albany to Utica?

8. A cabinetmaker worked $x$ days on two pieces of work. For one he received $v$ dollars, and for the other $w$ dollars. What were his average earnings per day for that time?

9. A train runs $x$ miles an hour and an automobile $x-y$ miles an hour. How much longer will it take the automobile to run $s$ miles than the train?

10. Indicate the result when $b$ is added to the numerator and subtracted from the denominator of the fraction $\frac{a}{c}$.

11. A farmer had $\frac{1}{x}$ of his crop in one field, $\frac{1}{y}$ in a second, and $\frac{1}{z}$ in a third. What part of his crop had he in these three fields?

12. A student spends $\frac{1}{m}$ of his income for room rent, $\frac{1}{n}$ for board, $\frac{1}{s}$ for books, and $\frac{1}{r}$ for clothing. If his income is $x$ dollars, how much has he left?



### Problems

**205.** Solve the following problems and verify each solution:

1. If a large lemon grown in Mexico had weighed $2\frac{1}{2}$ pounds less, its weight would have been $\frac{2}{3}$ of its actual weight. What was its actual weight?

2. The crew of the *Lusitania* numbers 800. If this is 200 less than $\frac{1}{3}$ the number of passengers and crew that may be accommodated, what is the passenger capacity?

3. The box of a Chinese sedan chair is $1\frac{1}{2}$ feet higher than it is long and the area of the floor is 4 square feet. Find its three dimensions, if the capacity of the box is 14 cubic feet.

4. The sum of the heaviest loads that can be carried by a man, a horse, and an elephant is 2900 pounds. The elephant can carry 10 times as much as the horse, and the horse $1\frac{2}{3}$ times as much as the man. What load can each carry?

5. The first issue of *The Sun* devoted $\frac{1}{3}$ of its columns to advertisements and $\frac{1}{4}$ to miscellaneous news. The rest of the paper, 5 columns, was devoted to poetry, finance, and shipping news. How many columns did it contain?

6. Of the world's supply of rubber one year, South America produced $\frac{1}{3}$ and Africa $\frac{1}{5}$. How much was produced by each, if the rest of the world produced 26,600 tons?

7. A large ear of seed corn exhibited at the Iowa Experiment Station sold for \$150. At the same rate, if it had weighed 8 ounces less, it would have sold for \$90. How much did it weigh?

8. The feathers that a Toulouse goose yields in a year are valued at \$2.80. If it yielded 4 ounces more, they would be worth \$3.50. Find the weight of the yield of feathers.

9. The number of vessels entering at New York in one year was 11,399. If $\frac{1}{4}$ of the number of steamships was 449 more than $\frac{1}{5}$ of the number of sailing vessels, how many of each were there?

10. Find the world's production of nickel in a year when the United States and Canada together produced $\frac{1}{2}$ of it, England $\frac{11}{60}$ of it, and the rest of the world 3800 tons.

11. In a recent year, the United States produced 2 times as much aluminium as Germany, $1\frac{1}{3}$ times as much as France, and 6200 tons more than England. If all these countries produced 19,800 tons, how much did the United States produce?

12. The largest log in a shipment of mahogany sent to New Orleans weighed 14,000 pounds. If the weight of the rest of the shipment had been 1 ton more, the weight of this log would have been $\frac{1}{13}$ of the total weight of the shipment. Find the weight of the shipment.

13. In constructing the Hall of Records building in New York City, 600,000 pounds of copper were used. The dome lacks 5250 pounds of having $\frac{1}{8}$ as much as the rest of the building. How many pounds of copper are there in the dome?

14. The freight charges on a car load of hay were $\frac{5}{7}$ as much as on a car load of apples. If there were 10 tons of hay and the charges on each ton were \$2 less than $\frac{1}{7}$ of the charges on all the apples, find the charges on each car load.

15. The number of pound cans of salmon in a case is 4 more than $\frac{1}{18}$ of the number of cans that can be packed in a minute in a Washington cannery. If 1000 cases can be packed in an hour, how many cans are there in a case?

16. A boy sold from his garden a certain number of bunches of beets. If he had sold 7 bunches more he would have received \$11 for them. If he had sold 5 bunches less he would have received \$9.80. How many bunches did he sell and at what price?

17. If the number of pounds of alligator teeth sold in a given year had been 50 less, the approximate number of teeth would have been 14,000; if 200 less, the number of teeth would have been 3500. Find the number of pounds sold and the average number of teeth in a pound.



## REVIEW

**206.** 1. What three signs are to be considered in connection with a fraction? What is the sign of a fraction?

2. Under what conditions may the sign of the numerator or of the denominator of a fraction be changed?

3. Show that $\frac{5}{9-x^2}=\frac{-5}{x^2-9}$.

4. What is the effect of changing the sign of an odd number of factors in either term of a fraction? an even number?

5. Show that $\frac{(x-y)(x-y)}{(z-w)(u-z)}=\frac{-x^2+2xy-y^2}{(z-w)(z-u)}$.

6. When is a fraction in its lowest terms? What principle applies to the reduction of fractions to higher or lower terms?

7. Reduce to lowest terms:

$$\frac{x(4a^2-9b^2)}{4a^2x^2+12abx^2+9b^2x^2} \text{ and } \frac{6lx+6sx-14ly-14sy}{36x^2-196y^2}.$$

8. In reducing a fraction to an equal fraction having a given denominator, how is the number found by which both terms are to be multiplied?

9. Reduce $\frac{3b}{a-2b}$ to a fraction whose denominator is $a^2-4ab+4b^2$.

10. Define highest common factor; lowest common multiple. Illustrate by finding the highest common factor and the lowest common multiple of $ax+ay$, $x^2-y^2$, and $x^2+2xy+y^2$.

11. What is the reciprocal of a fraction? of any number? Give the reciprocals of $\frac{x}{y}$, $\frac{a+b}{x-y}$, and $x$.

12. Define complex fraction and illustrate by writing one. Simplify the one you have written.

Reduce to an integral or a mixed expression:

13. $\dfrac{x^2-9y^2+7}{x-3y}$.
14. $\dfrac{4a^3+20a^2b+27ab^2+9b^3}{2a+3b}$.

Reduce the following mixed expressions to fractions:

15. $x^2-xy+y^2-\dfrac{2y}{x+y}$.
16. $\dfrac{m^2+n^2}{m+n}-n-m$.

Simplify:

17. $\dfrac{s+t}{2rl}+\dfrac{s-t}{4rl}-\dfrac{s^2+t^2}{4rl(s+t)}$.

18. $\dfrac{a+x}{x+2}+\dfrac{a-x}{2-x}-\dfrac{2(x^2-2a)}{x^2-4}$.

19. $\left(\dfrac{x^3-y^3}{xy+y^2}\times\dfrac{x^2+xy}{x-y}\right)\div\left(\dfrac{x^2y+xy^2}{x^2+2xy+y^2}\times\dfrac{x+y}{y^2}\right)$.

20. $\left(\dfrac{a^2+2ab+b^2}{a^2-2ab+b^2}-\dfrac{a+b}{a-b}\right)\times\left(\dfrac{3a-3b}{2b}\div\dfrac{6a+6b}{2a-2b}\right)$.

21. What is meant by clearing an equation of fractions? State the axiom upon which it is based.

22. What precautions must be taken to secure correct results in solving equations that involve fractions?

Solve, and verify each result:

23. $\dfrac{5x}{2}-\dfrac{7x}{4}=6$.
25. $\dfrac{x+7}{4}=\dfrac{x-3}{2}+1$.

24. $20x+\dfrac{10x}{3}=\dfrac{700}{6}$.
26. $\dfrac{3x-2}{10}-\dfrac{7-2x}{5}=\dfrac{x-5}{15}$.

27. $\dfrac{x+3}{x-3}-\dfrac{x}{4}=\dfrac{12}{24}-\dfrac{x-3}{4}$.

28. $\dfrac{x+7}{x+2}+\dfrac{x+8}{3}=\dfrac{3x-15}{9}-3$.

29. $2.04x-3.1-2.95x=8.12-5x+1.05$.

# SIMPLE EQUATIONS

## ONE UNKNOWN NUMBER

**207.** The student already knows what an equation is; he has solved several different kinds; and he knows some of the kinds by name. In this chapter and the next he will meet some of the same kinds with the treatment extended to a few new forms and some additional methods of solution.

**208.** An equation that does not involve an unknown number in any denominator is called an **integral equation.**

$x+5=8$ and $\dfrac{2x}{3}+5=8$ are integral equations. Though the second equation contains a fraction, the unknown number $x$ does not appear in the denominator.

**209.** An equation that involves an unknown number in any denominator is called a **fractional equation.**

$x+5=\dfrac{8}{x}$ and $\dfrac{2x}{x-1}=7$ are fractional equations.

**210.** Any number that satisfies an equation is called a **root** of the equation.

2 is a root of the equation $3x+4=10$.

**211.** Finding the roots of an equation is called **solving the equation.**

**212.** Two equations that have the same roots, each equation having all the roots of the other, are called **equivalent equations.**

$x+3=7$ and $2x=8$ are equivalent equations, each being satisfied for $x=4$ and for no other value of $x$.