96. The product of two binomials that have a common term.

Let $x+a$ and $x+b$ represent any two binomials having a common term, x. Multiplying $x+a$ by $x+b$,

$$\begin{array}{l} x+a \\ \underline{x+b} \\ x^2+ax \\ \underline{\qquad bx+ab} \\ x^2+(a+b)x+ab \end{array}$$

97. Principle. — *The product of two binomials having a common term is equal to the sum of: the square of the common term, the product of the sum of the unlike terms and the common term, and the product of the unlike terms.*

EXERCISES

98. 1. Expand $(x+2)(x+5)$ and test the result.

Solution

The square of the common term is x^2;
the sum of 2 and 5 is 7;
the product of 2 and 5 is 10;
$\therefore (x+2)(x+5)=x^2+7x+10.$

Test. — If $x=1$, we have $3 \cdot 6 = 1+7+10$, or $18=18$.

2. Expand $(a+1)(a-4)$ and test the result.

Solution

The square of the common term is a^2;
the sum of 1 and -4 is -3;
the product of 1 and -4 is -4;
$\therefore (a+1)(a-4)=a^2-3a-4.$

Test. — If $a=4$, we have $5 \cdot 0 = 16-12-4$, or $0=0$.

3. Expand $(n-2)(n-3)$ and test the result.

Solution

The square of the common term is n^2;
the sum of -2 and -3 is -5;
the product of -2 and -3 is 6;
$\therefore (n-2)(n-3)=n^2-5n+6.$

Test. — If $n=3$, we have $1 \cdot 0 = 9-15+6$, or $0=0$.

Expand by inspection, and test each result:

4. $(x+5)(x+6)$.
5. $(x+7)(x+8)$.
6. $(x-7)(x+8)$.
7. $(x+7)(x-8)$.
8. $(x-5)(x-4)$.
9. $(x-3)(x-2)$.
10. $(x-5)(x-1)$.
11. $(x+5)(x+8)$.
12. $(p-4)(p+1)$.
13. $(r-3)(r-1)$.
14. $(n-8)(n-12)$.
15. $(n-6)(n+15)$.
16. $(x^2+5)(x^2-3)$.
17. $(x^3-7)(x^3+6)$.
18. $(x^n-5)(x^n+4)$.
19. $(x^n-a)(x^n-2a)$.
20. $(y-2b)(y+3b)$.
21. $(z-4a)(z+3a)$.
22. $(2x+5)(2x+3)$.
23. $(2x-7)(2x+5)$.
24. $(3y-1)(3y+2)$.
25. $(4x^2+1)(4x^2-7)$.
26. $(ab-6)(ab+4)$.
27. $(x^2y^2-a)(x^2y^2+2a)$.
28. $(3xy+y^2)(y^2-xy)$.
29. $(b^2c^2+ef)(b^2c^2-ef)$.
30. $(5ab+2c^2)(5ab-2c^2)$.
31. $(3x^2+2y^2)(3x^2-2y^2)$.

By an extension of the method given above, the product of any two binomials having similar terms may be written.

32. Expand $(2x-5)(3x+4)$.

PROCESS

$$\begin{array}{c} 2x-5 \\ \times \\ \underline{3x+4} \\ 6x^2-7x-20 \end{array}$$

Explanation. — The product must have a term in x^2, a term in x, and a numerical, or **absolute**, term. The x^2-term is the product of $2x$ and $3x$; the x-term is the sum of the partial products $-5 \cdot 3x$ and $2x \cdot 4$, called the **cross-products**; and the absolute term is the product of -5 and 4.

The process should not be used except as an aid in explanation.

Expand by inspection, and test each result:

33. $(2x+5)(3x+4)$.
34. $(3x-2)(2x-3)$.
35. $(3a-4)(4a+3)$.
36. $(3x-y)(x-3y)$.
37. $(7z-a)(3z+2a)$.
38. $(2a+5b)(5a+2b)$.
39. $(7n^2-2p)(2n^2-7p)$.
40. $(ab^2-m^3)(ab^2-4m^3)$.
41. $(4r^m-3s^n)(2r^m-5s^n)$.
42. $(x^{t-1}+5y)(2x^{t-1}-3y)$.

SIMULTANEOUS EQUATIONS

99. If 4 bananas and 9 oranges cost 35¢, and 4 bananas and 6 oranges cost 26¢, and it is required to find the cost of 1 of each, we may simplify the problem thus:

	4 bananas and 9 oranges cost 35¢	(1)
	4 bananas and 6 oranges cost 26¢	(2)
Subtracting,	3 oranges cost 9¢	(3)

By thus *eliminating* the cost of the bananas, we have obtained a relation, (3), more simple than either of the two given relations, (1) and (2), for it involves only one unknown cost.

Or, let x represent the number of cents 1 banana costs, and y the number of cents 1 orange costs.

Then, 4 bananas will cost $4x$ cents, 9 oranges $9y$ cents, etc.

$$4x + 9y = 35 \quad (1)$$
$$4x + 6y = 26 \quad (2)$$

Eliminating the x's,
$$3y = 9 \quad (3)$$
$$y = 3, \text{ or 1 orange costs } 3¢.$$

Since $y = 3$, $9y$ in the first equation is equal to 27.

Substituting 3 for y in the first equation,

$$4x + 27 = 35, \quad (4)$$
$$x = 2, \text{ or 1 banana costs } 2¢.$$

100. In eliminating the x's in the preceding section, equal numbers, $4x + 6y$ and 26, were subtracted from the members of (1). Hence, Ax. 2, the results are equal, giving a true equation, $3y = 9$.

This method of elimination is called **elimination by subtraction.**

101. How must the equations $2x + 3y = 16$ and $5x - 3y = 19$ be combined to eliminate the y's?

$$2x + 3y = 16$$
$$5x - 3y = 19$$

Adding, Ax. 1,
$$7x \quad = 35$$

This method of elimination is called **elimination by addition.**

102. Equations like those discussed in § 99 or in § 101, in which the same unknown numbers have the same values, are called **simultaneous equations.**

EXERCISES

103. 1. If $2x + 3y = 18$ and $2x + y = 10$, what is the value of each unknown number?

SOLUTION

$$2x + 3y = 18 \quad (1)$$
$$2x + y = 10 \quad (2)$$

Subtracting, Ax. 2, $2y = 8$; $\therefore y = 4$.

Substituting 4 for y in (2), $2x + 4 = 10$; $\therefore x = 3$.

TEST. — Substituting 3 for x and 4 for y in (1) and (2),

(1) becomes $6 + 12 = 18$, or $18 = 18$;

(2) becomes $6 + 4 = 10$, or $10 = 10$.

NOTE. — The value of y may be substituted in *either* of the given equations.

Solve, and test results:

2. $\begin{cases} 5x + 2y = 22, \\ 5x + y = 21. \end{cases}$

3. $\begin{cases} 3x + 4y = 23, \\ 3x + 2y = 19. \end{cases}$

4. $\begin{cases} 4x + 5y = 32, \\ 2x + 5y = 26. \end{cases}$

5. $\begin{cases} 7x - 2y = 22, \\ 3x + 2y = 18. \end{cases}$

6. $\begin{cases} 6x + 7y = 13, \\ 6x + y = 7. \end{cases}$

7. $\begin{cases} 9x - 2y = 41, \\ 7x - 2y = 31. \end{cases}$

8. $\begin{cases} 5x + 3y = 16, \\ 2x + 3y = 10. \end{cases}$

9. $\begin{cases} 6x + 2y = 22, \\ 6x - 7y = 4. \end{cases}$

10. $\begin{cases} 3x - 4y = 16, \\ 5x + 4y = 48. \end{cases}$

11. $\begin{cases} 6x + 5y = 70, \\ x - 5y = 0. \end{cases}$

12. $\begin{cases} 5x - 4y = 8, \\ 3x - 4y = 0. \end{cases}$

13. $\begin{cases} 8x + 5y = 18, \\ 8x + 3y = 14. \end{cases}$

14. $\begin{cases} 7x - 3y = 39, \\ 5x - 3y = 27. \end{cases}$

15. $\begin{cases} 5y - 4x = 9, \\ 6y + 4x = 46. \end{cases}$

16. $\begin{cases} 8x - 3y = 39, \\ 8x - 4y = 36. \end{cases}$

17. $\begin{cases} 7x + 5y = 83, \\ 7x - 4y = 47. \end{cases}$

18. If $2x+3y=16$ and $5x+4y=33$, find x and y.

SOLUTION

$$2x+3y=16, \quad (1)$$
$$5x+4y=33. \quad (2)$$

We may eliminate either x or y. If we choose to eliminate x, we must first prepare the equations, so that x may have the same coefficient in each. Multiplying both members of (1) by 5, and both members of (2) by 2,

$$10x+15y=80 \quad (3)$$
and $$10x+8y=66 \quad (4)$$

Subtracting (4) from (3), $7y=14$; $\therefore y=2$.

Substituting 2 for y in (1), $2x+6=16$; $\therefore x=5$.

TEST. — These values, in (1) and (2), give $10+6=16$ and $25+8=33$.

NOTE. — To eliminate y instead of x, proceed as follows:

Multiplying (1) by 4, $8x+12y=64$.

Multiplying (2) by 3, $15x+12y=99$.

Subtracting the *upper equation from the lower*, thus avoiding negative coefficients,

$$7x=35; \ \therefore x=5.$$

Substituting 5 for x in (1), $10+3y=16$; $\therefore y=2$.

Solve, and test results:

19. $\begin{cases} 9x+2y=20, \\ 3x+y=7. \end{cases}$

20. $\begin{cases} 6x+5y=28, \\ 2x+3y=12. \end{cases}$

21. $\begin{cases} x+y=10, \\ x-y=2. \end{cases}$

22. $\begin{cases} 5x+2y=49, \\ 3x-2y=23. \end{cases}$

23. $\begin{cases} 4x-y=27, \\ x-y=3. \end{cases}$

24. $\begin{cases} 2x+y=13, \\ x+4y=17. \end{cases}$

25. $\begin{cases} 4y-3x=30, \\ 5y-6x=33. \end{cases}$

26. $\begin{cases} 10x+3y=62, \\ 6x+4y=46. \end{cases}$

27. $\begin{cases} 11x+8y=37, \\ 5x+6y=18. \end{cases}$

28. $\begin{cases} y+2x=18, \\ y-2x=2. \end{cases}$

29. $\begin{cases} 2y-3x=5, \\ 5y+4x=93. \end{cases}$

30. $\begin{cases} 4x-7y=12, \\ 3x+5y=50. \end{cases}$

31. $\begin{cases} 8x+7y=37, \\ 4x-3y=-1. \end{cases}$

32. $\begin{cases} 11x-5y=29, \\ 3x+2y=18. \end{cases}$

Problems

104. 1. The sum of two numbers is 8 and their difference is 2. Find the numbers.

SOLUTION

Let x = the larger number,

and y = the smaller number.

Then, $x+y=8$, (1)

and $x-y=2$. (2)

Adding (1) and (2), $2x=10$; $\therefore x=5$.

Subtracting (2) from (1), $2y=6$; $\therefore y=3$.

Hence, the numbers are 5 and 3.

Find two numbers related to each other as follows:

2. Sum = 14; difference = 8.

3. Sum of 2 times the first and 3 times the second = 34; sum of 2 times the first and 5 times the second = 50.

4. Sum = 18; sum of the first and 2 times the second = 20.

5. A cotton tent is worth \$10 less than a linen one of the same size, and 3 cotton ones cost \$2 more than 2 linen ones. Find the cost of each.

SOLUTION

Let x = the number of dollars a linen tent costs,

and y = the number of dollars a cotton tent costs.

Then, $x-y=10$, (1)

and $3y-2x=2$. (2)

Multiplying (1) by 2,

$$2x-2y=20. \quad (3)$$

Adding (2) and (3), $y=22$. (4)

Substituting (4) in (1),

$$x-22=10; \ \therefore x=32.$$

Hence, a linen tent costs \$32 and a cotton one \$22.

6. A steam train took 10 minutes longer to pass through the Simplon tunnel than an electric train. What was the time of each, if the steam train lacked 8 minutes of taking twice as long as the electric train?

7. During one month the number of arrivals and departures of vessels at the port of Seattle was 183. There were 5 more arrivals than departures. Find the number of each.

8. At one time the United States Navy had 17 coaling stations on the Atlantic and Pacific coasts. If the Atlantic had had 1 less, it would have had 3 times as many as the Pacific coast. How many coaling stations were there on each coast?

9. The length of the Grand Canal in China is 13 times the approximate length of the Panama Canal, and the difference in their lengths is 600 miles. Find the length of each.

10. A steam train is 25 tons heavier than an electric train that carries as many passengers. If 9 such steam trains weigh as much as 14 of the electric ones, find the weight of each train.

11. If at Christmas time 3 dozen carnations and 2 dozen orchids cost \$30, and 2 dozen carnations and 3 dozen orchids cost \$40, find the cost of each per dozen.

12. Small goldfish are worth \$4 per hundred less than large ones. If 3 hundred of the former and 2 hundred of the latter together cost \$18, find the cost of each per hundred.

13. A stock car will hold 45 more sheep than hogs. If the number of animals in 16 cars of sheep is the same as in 25 cars of hogs, find the number in a car load of each.

14. One sugar factory employs 2200 men in the factory and fields. If the number of field hands is 100 more than twice the number of factory workers, find the number of each.

15. The Roosevelt dam in Arizona is 17 feet lower than the Croton dam, and twice the height of the latter plus 3 times the height of the former is 1434 feet. Find the height of each.

16. The duty paid on 7 pianos entering Italy was \$173.70. If the duty paid on each upright piano was \$17.37, or $\frac{1}{2}$ that on each grand piano, how many pianos of each kind were imported?

17. Prospect Park is $326\frac{3}{4}$ acres smaller than Central Park, and twice the area of the former is $189\frac{1}{2}$ acres more than the area of the latter. Find the area of each.

18. A woman picked 5 crates of Brussels sprouts, containing in all 192 quarts. If the crates hold 32 and 48 quarts respectively, how many crates of each size did she pick?

19. One year a jeweler had 693 broken watch springs brought to him to renew. The number broken in summer lacked 39 of being twice the number broken in winter. How many were broken in summer? in winter?

20. An engine at Sharon, Pa., weighed 40 tons less than 5 times as much as two of its castings. The weight of the whole engine, minus twice the weight of the castings, was 314 tons. Find the weight of the whole engine and of the castings.

21. In a typewriting contest in Paris a woman in a given time wrote 500 words less than a man, and twice the number that the man wrote is 15,500 less than 3 times the number that the woman wrote. Find the number of words written by each.

22. In United States money, 2 marks, German money, and 3 francs, French money, are valued at \$1.055, and 1 mark and 5 francs at \$1.203. What is the value in United States money of a mark? of a franc?

23. The expense of running a small automobile is estimated at 51 ¢ a week more than the expense of keeping a horse and carriage. The former can be run for 3 weeks for \$2.22 less than the latter can be kept for 4 weeks. What is the weekly expense of each?

SUGGESTION. — Both equations should be expressed in terms of cents, or both in terms of dollars.

24. It cost 42 cents to stop a certain train and get it back to its former speed. Another train of less speed cost 35 cents to stop and start. If in all both trains made 5 stops, at a cost of \$1.89, find the number of stops made by each.

DIVISION

105. Sign of the quotient.

Since division is the inverse of multiplication, the following are direct consequences of the law of signs for multiplication given in (§ 78):

$$+a\times+b=+ab;\ \therefore\ +ab\div+a=+b.$$
$$+a\times-b=-ab;\ \therefore\ -ab\div+a=-b.$$
$$-a\times+b=-ab;\ \therefore\ -ab\div-a=+b.$$
$$-a\times-b=+ab;\ \therefore\ +ab\div-a=-b.$$

Hence, for division:

106. Law of signs. — *The sign of the quotient is + when the dividend and divisor have* **like signs,** *and − when they have* **unlike signs.**

EXERCISES

107. 1. Divide each of the following numbers by 2:

6, −6, 10, −10, 14, −12, −18, 22, −8.

2. Divide each of the foregoing numbers by −2.

Perform the indicated divisions:

3. $7\overline{)-14}$	4. $-3\overline{)15}$	5. $-3\overline{)-12}$	6. $-1\overline{)9}$
7. $4\div(-4)$.	8. $22\div(-2)$.	9. $-1\div(-1)$.	10. $-6\div3$.
11. $9\div(-3)$.	12. $-21\div3$.	13. $45\div(-5)$.	14. $-8\div2$.
15. $\frac{36}{4}$.	16. $\frac{28}{-7}$.	17. $\frac{-42}{6}$.	18. $\frac{-20}{-5}$.
19. $\frac{-99}{11}$.	20. $\frac{48}{-6}$.	21. $\frac{-63}{-7}$.	22. $\frac{72}{8}$.

108. To divide when the numbers are either positive or negative.

Division, when the numbers involved are positive, was treated in §§ 30–38. The student is now prepared to divide whether the numbers are positive or negative, since the only new point involved is the matter of signs, just discussed.

EXERCISES

109. 1. Divide $-8\,a^4x^6$ by $2\,a^3x^4$.

PROCESS

$$2\,a^3x^4\overline{)-8\,a^4x^6}$$
$$-4\,ax^2$$

EXPLANATION. — Since the signs of dividend and divisor are unlike, the sign of the quotient is − (Law of Signs, § 106).

$8\div2=4$ (Law of Coefficients, § 33).

$a^4\div a^3=a^{4-3}=a^1=a$ (Law of Exponents, § 32).

$x^6\div x^4=x^{6-4}=x^2$ (Law of Exponents).

Hence, the quotient is $-4\,ax^2$.

Divide:

2. $30\,m^2n^2$ by $5\,mn$.	6. $42\,n^2x^2$ by $-6\,n^2$.
3. $-24\,x^2y^2z^3$ by $8\,x^2y$.	7. $-12\,p^3q^3$ by $12\,pq$.
4. $21\,ax^2y$ by $-7\,ay$.	8. $-20\,r^2s^4$ by $-10\,rs^2$.
5. $-9\,abc^3$ by $-3\,abc$.	9. $40\,mnv^2$ by $-8\,mnv$.

10. Divide $4\,a^3b-6\,a^2b^2+4\,ab^3$ by $2\,ab$; by $-2\,ab$.

PROCESS

$$2\,ab\overline{)4\,a^3b-6\,a^2b^2+4\,ab^3}$$
$$2\,a^2\ -3\,ab\ +2\,b^2$$

PROCESS

$$-2\,ab\overline{)\ 4\,a^3b-6\,a^2b^2+4\,ab^3}$$
$$-2\,a^2\ +3\,ab\ -2\,b^2$$

TEST OF SIGNS. — When the divisor is positive, the signs of the quotient should be *like* those of the dividend. When the divisor is negative, the signs of the quotient should be *unlike* those of the dividend.

Divide:

11. $a^2y-2\,ay^2$ by ay; by $-ay$.
12. $9\,x^2y^2+15\,xy^2$ by $3\,xy^2$; by $-3\,xy^2$.
13. $-xz^3-3\,xz+x^2z^2$ by $-xz$; by xz.
14. $3\,x^3-6\,x^5+9\,x^7-12\,x^9$ by $3\,x^2$; by $-3\,x^2$.
15. $30\,r^3s^3+15\,r^2s^2-45\,rs^4+75\,r$ by $15\,r$; by $-15\,r$.
16. $-t^5u-t^4uv+tu^4v-t^3u^3v^2+tu^2v^3$ by tu; by $-tu$.
17. $x^a+2\,x^{a+1}-5\,x^{a+2}-x^{a+3}+3\,x^{a+4}$ by x^a; by $-x^a$.
18. $x^n-x^{n-1}+x^{n-2}-x^{n-3}+x^{n-4}-x^{n-5}$ by x^2; by $-x^2$.
19. $y^{n+1}-2\,y^{n+2}+y^{n+3}-3\,y^{n+4}+y^{n+5}$ by y^{n+1}; by $-y^{n+1}$.
20. $a(x+y)^2-ab(x+y)^3+a^2b^2(x+y)^4$ by $-a$; by $a(x+y)^2$.

21. Divide $81 + 9a^2 + a^4$ by $a^2 - 3a + 9$; test the result.

PROCESS | TEST

$$\begin{array}{l|l}
a^4 + 9a^2 + 81 & a^2 - 3a + 9 \\
\underline{a^4 - 3a^3 + 9a^2} & \overline{a^2 + 3a + 9} \\
\quad 3a^3 + 81 & \\
\quad \underline{3a^3 - 9a^2 + 27a} & \\
\qquad 9a^2 - 27a + 81 & \\
\qquad \underline{9a^2 - 27a + 81} &
\end{array}
\qquad
\begin{array}{l}
91 \div 7 \\
= 13
\end{array}$$

NOTE. — The test is made by substituting 1 for a; similarly, the result may be tested by substituting any other value for a, except such as gives for the result $0 \div 0$ or any number divided by 0, because we are unable to determine the numerical value of such results.

Divide, and test:

22. $a^4 + 16 + 4a^2$ by $2a + a^2 + 4$.

23. $x^5 - 61x - 60$ by $x^2 - 2x - 3$.

24. $a^5 - 41a - 120$ by $a^2 + 4a + 5$.

25. $25x^5 - x^3 - 8x - 2x^2$ by $5x^2 - 4x$.

26. $a^8 + a^6 + a^4 + a^2 + 3a - 1$ by $a + 1$.

27. $4y^4 - 9y^2 - 1 + 6y$ by $3y + 2y^2 - 1$.

28. $2a^4 - 5a^3b + 6a^2b^2 - 4ab^3 + b^4$ by $a^2 - ab + b^2$.

PROCESS | TEST

$$\begin{array}{l|l}
2a^4 - 5a^3b + 6a^2b^2 - 4ab^3 + b^4 & a^2 - ab + b^2 \\
\underline{2a^4 - 2a^3b + 2a^2b^2} & \overline{2a^2 - 3ab + b^2} \\
\quad -3a^3b + 4a^2b^2 - 4ab^3 & \\
\quad \underline{-3a^3b + 3a^2b^2 - 3ab^3} & \\
\qquad a^2b^2 - ab^3 + b^4 & \\
\qquad \underline{a^2b^2 - ab^3 + b^4} &
\end{array}
\qquad
\begin{array}{l}
0 \div 1 \\
= 0
\end{array}$$

NOTE. — It will be observed from the test that $0 \div 1 = 0$. In general, $0 \div a = 0$; that is, zero divided by any number equals zero.

29. Divide $ax^3 - a^2x^2 - bx^2 + b^2$ by $ax - b$.

30. Divide $20x^2y - 25x^3 - 18y^3 + 27xy^2$ by $6y - 5x$.

31. Divide $a^4 - 4a^3x + 6a^2x^2 - 4ax^3 + x^4$ by $a^2 - 2ax + x^2$.

32. Divide $c^3 - 8$ by $c + 2$.

PROCESS

$$\begin{array}{l|l}
c^3 - 8 & c + 2 \\
\underline{c^3 + 2c^2} & \overline{c^2 - 2c + 4 + \frac{-16}{c+2}} \\
\quad -2c^2 - 8 & \\
\quad \underline{-2c^2 - 4c} & \\
\qquad 4c - 8 & \\
\qquad \underline{4c + 8} & \\
\qquad\quad -16 &
\end{array}$$

Divide, and test results:

33. $x^5 + 32$ by $x + 2$.

34. $x^6 - y^6$ by $x^2 + y^2$.

35. $m^5 - n^5$ by $m + n$.

36. $m^5 + n^5$ by $m + n$.

37. $x^7 + 2x^6 - 2x^4 + 2x^2 - 1$ by $x + 1$.

38. $y^5 + 3y^4 + 5y^3 + 3y^2 + 3y + 5$ by $y + 1$.

39. $2n^5 - 4n^4 - 3n^3 + 7n^2 - 3n + 2$ by $n - 2$.

40. $y^4 + 7y - 10y^2 - y^3 + 15$ by $y^2 - 2y - 3$.

41. $7x^3 + 2x^4 - 27x^2 + 16 - 8x$ by $x^2 + 5x - 4$.

42. $28x^4 + 6x^3 + 6x^2 - 6x - 2$ by $2 + 2x + 4x^2$.

43. $25v^2 - 20v^3 + 3v^4 + 16v - 6$ by $3v^2 - 8v + 2$.

44. $4 - 18x + 30x^2 - 23x^3 + 6x^4$ by $2x^2 - 5x + 2$.

45. $32x^3 + 24x^4 - 25x - 4 - 16x^2$ by $6x^2 - x - 4$.

46. 1 by $1 + x$ to five terms of the quotient.

47. 1 by $1 - x$ to five terms of the quotient.

48. $a^3 - 6a^2 + 12a - 8 - b^3$ by $a - 2 - b$.

49. $y^5 + 32x^5$ by $16x^4 + y^4 - 2xy^3 - 8x^3y + 4x^2y^2$.

50. $2 - 3n^x + 13n^{2x} + 23n^{3x} - 11n^{4x} + 6n^{5x}$ by $2 + 3n^x$.

51. $6a^{2m} + 5a^{2m-1} - 10a^{2m-2} + 20a^{2m-3} - 16a^{2m-4}$ by $2a^m + 3a^{m-1} - 4a^{m-2}$.

Special Cases in Division

110. 1. By actual division,

$(x^2-y^2)\div(x-y)=x+y.$
$(x^3-y^3)\div(x-y)=x^2+xy+y^2.$
$(x^4-y^4)\div(x-y)=x^3+x^2y+xy^2+y^3.$

Observe that the *difference* of the same powers of two numbers is *divisible* by the *difference* of the numbers.

'Divisible' means 'exactly divisible.'

2. By actual division,

$(x^2-y^2)\div(x+y)=x-y.$
$(x^3-y^3)\div(x+y)=x^2-xy+y^2$, rem., $-2y^3$.
$(x^4-y^4)\div(x+y)=x^3-x^2y+xy^2-y^3.$
$(x^5-y^5)\div(x+y)=x^4-x^3y+x^2y^2-xy^3+y^4$, rem., $-2y^5$.

Observe that the *difference* of the same powers of two numbers is *divisible* by the *sum* of the numbers *only when the powers are even.*

3. By actual division,

$(x^2+y^2)\div(x-y)=x+y$, rem., $2y^2$.
$(x^3+y^3)\div(x-y)=x^2+xy+y^2$, rem., $2y^3$.
$(x^4+y^4)\div(x-y)=x^3+x^2y+xy^2+y^3$, rem., $2y^4$.

Observe that the *sum* of the same powers of two numbers is *not divisible* by the *difference* of the numbers.

4. By actual division,

$(x^2+y^2)\div(x+y)=x-y$, rem., $2y^2$.
$(x^3+y^3)\div(x+y)=x^2-xy+y^2.$
$(x^4+y^4)\div(x+y)=x^3-x^2y+xy^2-y^3$, rem., $2y^4$.
$(x^5+y^5)\div(x+y)=x^4-x^3y+x^2y^2-xy^3+y^4.$

Observe that the *sum* of the same powers of two numbers is *divisible* by the *sum* of the numbers *only when the powers are odd.*

111. Hence, the preceding conclusions may be summarized as follows, n being a positive integer:

PRINCIPLES. — 1. *x^n-y^n is always divisible by $x-y$.*

2. *x^n-y^n is divisible by $x+y$ only when n is even.*

3. *x^n+y^n is never divisible by $x-y$.*

4. *x^n+y^n is divisible by $x+y$ only when n is odd.*

112. The following **law of signs** may be inferred readily:

When $x-y$ is the divisor, the signs in the quotient are plus.

When $x+y$ is the divisor, the signs in the quotient are alternately plus and minus.

113. The following **law of exponents** also may be inferred:

In the quotient the exponent of x decreases and that of y increases by 1 in each successive term.

EXERCISES

114. Find quotients by inspection:

1. $\frac{a^3-b^3}{a-b}$. 2. $\frac{m^3-n^3}{m-n}$. 3. $\frac{a^3-8}{a-2}$.

4. Devise a rule for dividing the difference of the cubes of any two numbers by the difference of the numbers.

Find quotients by inspection:

5. $\frac{a^3+b^3}{a+b}$. 6. $\frac{m^3+n^3}{m+n}$. 7. $\frac{c^3+27}{c+3}$.

8. Devise a rule for dividing the sum of the cubes of any two numbers by the sum of the numbers.

Find quotients by inspection:

9. $\frac{a^3-125}{a-5}$. 12. $\frac{r^4-s^4}{r-s}$. 15. $\frac{1+a^5}{1+a}$.

10. $\frac{n^3+64}{n+4}$. 13. $\frac{n^6-1}{n-1}$. 16. $\frac{x^5-32}{x-2}$.

11. $\frac{m^5+n^5}{m+n}$. 14. $\frac{x^6-1}{x+1}$. 17. $\frac{a^4-81}{a+3}$.

PARENTHESES

115. The student has seen how *parentheses*, ( ), are used to group numbers that are to be regarded as a single number. Other signs used in the same way are *brackets*, [ ]; *braces*, { }; the *vinculum*, $\overline{\quad}$; and the *vertical bar*, |.

Thus, all of the forms, $(a+b)c$, $[a+b]c$, $\{a+b\}c$, $\overline{a+b}\cdot c$, and $\left.\begin{matrix}a\\+b\end{matrix}\right| c$, have the same meaning.

These signs have the general name, **signs of aggregation.**

When numbers are included by any of the signs of aggregation, they are commonly said to be *in parenthesis*, *in a parenthesis*, or *in parentheses.*

116. Removal of parentheses preceded by + or −.

EXERCISES

1. Remove parentheses and simplify $3a+(b+c-2a)$.

SOLUTION

The given expression indicates that $(b+c-2a)$ is to be *added* to $3a$. This may be done by writing the terms of $(b+c-2a)$ after $3a$ in succession, each with its proper sign, and uniting terms.

$$\therefore 3a+(b+c-2a)=3a+b+c-2a=a+b+c.$$

2. Remove parentheses and simplify $4a-(2a-2b)$.

SOLUTION

The given expression indicates that $(+2a-2b)$ is to be *subtracted* from $4a$. Proceeding as in subtraction, that is, changing the sign of each term of the subtrahend and adding, we have

$$4a-(2a-2b)=4a-2a+2b=2a+2b.$$

PRINCIPLES. — 1. *A parenthesis preceded by a plus sign may be removed from an expression without changing the signs of the terms in parenthesis.*

2. *A parenthesis preceded by a minus sign may be removed from an expression, if the signs of all the terms in parenthesis are changed.*

Simplify by removing parentheses:

3. $a+(b-c)$.
4. $x-(y-z)$.
5. $x-(-y+z)$.
6. $m-n-(-a)$.
7. $m-(n-2a)$.
8. $5x-(2x+y)$.
9. $a-m+(n-m)$.
10. $a-b-(c-d)$.
11. $5a-2b-(a-2b)$.
12. $a-(b-c+a)-(c-b)$.
13. $2xy+3y^2-(x^2+xy-y^2)$.
14. $m+(3m-n)-(2n-m)+n$.

When an expression contains parentheses within parentheses, they may be removed *in succession*, beginning with either the outermost or the innermost, preferably the latter.

15. Simplify $6x-\{3a+(9b-2a)+4x-10b\}$.

SOLUTION

	$6x-\{3a+(9b-2a)+4x-10b\}$
Prin. 1,	$=6x-\{3a+9b-2a+4x-10b\}$
Uniting terms,	$=6x-\{a-b+4x\}$
Prin. 2.	$=6x-a+b-4x$
Uniting terms,	$=2x-a+b.$

Simplify:

16. $4a+b-\{x+4a+b-2y-(x-y)\}$.
17. $ab-\{ab+ac-a-(2a-ac)+\overline{2a-2ac}\}$.
18. $a+[y-\{5+4a-6y-3\}-(7y-4a-1)]$.
19. $4m-[p+3n-(m+n)+3-(6p-3n-5m)]$.
20. $ab-\{5+x-(b+c-ab+x)\}+[x-(b-c-7)]$.
21. $1-x-\{1-x-[1-x-(1-x)-(x-1)]-x+1\}$.
22. Simplify $a^2+a(b-a)-b(2b-3a)$.

SOLUTION

The expression indicates the sum of a^2, $a(b-a)$, and $-b(2b-3a)$.

Expanding, $a(b-a)=ab-a^2$ and $-b(2b-3a)=-2b^2+3ab$.

Therefore, writing the terms in order with their proper signs,

$$a^2+a(b-a)-b(2b-3a)=a^2+ab-a^2-2b^2+3ab=4ab-2b^2.$$

Simplify:

23. $x^2 + x(y - x)$.
24. $c^2 - c(c - d)$.
25. $5 - 2(x - 3)$.
26. $x^2 - y^2 - (x - y)^2$.
27. $c(a - b) - c(a + b)$.
28. $a^3 - b^3 - 3ab(a - b)$.
29. $-2(x^2y - xy^2) - 5(xy^2 - x^2y)$.
30. $(3a - 2)(2a - 3) - 6(a - 2)(a - 1)$.
31. $(3m - 1)(m + 2) - 3m(m + 3) + 2(m + 1)$.
32. $(a - b)^2 - 2(a^2 - b^2) - 2a(-a - b) - 4b^2$.
33. $(x^2 + 2xy + y^2)(x^2 - 2xy + y^2) - (x^2 + y^2)(x^2 + y^2)$.
34. $y^3 - [2x^3 - xy(x - y) - y^3] + 2(x - y)(x^2 + xy + y^2)$.
35. When $a = -2$, $b = 3$, $c = 4$, find the value of

$$a^2 - (a - c)(b + c) + 2b.$$

SOLUTION

$$\begin{aligned} a^2 - (a - c)(b + c) + 2b &= (-2)^2 - (-2 - 4)(3 + 4) + 2 \cdot 3 \\ &= 4 - (-6)(7) + 6 \\ &= 4 - (-42) + 6 \\ &= 4 + 42 + 6 = 52. \end{aligned}$$

When $x = 3$, $y = -4$, $z = 0$, $m = 6$, $n = 2$, find the value of

36. $m(x - y) + z^2$.
37. $z + m^2 - (y^3 - 1)$.
38. $x^2 - y^2 - m^2 + n^2$.
39. $(x + y)(m - n) + 3z$.
40. $(m + x)^2 - (n - y)^2 - y^4$.
41. $xyz - n(x - m)^3 - (nx)^3$.
42. $3m(m - n) + 4n(y - x) - 7(y + z)$.
43. $x^2y^2(m - n)^2(m + n) + (m + n)^2(m - n)$.
44. $3m(x - y - n)^2 - (y - n - x)(n - x - y)$.
45. $(x + y + z)^2 - xy(y + z - x)(x + z - y) - z(x + y - z)$.
46. $(2x + y)^n - (x^2 - 2y)^x - (m + n)^2(x + y + z)^3$.
47. $(m + n + x)^n - (m + n - x)^n - (m - n + x)^n(-m + n + x)^n$.
48. Show that $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2ac - 2bc$, when $a = 1$, $b = 2$, and $c = 3$; when $a = 4$, $b = 2$, and $c = -1$.

117. Grouping terms by means of parentheses.

It follows from § 116 that:

PRINCIPLES. — 1. *Any number of terms of an expression may be inclosed in a parenthesis preceded by a plus sign without changing the signs of the terms to be inclosed.*

2. *Any number of terms of an expression may be inclosed in a parenthesis preceded by a minus sign, if the signs of the terms to be inclosed are changed.*

EXERCISES

118. 1. In $a^2 + 2ab + b^2$, group the terms involving b.

SOLUTION

$$a^2 + 2ab + b^2 = a^2 + (2ab + b^2).$$

2. In $a^2 - x^2 - 2xy - y^2$, group as a subtrahend the terms involving x and y.

SOLUTION

$$a^2 - x^2 - 2xy - y^2 = a^2 - (x^2 + 2xy + y^2).$$

3. In $ax^2 + ab + 2x^2 + 2b$, group the terms involving x^2, and also the terms involving b, as addends.

4. In $a^3 + 3a^2b + 3ab^2 + b^3$, group the first and fourth terms, and also the second and third terms, as addends.

In each of the following expressions group the last three terms as a subtrahend:

5. $a^2 - b^2 - 2bc - c^2$.
6. $a^2 - b^2 + 2bc - c^2$.
7. $a^2 + 2ab + b^2 - c^2 + 2cd - d^2$.
8. $a^2 - 2ab + b^2 - c^2 - 2cd - d^2$.

9. In $a^2 + 2ab + b^2 - 4a - 4b + 4$, group the first three terms as an addend and the fourth and fifth as a subtrahend.

Errors like the following are common. Point them out.

10. $x^3 - x^2 + x - 1 = (x^3 - 1) - (x^2 + x)$.

11. $x^2 - y^2 + 2yz - z^2 = x^2 - (y^2 + 2yz - z^2)$.

119. The use of parentheses in grouping numbers enables us to extend the application of certain cases in multiplication.

Thus, in § 94 and in § 97, one or both numbers may consist of more than one term.

EXERCISES

120. 1. Expand $(a+m-n)(a-m+n)$.

SOLUTION

$$a+m-n=a+(m-n) \text{ and } a-m+n=a-(m-n).$$

$$\therefore [a+m-n][a-m+n]=[a+(m-n)][a-(m-n)]$$

§ 94, $= a^2-(m-n)^2$

§ 88, $= a^2-(m^2-2mn+n^2)$

$= a^2-m^2+2mn-n^2.$

Expand:

2. $(r+p-q)(r-p+q)$.
3. $(r+p+q)(r-p-q)$.
4. $(x+b+n)(x-b-n)$.
5. $(x^2+2x+1)(x^2+2x-1)$.
6. $(x^2+2x-1)(x^2-2x+1)$.
7. $(x^2+3x-2)(x^2-3x+2)$.
8. $[(a+b)+(c+d)][(a+b)-(c+d)]$.
9. $(a+b+x+y)(a+b-x-y)$.
10. $(a+b+m-n)(a+b-m+n)$.
11. $(x-m+y-n)(x-m-y+n)$.
12. $(a-m-b-n)(a+m-b+n)$.

13. Expand $(x+y+1)(x+y-3)$.

SOLUTION

$$(x+y+1)(x+y-3)=(\overline{x+y}+1)(\overline{x+y}-3)$$

§ 97, $=(x+y)^2-2(x+y)-3$

§ 85, $=x^2+2xy+y^2-2x-2y-3.$

Expand:

14. $(x-y-2)(x-y-8)$.
15. $(x^2+x-1)(x^2+x+3)$.
16. $(m-n+2)(m-n-4)$.
17. $(t^4-2t^2-5)(t^4-2t^2+2)$.
18. $(2s+3r+4)(2s+3r-3)$.
19. $(2a+5b+6)(2a+5b-8)$.

121. Collecting literal coefficients.

EXERCISES

Add:

1. ax bx $(a+b)x$	2. bm $-cm$ $(b-c)m$	3. $-cx$ $-dx$ $-(c+d)x$	4. $(t+r)x$ $(t+2r)x$ $(2t+3r)x$
5. ax nx	6. cy $-dy$	7. $-mp$ $-np$	8. $(a+b)x$ $(2a+c)x$

Subtract the lower expression from the upper one:

9. mx nx	10. $dy+az$ $ey-bz$	11. $ax-by$ $2x-cy$
12. a^2x+aby b^2x+aby	13. $mx-ny$ $nx-my$	14. $(2r-s)y$ $(r+2s)y$

15. Collect the coefficients of x and of y in $ax-ay-bx-by$.

SOLUTION. — The total coefficient of x is $(a-b)$; the total coefficient of y is $(-a-b)$, or $-(a+b)$.

$$\therefore ax-ay-bx-by=(a-b)x-(a+b)y.$$

Collect in alphabetical order the coefficients of x and of y in each of the following, giving each parenthesis the sign of the first coefficient to be inclosed therein:

16. $ax-by-bx-cy+dx-ey$.
17. $5ax+3ay-2dx+ny-5x-y$.
18. $cx-2bx+7ay+3ax-lx-ty$.
19. $bx+cy-2ax+by-cx-dy$.
20. x^2+ax-y^2+ay.
21. $x^2-ay-ax-y^2$.
22. $bx-cy-2ay+by$.
23. $rx-ay-sx+2cy$.

Group the same powers of x in each of the following:

24. $ax^3+bx^2-cx+ex^3-dx^2-fx$.
25. $x^3+3x^2+3x-ax^2-3ax^3+bx$.
26. $x^2-abx-x^3-bx^2-cx-mnx^3+dx$.
27. $ax^4-x^4-ax^2+x^2+ax-x-abx^3+x^3$.

EQUATIONS AND PROBLEMS

122. 1. Given $5(2x-3)-7(3x+5)=-72$, to find the value of x.

SOLUTION

$$5(2x-3)-7(3x+5)=-72.$$

Expanding, $10x-15-21x-35=-72.$

Transposing, $10x-21x=15+35-72.$

Uniting terms, $-11x=-22.$

Multiplying by -1, $11x=22.$

$$\therefore x=2.$$

VERIFICATION. — Substituting 2 for x in the given equation,

$$5(4-3)-7(6+5)=-72.$$
$$5-77=-72.$$

Hence, 2 is a true value of x.

Find the value of x, and verify the result, in:

2. $2=2x-5-(x-3)$.
3. $10x-2(x-3)=22$.
4. $1=5(2x-4)+5x+6$.
5. $7(5-3x)=3(3-4x)-1$.
6. $4x-x^2=x(2-x)+2$.
7. $7(2x-3)=2-3(2x+1)$.
8. $3(2-4x)-(x-1)=-6$.
9. $4x^3-4(x^3-x^2+x-2)=4x^2$.
10. $5+7(x-5)=15(x+2-36)$.
11. $2(x-5)+7=x+30-9(x-3)$.
12. $(x-2)(x-2)=(x-3)(x-3)+7$.
13. $(x-4)(x+4)=(x-6)(x+5)+25$.
14. $x^2-(2x+3)(2x-3)+(2x-3)^2=(x+9)(x-2)-2$.
15. $3(4-x)^2-2(x+3)=(2x-3)^2-(x+2)(x-2)+1$.
16. $20(2-x)+3(x-7)-2[x+9-3\{9-8+28\}]=23$.
17. $(2x-4)^2-2\{x-6-3x(4+5)\}=4(x+2)^2+72$.
18. $3(x-7)-(x-9)+136=x-2[3x+4-(2x+6+9x)]$.

Literal Equations

123. 1. Find the value of x in the equation $bx-b^2=cx-c^2$.

SOLUTION

$$bx-b^2=cx-c^2.$$

Transposing, $bx-cx=b^2-c^2.$

Collecting coefficients of x, $(b-c)x=b^2-c^2.$

Dividing by $b-c$, $x=\dfrac{b^2-c^2}{b-c}=b+c.$

2. Find the value of x in the equation $x-a^3=2-ax$.

SOLUTION

$$x-a^3=2-ax.$$

Transposing, $ax+x=a^3+2.$

Collecting coefficients of x, $(a+1)x=a^3+2.$

Dividing by $a+1$, $x=\dfrac{a^3+2}{a+1}=a^2-a+1+\dfrac{1}{a+1}.$

Find the value of x in:

3. $3(x-a-2b)=3b$.
4. $5b=3(2x-b)-4b$.
5. $cx-c^3-d^3+dx=0$.
6. $x-1-c=cx-c^3-c^4$.
7. $a^2-ax+5x=7a-10$.
8. $2m^3-mx+nx-2n^3=0$.
9. $a^2-ax-2ab+bx+b^2=0$.
10. $2n^2+5n+x=n^3-nx-2$.
11. $3ab-a^2-2bx=2b^2-ax$.
12. $a^2x-a^3+2a^2+5x-5a+10=0$.
13. $ax-2bx+3cx=ab-2b^2+3bc$.
14. $cx-c^4-2c^3-2c^2=2c-x+1$.
15. $9a^2+4mx=-(3ax-16m^2)$.
16. $x+6n^4-4n^3=1-3nx+2n-n^2$.
17. $n^2x-3m^2n^3+nx+3m^2+x=0$.
18. $x-3b^2-192b^2c^3-4cx+16c^2x=0$.
19. $r(x-sx-1)+r^2(x-r+s^2)=-1-x-r(sx+1)+r^2s^2$.
20. $a^4-c-ax-bx+cx-b^4c=2a^2b^2+c(x-1)-b^4(1+c)$.

Algebraic Representation

124. 1. Find the value of x that will make $6x$ equal to 48.

2. Indicate the product when the sum of x, y, and $-d$ is multiplied by xy.

3. If a man earns a dollars per month, and his expenses are b dollars per month, how much will he save in a year?

4. Indicate the sum of x and z multiplied by m times the sum of x and y.

5. From x subtract m times the sum of the squares of $(a+b)$ and $(a-b)$.

6. A number x is equal to $(y-c)$ times $(d+c)$. Write the equation.

7. What is the number of square rods in a rectangular field whose length is $(a+b)$ rods and width $(a-b)$ rods?

8. At a factory where N persons were employed, the weekly pay roll was P dollars. Find the average earnings of each person per week.

9. How many seconds are x days $+ c$ hours $+ d$ minutes?

10. Express in cents the interest on y dollars for x years, if the interest for one month is z cents on one dollar.

11. If it takes b men c days to dig part of a well, and d men e days to finish it, how long will it take one man to dig the well alone?

12. Find an expression for 5 per cent of x; y per cent of z.

13. A train ran M miles in H hours and m miles in the succeeding h hours. Find its average rate per hour during each period and during the whole time.

14. A farmer has hay enough to last m cows for n days. How long will it last $(a-b)$ cows?

15. A dealer bought n 50-gallon barrels of paint at c cents per gallon. He sold the paint and gained g dollars. Find the selling price per gallon.

Problems

125. Solve the following problems and verify the solutions:

1. I bought 40 stamps for 95 cents. If part of them were 2-cent stamps and part 3-cent stamps, how many of each did I buy?

SOLUTION

Let x = the number of 2-cent stamps.

Then, $40 - x$ = the number of 3-cent stamps.

$\therefore 2x + 3(40 - x) = 95.$

Solving, $x = 25$, the number of 2-cent stamps,

and $40 - x = 15$, the number of 3-cent stamps.

VERIFICATION. — The results obtained may satisfy the *equation of the problem* and still be incorrect, because the equation may be incorrect. If, however, the results satisfy the *conditions of the problem*, the solution is presumably correct.

1st condition: The whole number of stamps bought is 40.

$$25 + 15 = 40.$$

2d condition: The total cost of the stamps = 95¢.
The cost of 25 stamps @ 2¢ + 15 stamps @ 3¢ = 95¢.

2. A certain paper mill produces 350 tons of paper from sawdust each week. Of this 50 tons more is used for newspapers than for wrapping paper. How many tons are used for each?

3. The roadway of the Connecticut Avenue concrete bridge in Washington, D.C., together with two sidewalks, is 52 feet wide. How wide is the roadway, if it is 8 feet less than twice the combined width of the sidewalks?

4. One year the box factories of New England used 6,000,000 feet of boards. The amount of white pine used less 1,200,000 feet was 3 times that of the other timber. How much white pine was used?

5. It costs $2\frac{1}{2}$¢ more a day to feed an immigrant than it does to feed a United States private soldier. If it costs as much to feed 44 immigrants as it does to feed 49 privates, find the cost of the daily rations of each.

6. Of the 160,000 inhabitants of Hawaii, twice as many were Japanese as Chinese. The rest of the inhabitants, or $\frac{1}{4}$ of the total, were Americans and Europeans. Find the number of Chinese.

7. The combined capacity of two ice factories is 264 tons a day. If the capacity of the smaller one is increased 57 tons, its capacity will be half that of the larger one. Find the capacity of each.

8. It cost a man 60 ¢ to send a telegram at '30–2', that is, 30¢ for the first 10 words and 2¢ for each additional word. How many words did the message contain?

Suggestion. — Let x be the number of words in the message.
Then, $x - 10$ will represent the number of words in excess of 10 words.

$$\therefore 30 + 2(x - 10) = 60$$

9. How many words can be sent by telegraph from New Haven to New York for 75 ¢ at the day rate, '25–2'?

10. A long-distance telephone message cost me $1.25. The rate was 50 ¢ for the first 3 minutes and 15 ¢ for each additional minute. How long did the conversation last?

11. The day rate for a telegram between New Orleans and New York is '60–4' and the night rate is '40–3.' A message of a certain number of words cost 25¢ less to send, at night than in the daytime. Find the number of words.

12. A boy was twice as old as his sister 4 years ago. Now his sister is $\frac{2}{3}$ as old as he is. Find the age of each.

13. During one month the Dead Letter Office received 1,000,000 pieces of mail matter. If the number remaining in the office was $\frac{2}{3}$ as many as the number returned to the senders, how many pieces were returned?

14. An eighteen-hour train between New York and Chicago was late 91 times during its first year's run. It was late at Chicago 10 times more than 50 % as many times as it was late at Jersey City. How many times was it late at Jersey City?

15. In China, one woman earned 3 ¢ and another 8 ¢ a day by embroidering. The former worked 28 days on a piece of work, and then the two finished it. If the labor cost $5.02, how long did each work?

16. The shed that sheltered an airship was 544 feet in perimeter. If twice its length was 52 feet more than 4 times its width, what was its width? its length?

17. The average life of 5-dollar bills is $\frac{3}{4}$ of a year longer than that of 1-dollar bills, and $\frac{2}{3}$ as long as that of 10-dollar bills. If a 10-dollar bill lasts $1\frac{3}{4}$ years longer than a 1-dollar bill, find the average life of a bill of each denomination.

18. A farmer's net receipts from hens in a year were $ 90.15. The eggs sold for $ 92.55 more than the chickens, and the expenses were $ 72.65 less than the selling price of the eggs. What did the eggs sell for? the chickens?

19. Upon the floor of a room 4 feet longer than it is wide is laid a rug whose area is 112 square feet less than the area of the floor. There are 2 feet of bare floor on each side of the rug. What is the area of the floor? of the rug?

20. A party of 8 traveled second class from London to Paris for $5.70 less than twice the amount paid by a party of 3 traveling first class. If a first-class ticket cost $4.15 more than a second-class ticket, find the price of each.

21. A military cable and telegraph system between Seattle and Alaska covers 4044 miles. The length of the submarine cable is 272 miles less than twice that of the land telegraph. The land telegraph is 12 miles longer than 13 times the wireless. How long is the wireless?

22. The United States has 280 life-saving stations, 1 being situated at the falls of the Ohio River. Of the remainder, the Atlantic coast has $11\frac{1}{6}$ times as many as the Pacific. Find the number on the Pacific coast, if it lacks 2 of being $\frac{1}{3}$ the number on the Great Lakes.