16. Write three similar monomials; four dissimilar monomials.

17. If n is a whole number greater than 1 and a is any number, what is the meaning of a^n?

Find the value of each of the following expressions when $a=5$, $b=4$, $c=3$, $d=2$, $e=1$, and $n=3$.

18. $6\,ab$; $2\,cd$; $4\,nbd$; $\frac{1}{2}\,ea$; nd^{n+1}.

19. $3\,a^2b$; $3\,ab^2$; $3(ab)^2$; d^2n^3; $(dn)^3$; d^{n-2}.

20. $a+b\div d-n\div e$.

21. $a(b-d)+a-n\div c$.

22. $10\div d+3\div n-e$.

23. $10\div(d+3)+ac\div n$.

24. $c^5+c^4+2\,c^3-2\,c^2-3\,c+3$.

25. $d^7+d^6+3\,d^5-5\,d^4+2\,d^3-4\,d^2+8\,d-1$.

26. For what value of x is $12\,x$ equal to 72?

Write '$12\,x$ is equal to 72' as an equation. Solve the equation.

Express in algebraic form; solve equations when you can:

27. Three times a certain number, x, is 21.

28. The sum of a certain number and three times the number is 40.

29. Six times a number, less 4 times that number, is 13.

30. The distance around a square lot, each side a feet long, is 1280 feet.

31. Half of a certain number is 17.

32. Twice a certain number, less $\frac{1}{3}$ of the number, equals 15.

33. Mary had m books and James had twice as many, the two together having 18 books.

34. John had 50 cents, spent c cents, and earned d cents. How much money had he then?

35. I bought 2 bottles of olives at b cents per bottle, 3 packages of crackers at p cents per package, and a small cheese for c cents. How much did I expend for all? How much money had I left out of a dollar?

FUNDAMENTAL OPERATIONS

16. In this chapter the student will use numbers he has used in arithmetic and letters to represent such numbers. He will notice that the processes of addition, subtraction, multiplication, and division here are performed as in arithmetic.

ADDITION

17. To add monomials.

1. How many are 2 plus 5? How many times a number are 2 times the number plus 5 times the number?

2. If n stands for a number, how many times n are 2 times n plus 5 times n? $2\,n+5\,n=?$

3. $2x+5x=?$ 4. $2r+5r=?$ 5. $2t+5t=?$

6. How many are $3+4+6$?

7. How many days are 3 days + 4 days + 6 days?

8. $3d+4d+6d=?$ 9. $3y+4y+6y=?$

EXERCISES

18. 1. Add $4\,a$ and $3\,a$.

PROCESS

$$\begin{array}{r} 4\,a \\ 3\,a \\ \hline 7\,a \end{array}$$

EXPLANATION. — Just as $3\,a$'s and $4\,a$'s are $7\,a$'s, so $3\,a+4\,a=7\,a$; that is, when the monomials are similar the sum may be obtained by adding the numerical coefficients and annexing to their sum the common literal part.

Add:

2. $\begin{array}{r} 3 \\ 6 \\ \hline \end{array}$ 3. $\begin{array}{r} 3\,x \\ 6\,x \\ \hline \end{array}$ 4. $\begin{array}{r} 7 \\ 1 \\ \hline \end{array}$ 5. $\begin{array}{r} 7\,m \\ m \\ \hline \end{array}$ 6. $\begin{array}{r} 3\,y \\ 4\,y \\ \hline \end{array}$

Add:

7. $$\begin{array}{r} 2\,n \\ \underline{5\,n} \end{array}$$

8. $$\begin{array}{r} 3\,x \\ \underline{8\,x} \end{array}$$

9. $$\begin{array}{r} 4\,xy \\ \underline{7\,xy} \end{array}$$

10. $$\begin{array}{r} 3\,mn^2 \\ \underline{9\,mn^2} \end{array}$$

11. $$\begin{array}{r} 5\,r \\ 2\,r \\ \underline{4\,r} \end{array}$$

12. $$\begin{array}{r} 9\,n \\ 4\,n \\ \underline{6\,n} \end{array}$$

13. $$\begin{array}{r} 2\,ab \\ ab \\ \underline{4\,ab} \end{array}$$

14. $$\begin{array}{r} 6\,c^2d^3 \\ 8\,c^2d^3 \\ \underline{c^2d^3} \end{array}$$

Perform the additions indicated:

15. $8\,a+2\,a+a+3\,a+a+7\,a.$

16. $5\,y+3\,y+8\,y+10\,y+5\,y+y+2\,y.$

17. $8\,m+3\,m+5\,m+2\,m+6\,m+4\,m.$

18. $7\,bc+bc+4\,bc+5\,bc+8\,bc+3\,bc.$

19. $4\,x^2y^2+5\,x^2y^2+3\,x^2y^2+x^2y^2+10\,x^2y^2+6\,x^2y^2.$

20. $3(ab)^2+9(ab)^2+(ab)^2+7(ab)^2+9(ab)^2+2(ab)^2+(ab)^2.$

21. $5(x+y)+2(x+y)+3(x+y)+8(x+y)+2(x+y)+(x+y).$

22. $4(a+b)^2+11\,(a+b)^2+7(a+b)^2+2(a+b)^2+5(a+b)^2.$

Only similar terms can be united into a single term. Dissimilar terms are considered to have been added when the addition is indicated.

23. Add $6\,a$, $5\,b$, $2\,a$, $3\,b$, $2\,c$, and a.

SOLUTION. — Sum $= 6\,a+2\,a+a+5\,b+3\,b+2\,c = 9\,a+8\,b+2\,c.$

Add:

24. $2\,x$, $4\,a$, $3\,x$, and a.

25. m, $3\,c$, $6\,m$, and $4\,c$.

26. $4\,u$, v, $3\,u$, and $10\,v$.

27. $5\,r$, $\frac{3}{4}\,t$, $2\,r$, and $\frac{1}{4}\,t$.

28. $\frac{1}{2}\,p$, $\frac{2}{3}\,q$, $\frac{1}{4}\,p$, and $\frac{1}{6}\,q$.

29. d, $.4\,b$, $.5\,d$, and $.6\,b$.

30. $2\,m$, mn, n, $2\,mn$, $3\,m$, $4\,n$, and $5\,mn$.

31. $3\,b$, $2\,a$, $2\,b$, $2\,c$, $2\,d$, a, c, b, $4\,d$, and $3\,c$.

32. rs, $3\,r^2s$, $4\,rs^2$, $2\,rs$, rs^2, $4\,r^2s$, $2\,rs^2$, and $3\,rs$.

33. $3\,xy$, $2\,pq$, $7\,cd$, pq, $2\,cd$, $8\,pq$. $4\,cd$, and $2\,xy$.

34. x^2, $4\,xy$, $7\,y^2$, $2\,xy$, $3\,y^2$, $6\,x^2$, y^2, xy, $5\,x^2$, and $4\,y^2$.

19. To add polynomials.

EXERCISES

1. Add $x+2\,y+3\,z$, $x+y$, and $x+4\,y+z$.

PROCESS

$$\begin{array}{r} x+2\,y+3\,z \\ x+y \\ \underline{x+4\,y+z} \\ 3\,x+7\,y+4\,z \end{array}$$

EXPLANATION. — For convenience, similar terms may be written in the same column.

The sum of the first column is $3\,x$, of the second $7\,y$, of the third $4\,z$; the sum of these dissimilar terms is then indicated.

Add:

2. $$\begin{array}{r} 2\,a+4\,b \\ 6\,a+2\,b \\ \underline{a+3\,b} \end{array}$$

3. $$\begin{array}{r} 4\,r+3\,s \\ r+s \\ \underline{3\,r+2\,s} \end{array}$$

4. $$\begin{array}{r} x^2+2\,xy+y^2 \\ x^2+y^2 \\ \underline{3\,xy+y^2} \end{array}$$

5. Add $2\,c+5\,d$, $7\,c+d$, $d+4\,c$, and $2\,d+c$.

6. Add $6\,m+4\,n$, $2\,m+3\,n$, $5\,n+7\,m$, and $2\,n+3\,m$.

7. Add $ab+a^2c+5$, $3\,ab+3\,a^2c+7$, and $2\,a^2c+2\,ab+3$.

Express in simplest form:

8. $2\,a+2\,b+3\,c+4\,b+4\,a+6\,a+2\,c.$

9. $3\,w+4\,x+7\,y+2\,v+2\,w+x+3\,y+4\,v+3\,x+4\,w+v.$

10. $x^2z+5\,xz^2+7\,xy+6\,xz^2+2\,x^2z+4\,xy+4\,x^2z+xz^2+xy.$

Add:

11. $6\,m+8\,n+x+y$, $2\,m+2\,n+3\,x+4\,y$, and $m+x+y$.

12. $3\,x+7\,y+4\,z+6\,w$, $7\,z+4\,x+2\,y+w$, and $x+y+z+w$.

13. $x^2+2\,xy+y^2$, $2\,x^2+xy+y^2$, x^2+xy+y^2, $3\,xy+y^2+x^2$, $2\,x^2+3\,xy+y^2$, $x^2+xy+2\,y^2$, and $2\,xy+3\,x^2+4\,y^2$.

14. $2\,c+7\,d+6\,n$, $11\,m+3\,c+5\,n$, $7\,n+2\,d+8\,c$, $d+3\,m+10\,c$, $4\,d+3\,n+8\,m$, $m+6\,n$, and $2\,m+3\,d$.

15. $3\,x^m+2\,y^n$, $4\,x^m+5\,y^n$, $2\,x^m+7\,y^n$, and $2\,x^m+y^n$.

16. $4\,y^a+z^c+w^b$, $y^a+2\,w^b+3\,z^c$, $5\,z^c+3\,w^b$, $2\,y^a+w^b$, and $y^a+4\,z^c+2\,w^b$.

SUBTRACTION

20. **1.** How many are 8 less 3? How many times a number are 8 times the number less 3 times the number?

2. Letting n stand for a number, how many times n are 8 times n less 3 times n? $8n-3n=?$

3. $8z-3z=?$ **4.** $8s-3s=?$ **5.** $8a-3a=?$

EXERCISES

21. **1.** From $10a$ subtract $4a$.

PROCESS

$$\begin{array}{r} 10a \\ \underline{4a} \\ 6a \end{array}$$

EXPLANATION. — Just as 10 a's less 4 a's are 6 a's, so $10a-4a=6a$; that is, when terms are similar their difference may be obtained by subtracting the numerical coefficients and annexing the common literal part.

	2.	3.	4.	5.	6.	7.
From	9	$9x$	7	$7ab$	$18m^2$	$20xy$
Take	4	$4x$	3	$3ab$	$13m^2$	$16xy$

	8.	9.	10.	11.
From	$16ax^2$	$14r^2s^3$	$8x^2y^2z$	$21(a+b)$
Take	$9ax^2$	$7r^2s^3$	$6x^2y^2z$	$11(a+b)$

	12.	13.	14.	15.
From	$3p+8q$	$4l+2t$	$9x+7y$	$5r+8s$
Take	$2p+4q$	$4l+t$	$2x+3y$	$2r+5s$

	16.	17.	18.
From	$n+5n^2+2n^3$	$3r+2s+t$	$8a^2+2ab+3b^2$
Take	$n+n^2+n^3$	$r+s+t$	$5a^2+ab+2b^2$

19. From $12x+7y$ subtract $8x+3y$.

20. From $10ab+3c$ subtract $5ab+2c$.

21. From $7r+5s+6t$ subtract $3r+2s+5t$.

22. From $9x^2+8y^2+6xy$ subtract $5x^2+3y^2+2xy$.

23. From $5m+7n+8l+6$ subtract $5m+4n+4l+5$.

24. From $7x^2+3y+6z+4v$ subtract $3x^2+2y+5z+3v$.

Subtract:

25. $2x^3+y^3+3r^3$ from $4x^3+7y^3+5r^3$.

26. $4ab+2b^2+2cd$ from $6ab+3b^2+6cd$.

27. $3x^2y+xy+5$ from $9x^2y+6xy^2+xy+8$.

28. $2v^3w^3+2vw+4v^5$ from $12v^5+9v^3w^3+6vw$.

29. $5m^2nx^2+abd$ from $18m^2nx^2+12a^2b^2c^2+4abd$.

30. $4x^m+2x^my^n+5y^n$ from $7x^m+2x^my^n+9y^n$.

31. $6m^s+11m^sn^t+5n^t$ from $10m^s+11m^sn^t+8n^t$.

32. $a^{m+n}+3b^{m-n}+7c^{2n}$ from $3a^{m+n}+5b^{m-n}+9c^{2n}$.

33. $10(m+n^2)+5(m^2+n)$ from $12(m+n^2)+8(m^2+n)$.

Simplify, adding or subtracting in order as signs indicate:

34. $9x-4x+6x$.

35. $5n+3n-7n$.

36. $8a-5a-3a$.

37. $2s+8s-5s$.

38. $3b-2b+7b$.

39. $8r-6r+5r-2r$.

40. $7y+8y-6y+7y$.

41. $5z+7z-2z-4z$.

42. $9v-3v+2v-5v$.

43. $7n-2n-3n+4n$.

44. $8x+7x-3x+4x-2x-3x+6x$.

45. $2y+3y-y+7y-3y+9y+2y-6y$.

46. $9z-5z+6z-3z+4z+2z-7z+3z$.

47. $5v+6v+2v-5v+4v-6v+9v-5v-3v$.

48. $7m+6n-3m+5n+7m-4n+3m+4n+5m$.

49. $9r+8s+7r-2s+9s-3r+2r-7s+6s-5r+4r$.

50. $2l+9t+3l-l+3t+2t+8l-5t+9l-6l+2t-4t+7t$.

51. $10(a-x)+15(a-x)+7(a-x)-13(a-x)-12(a-x)$.

MULTIPLICATION

22. Product of two monomials.

In algebra, as in arithmetic, the product of two numbers contains all the factors of both numbers, arranged or grouped in any way we please.

Then, since $a^2 = aa$ and $a^3 = aaa$,

$$a^2 \cdot a^3 = (aa)(aaa) = aaaaa = a^5.$$

That is, $a^2 \cdot a^3 = a^{2+3} = a^5$. (Add exponents)

Similarly, $3\,a^2 \cdot 5\,a^3 = (3 \cdot 5)(a^2 \cdot a^3) = 15\,a^5$. (Multiply coefficients)

Again, $3\,a^2b \cdot 5\,a^3b^2 = (3 \cdot 5)(a^2 \cdot a^3)(b^1 \cdot b^2) = 15\,a^5b^3$.

Hence, for multiplication:

23. Law of exponents. — *The exponent of a number in the product is equal to the sum of its exponents in multiplicand and multiplier.*

24. Law of coefficients. — *The coefficient of the product is equal to the product of the coefficients of multiplicand and multiplier.*

EXERCISES

25. Tell products quickly:

	1.	2.	3.	4.
	$7\,a$	$3\,x$	$5\,m$	$8\,ab^2x^3$
	$3\,a$	$4\,y$	$2\,m^3n$	b^4cx^4
	$21\,a^2$	$12\,xy$	$10\,m^4n$	$8\,ab^6cx^7$

5.	6.	7.	8.
$3\,y$	$4\,x^2$	$8\,av$	$12\,a^3bc$
$4\,y$	$7\,x^2$	$3\,aw$	$3\,a^2b^2d^2$

9.	10.	11.	12.
ab^2	$3\,xy^3$	$2\,ax$	$16\,c^2d^4m$
a^3b	$9\,xz$	$2\,by$	$2\,c^5d^3n$

13.	14.	15.	16.
x^4y	$7\,pq^2$	$8\,c^2d$	$2\,axy^5t$
xy	p^5q	$4\,d^3z$	$6\,a^6yz^2s$

26. To multiply a polynomial by a monomial.

Multiplying as in arithmetic, we have:

1.	43	$40 + 3$	4 tens $+ 3$ units	$4\,t + 3\,u$
	2	2	2	2
	86	$80 + 6$	8 tens $+ 6$ units	$8\,t + 6\,u$
2.	321	$300 + 20 + 1$	$3\,h + 2\,t + u$	$x + y + z$
	3	3	3	a
	963	$900 + 60 + 3$	$9\,h + 6\,t + 3\,u$	$ax + ay + az$

27. *The product of a polynomial by a monomial is equal to the sum of the partial products obtained by multiplying each term of the polynomial by the monomial.*

EXERCISES

28. Multiply:

1.	2.	3.
$x + 2$	$ax^2 + y$	$5\,m^2s + 2\,t$
4	ax	$3\,st^2$

4.	5.	6.
$m^2 + n^3$	$x^2 + 2\,xy + y^2$	$xy + yz + xz$
mn	xy	xyz

7.	8.
$1 + 2\,x + 6\,x^2 + 4\,x^3$	$4\,x^3 + 6\,x^2 + 2\,x + 1$
x^3	$3\,x^2$

In exercise 7, the multiplicand is **arranged** according to the **ascending powers** of x; in exercise 8, according to the **descending powers** of x.

Arrange according to the ascending or descending powers of some letter and perform the multiplications indicated:

9. $ab(6\,a^2 + a^4 + 1 + 4\,a^3 + 4\,a)$.
10. $2\,xy(8\,x^3y + 2\,x^4 + 2\,y^4 + 12\,x^2y^2 + 8\,xy^3)$.
11. $a^2bc(3\,a^4 + 16\,b^4 + 2\,ab^3 + 4\,a^3b + 5\,a^2b^2)$.
12. $8\,t^3s^3(t^6 + 6\,ts^5 + 20\,t^3s^3 + 15\,t^4s^2 + s^6 + 15\,t^2s^4)$.
13. $5\,x^2y^3(x^5 + y^5 + 5\,x^4y + 5\,xy^4 + 10\,x^3y^2 + 10\,x^2y^3)$.

29. To multiply a polynomial by a polynomial.

EXERCISES

1. Multiply $x+5$ by $x+2$; test the result.

	PROCESS	TEST
	$x+5$	$=6$ when $x=1$
	$x+2$	$=3$
x times $(x+5)=$	x^2+5x	
2 times $(x+5)=$	$2x+10$	
$(x+2)$ times $(x+5)=$	$x^2+7x+10$	$=18$

TEST. — The product must equal the multiplicand multiplied by the multiplier, regardless of what value x may represent. To test the result, therefore, we may assign to x any value we choose and observe whether, for that value, *product obtained = multiplicand × multiplier*. When $x=1$, multiplicand $=6$, multiplier $=3$, and $x^2+7x+10=18$; since $6\times 3=18$, it may be assumed that $x^2+7x+10$ is the correct product.

RULE. — *Multiply the multiplicand by each term of the multiplier and find the sum of the partial products.*

2. Multiply $x+4$ by $x+6$; test the result when $x=1$.
3. Multiply $x+1$ by $x+2$; test the result when $x=5$.
4. Multiply $2x+3$ by $4x+1$; test the result when $x=1$.
5. Multiply x^2+x+1 by $x+1$; test the result when $x=2$.
6. Multiply $2a+b+c$ by $3a+b$; test the result when $a=1$, $b=1$, and $c=1$.

In like manner the multiplication of any two literal expressions may be tested arithmetically by assigning any values we please to the letters.

While it is usually most convenient to substitute 1 for each letter, since this may be done readily by adding the numerical coefficients, the student should bear in mind that this really tests the coefficients and not necessarily the exponents, for any power of 1 is 1.

Multiply, and test each result:

7. $\begin{array}{r} x+y \\ \underline{x+y} \end{array}$ 8. $\begin{array}{r} x+4 \\ \underline{x+9} \end{array}$ 9. $\begin{array}{r} 2x+1 \\ \underline{3x+5} \end{array}$ 10. $\begin{array}{r} 5y+3z \\ \underline{4y+z} \end{array}$

Multiply, and test each result:

11. $2x+3$ by $x+2$.
12. $4x+1$ by $3x+4$.
13. $5n+1$ by $4n+5$.
14. $h+2k$ by $3h+k$.
15. $3l+5t$ by $2l+6t$.
16. $4y+6b$ by $2y+b$.
17. $2b+5c$ by $5b+2c$.
18. $ax+by$ by $ax+by$.

An indicated product is said to be **expanded** when the multiplication is performed.

Expand, and test each result:

19. $(c^2+d^2)(c^2+d^2)$.
20. $(3a+b)(3a+b)$.
21. $(2n^2+l)(n^2+2l)$.
22. $(2b+5c)(3b+8c)$.
23. $(\frac{1}{2}a+\frac{1}{3}b)(\frac{1}{2}a+\frac{1}{3}b)$.
24. $(\frac{2}{3}x+\frac{1}{4}y)(\frac{3}{4}x+\frac{1}{2}y)$.
25. $(2xy+3y)(4xy+7y)$.
26. $(4ax+3by)(4ax+3by)$.

Multiply, and test each result:

27. $\begin{array}{r} x^2+2xy+y^2 \\ \underline{x+y} \end{array}$ 28. $\begin{array}{r} a+b+c \\ \underline{a+b+c} \end{array}$ 29. $\begin{array}{r} 2t+3s+6 \\ \underline{t+2s+1} \end{array}$

30. a^2+ay+y^2 by $a+y$.
31. x^3+3x^2+x by $x+1$.
32. $(3l+n+1)(1+l+n)$.
33. $(r^2+2rs+s^2)(r+s+1)$.
34. $2a^2+3b^2+ab$ by $3a^2+4ab+5b^2$.
35. $3n^2+3m^2+mn$ by $m^3+2mn^2+m^2n$.
36. $a^5+a^4+4a^3+a^2+a$ by $a+1$.
37. $31x^3+27x^2+9x+3$ by $3x+1$.
38. $4x^3+3x^2y+5xy^2+6y^3$ by $5x+6y$.
39. $a^2+b^2+c^2+ab+ac+bc$ by $a+b+c$.
40. $m^8+m^6n^3+m^4n^6+m^2n^9+n^{12}$ by m^2+n^3.
41. Multiply $a^{6n}+a^{4n}b^{2c}+a^{2n}b^{4c}+b^{6c}$ by $a^{2n}+b^{2c}$.
42. Multiply $x^{2n}+2x^ny^n+y^{2n}$ by $x^{2n}+2x^ny^n+y^{2n}$.

DIVISION

30. In multiplication two numbers are given and their product is to be found. In division the product of two numbers and one of the numbers are given, and the other number is to be found.

Division is thus the **inverse** of multiplication.

Thus, $3 \times 4 = 12$ illustrates multiplication;
but $12 \div 4 = 3$ illustrates division, the *inverse* process.

31. To divide a monomial by a monomial.

Because $a^2 \cdot a^3 = a^{2+3} = a^5$,

$a^5 \div a^3 = a^{5-3} = a^2$. (Subtract exponents)

Similarly, because

$3a^2 \cdot 5a^3 = (3 \cdot 5)\, a^{2+3} = 15\, a^5$,

$15\, a^5 \div 5\, a^3 = (15 \div 5)\, a^{5-3} = 3\, a^2$. (Divide coefficients)

The quotient may be obtained, just as in arithmetic, by removing equal factors from dividend and divisor by cancellation, thus:

$$\frac{15\, a^5}{5\, a^3} = \frac{3 \cdot \not{5} \cdot \not{a} \cdot \not{a} \cdot \not{a} \cdot a \cdot a}{\not{5} \cdot \not{a} \cdot \not{a} \cdot \not{a}} = 3\, a^2.$$

Again, $$\frac{21\, a^5b^3}{3\, a^3b^2} = \frac{\not{3} \cdot 7 \cdot \not{a} \cdot \not{a} \cdot \not{a} \cdot a \cdot a \cdot \not{b} \cdot \not{b} \cdot b}{\not{3} \cdot \not{a} \cdot \not{a} \cdot \not{a} \cdot \not{b} \cdot \not{b}} = 7\, a^2b,$$

or $$\frac{21\, a^5b^3}{3\, a^3b^2} = \frac{21}{3}\, a^{5-3}b^{3-2} = 7\, a^2b^1 = 7\, a^2b.$$

Hence, for division:

32. Law of exponents. — *The exponent of a number in the quotient is equal to its exponent in the dividend minus its exponent in the divisor.*

Since a number divided by itself equals 1, $a^5 \div a^5 = a^{5-5} = a^0 = 1$; that is, a number whose exponent is 0 is equal to 1.

33. Law of coefficients. — *The coefficient of the quotient is equal to the coefficient of the dividend divided by the coefficient of the divisor.*

EXERCISES

34. Tell quotients quickly:

1. $5)\underline{5^3}$ $\quad 5^2$
2. $7\, c^2d^4)\underline{35\, c^4d^5}$ $\quad 5\, c^2d$
3. $2\, a^3)\underline{a^5x}$ $\quad \frac{1}{2}\, a^2x$
4. $2^2)\underline{2^3}$
5. $3^4 \div 3^4$.
6. $4^0\, m^3)\underline{4^5\, m^4n^2}$
7. $\dfrac{12\, a^3b^3}{4\, ab^2}$.
8. $\dfrac{18\, x^4y^4}{3\, x^2y}$.
9. $\dfrac{21\, ab^3c^2}{7\, b^3}$.
10. $\dfrac{28\, a^4b^2c}{4\, abc}$.
11. $\dfrac{16\, x^3y^3z^3}{4\, xy^2z}$.
12. $\dfrac{24\, x^2y^2z^3}{8\, x^2z^2}$.
13. $\dfrac{20\, a^4b^5y^2}{4\, a^2b^2y^2}$.
14. $\dfrac{36\, a^4y^2z^3}{9\, a^4z^2}$.
15. $\dfrac{3\, ab(a+b)^2}{2\,(a+b)}$.
16. $\dfrac{4\, a^4b^3c^5}{20\, a^2bc^3}$.
17. $\dfrac{4\, x^6y^3z^4}{32\, x^4y^2z^3}$.
18. $\dfrac{2\, a^2(x-y)^3}{a\,(x-y)^2}$.

35. To divide a polynomial by a monomial.

Dividing as in arithmetic, we have

1. $2)\underline{86}$ $\quad 43$ $\qquad 2)\underline{80+6}$ $\quad 40+3$ $\qquad 2)\underline{8 \text{ tens} + 6 \text{ units}}$ $\quad 4 \text{ tens} + 3 \text{ units}$ $\qquad 2)\underline{8\,t + 6\,u}$ $\quad 4\,t + 3\,u$

2. Since, § 27, $(a+b)\,x = ax + bx$,

if $ax + bx$ is regarded as the dividend and x as the divisor,

$(ax + bx) \div x = a + b$; that is,

36. *The quotient of a polynomial divided by a monomial is equal to the sum of the partial quotients obtained by dividing each term of the polynomial by the monomial.*

EXERCISES

37. 1. Divide $9\, x^2y^2 + 15\, xy^2z^2$ by xy^2; by $3\, xy$.

PROCESS

$xy^2)\underline{9\, x^2y^2 + 15\, xy^2z^2}$
$\quad 9\, x \quad + 15\, z^2$

PROCESS

$3\, xy)\underline{9\, x^2y^2 + 15\, xy^2z^2}$
$\quad 3\, xy \quad + \quad 5\, yz^2$

Find quotients:

2. $4\,cd\overline{)4\,c^2d + 20\,cd^2}$

3. $\dfrac{xz^2 + 3\,xz + x^2z^2}{xz}$.

4. $\dfrac{5\,x^2y + 10\,x^2y^2 + 15\,xy^2}{5\,xy}$.

5. $\dfrac{4\,a^2b^3 + 12\,a^3b^2 + 16\,a^4b}{4\,a^2b}$.

6. $\dfrac{24\,a^6b^2 + 32\,a^5b^3 + 40\,a^4b^4}{8\,a^4b^2}$.

7. $7\,ab\overline{)14\,a^4b^3 + 49\,a^2b}$

8. $\dfrac{35\,x^2y^3z^4 + 45\,x^4y^3z^2}{5\,x^2y^2z}$.

9. $\dfrac{36\,a^3b^4c^6 + 60\,a^2b^5c^7}{12\,a^2b^4c^6}$.

10. $\dfrac{24\,r^3s^2 + 30\,r^2s^2 + 42\,r^2s^3}{6\,r^2s^2}$.

11. $\dfrac{9\,x^2yz + 36\,xy^2z^3 + 45\,xyz^5}{9\,xyz}$.

12. $(8\,a^7b^3 + 28\,a^6b^4 + 16\,a^5b^5 + 4\,a^4b^6) \div 4\,a^4b^3$.

13. $(3\,x^3yz^2 + 15\,x^5y^2z^3 + 6\,x^4yz^3 + 18\,x^6y^2z) \div 3\,x^2yz$.

38. To divide a polynomial by a polynomial.

EXERCISES

1. Divide $3\,x^2 + 35 + 22\,x$ by $x + 5$; test the result.

PROCESS | TEST

$$
\begin{array}{lrl}
 & 3\,x^2 + 22\,x + 35 & \underline{\;x + 5\;} \\
3\,x \text{ times } (x+5) & \underline{3\,x^2 + 15\,x} & 3\,x + 7 \\
 & 7\,x + 35 & \\
7 \text{ times } (x+5) & \underline{7\,x + 35} &
\end{array}
\qquad
\begin{array}{l}
60 \div 6 \\
\;\;= 10
\end{array}
$$

EXPLANATION. — For convenience, the divisor is written at the right of the dividend and the quotient below the divisor. Both dividend and divisor are arranged according to the descending powers of x.

Since the dividend is the product of the quotient and divisor, it is the sum of all the partial products formed by multiplying each term of the quotient by each term of the divisor. Hence, if $3\,x^2$, the first term of the dividend as arranged, is divided by x, the first term of the divisor, the result, $3\,x$, is the first term of the quotient.

Subtracting $3\,x$ times $(x + 5)$ from the dividend, leaves $7\,x + 35$, the part of the dividend still to be divided.

Proceeding, then, as before we find, $7\,x \div x = 7$, the next term of the quotient. 7 times $(x + 5)$ equals $7\,x + 35$. Subtracting, we have no remainder. Hence, the quotient is $3\,x + 7$.

TEST. — When $x = 1$, the dividend equals 60 and the divisor 6. The quotient then should equal $60 \div 6$, or 10. On substituting 1 for x, we find that the quotient is equal to 10. Presumably, then, the result is correct.

2. Divide $x^3 + 6\,x^2 + 12\,x + 10$ by $x + 2$.

PROCESS | TEST

$$
\begin{array}{r|l}
x^3 + 6\,x^2 + 12\,x + 10 & x + 2 \\ \cline{2-2}
\underline{x^3 + 2\,x^2 \qquad\qquad} & x^2 + 4\,x + 4 + \dfrac{2}{x+2} \\
4\,x^2 + 12\,x \qquad & \\
\underline{4\,x^2 + \;\;8\,x \qquad} & \\
4\,x + 10 & \\
\underline{4\,x + \;\;8} & \\
2 &
\end{array}
\qquad
\begin{array}{l}
29 \div 3 \\
\;\;= 9\frac{2}{3}
\end{array}
$$

As in arithmetic, the whole of the undivided part of the dividend is not brought down for each division, but only so much of it as may be needed each time.

The remainder 2 is written over the divisor in the form of a fraction which is then added to the quotient as in arithmetic.

RULE. — *Arrange both dividend and divisor according to the ascending or the descending powers of a common letter.*

Divide the first term of the dividend by the first term of the divisor, and write the result for the first term of the quotient.

Multiply the whole divisor by this term of the quotient, and subtract the product from the dividend. The remainder will be a new dividend.

Divide the new dividend as before, and continue to divide in this way until the first term of the divisor is not contained in the first term of the new dividend.

If there is a remainder after the last division, write it over the divisor in the form of a fraction, and add the fraction to the part of the quotient previously obtained.

Divide, and test each result:

3. x^2+2x+1 by $x+1$.
4. a^2+5a+6 by $a+2$.
5. $5r+r^2+4$ by $r+4$.
6. $3+7y^2+2y^4$ by y^2+3.
7. $6x^3+x^6+7$ by x^3+1.
8. $6t^2+20t+23$ by $3t+7$.
9. y^3+3y^2+3y+1 by $y+1$.
10. $6z^4+4+10z^2+4z^6$ by $4z^2+2$.
11. $b^9+6b^6+b^4+9b^3+4b+8$ by b^3+4.
12. Divide $a^4+6a^3+27a^2+54a+81$ by a^2+3a+9.

PROCESS

$$\begin{array}{l|l}
a^4+6a^3+27a^2+54a+81 & a^2+3a+9 \\
\underline{a^4+3a^3+\ \ 9a^2} & \overline{a^2+3a+9} \\
\quad 3a^3+18a^2+54a & \\
\quad \underline{3a^3+\ \ 9a^2+27a} & \\
\qquad 9a^2+27a+81 & \\
\qquad \underline{9a^2+27a+81} &
\end{array}$$

TEST

$169 \div 13$
$= 13$

Divide, and test each result:

13. $x^4+4x^3+12x^2+16x+16$ by x^2+2x+4.
14. $4l^8+4l^6+13l^4+6l^2+9$ by $2l^4+l^2+3$.
15. $4y^4+5y^3+y^6+11y+3y^2+6$ by y^3+3y+2.
16. $6r^4+26r^2+18+15r+7r^3$ by $2r+3r^2+3$.
17. $x^5+x^4y+2x^3y^2+2x^2y^3+xy^4+y^5$ by $x+y$.
18. $2a^5+6a^3+3a^2+2a^4+5a+2$ by a^2+a+2.
19. $x^7+2x^6y+4x^5y^2+3x^4y^3+2x^3y^4$ by x^2+xy+y^2.
20. $z^7+8z^6+3z^9+z^4+6z+20z^3+30$ by $z+3z^3+5$.
21. $5s^3t+4s^2t^2+t^4+3s^4+3st^3$ by $3s^3+2s^2t+2st^2+t^3$.
22. $4a^4+28a^3b+61a^2b^2+45ab^3+12b^4$ by $2a^2+7ab+3b^2$.
23. $p^6+4p^4q^2+9p^2q^4+6q^6+2p^5q+6p^3q^3+12pq^5$ by $p^4+3p^2q^2+6q^4$.

EQUATIONS AND PROBLEMS

39. How many pounds added to 25 pounds will give 30 pounds?

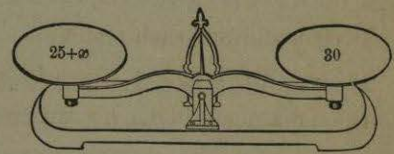
The statement of the problem may be condensed to

$$\begin{array}{r} 25 \text{ pounds} \\ +\ \underline{?} \text{ pounds} \\ 30 \text{ pounds} \end{array} \quad \text{or} \quad \begin{array}{r} 25 \text{ pounds} \\ +\ \underline{x} \text{ pounds} \\ 30 \text{ pounds} \end{array} \quad \text{or} \quad 25+x=30$$

The letter x is only a convenient symbol for the **unknown number** (of pounds), or the number (of pounds) to be found. 25 and 30, on the other hand, are **known numbers.**

The equation, $25+x=30$, is the briefest possible statement of the relation between the known and unknown numbers in the problem. Finding the value of x is called **solving** the equation, $25+x$ is the **first member** of the equation, and 30 is the **second member.**

Equations



40. 1. If 25 pounds are taken from the weight in each scale pan, the balance will be preserved.

In the same way, if 25 is subtracted from each member of the equation $25+x=30$, the equality will be preserved.

$$\begin{array}{r} 25+x=30 \\ \underline{25 \qquad 25} \\ x=\ \ 5 \end{array}$$

2. What number subtracted from $x+10$ will give x?

If the first member of $x+10=12$ is decreased to x by subtracting 10, what must be done to the second member to preserve the equality?

Tell how the equation $x+10=12$ may be solved.

3. Suppose that $x-4=3$ and we wish to find the value of x. How much greater is x than $x-4$?

If the first member of $x-4=3$ is increased to x by adding 4, what must be done to the second member to preserve the equality? Tell how the equation may be solved.

The same number may be added to both members of an equation, or subtracted from both, without destroying the equality.

EXERCISES

41. State what must be done to both members to change one member to x without destroying the equality; solve:

1. $x+6=8.$	5. $x+2=10.$	9. $12=10+x.$
2. $x-3=2.$	6. $x-5=11.$	10. $15=11+x.$
3. $x-4=5.$	7. $x+1=12.$	11. $30=20+x.$
4. $x+7=9.$	8. $x-7=10.$	12. $14=x+10.$

42. 1. If $x=8$, what is the value of $2x$? of $3x$? of $\frac{5}{2}x$?

2. If $6x=12$, what is the value of $1x$, or of x?

3. If $\frac{1}{3}x=10$, what is the value of 3 times $\frac{1}{3}x$, or of x?

4. What must be done to both members of each of the following equations to give an equation whose first member is x?

$$\tfrac{1}{2}x=3 \qquad \tfrac{1}{3}x=5 \qquad 4x=12 \qquad 5x=35$$

Both members of an equation may be multiplied or divided by the same number without destroying the equality.

EXERCISES

43. State what must be done to both members to change one member to x without destroying the equality; solve:

1. $2x=6.$	5. $\frac{1}{2}x=5.$	9. $\frac{1}{6}x=5.$
2. $5x=5.$	6. $\frac{1}{3}x=2.$	10. $\frac{1}{8}x=4.$
3. $4x=8.$	7. $\frac{1}{4}x=3.$	11. $8x=24.$
4. $3x=9.$	8. $\frac{1}{5}x=7.$	12. $9x=18.$

44. The equations solved so far in this chapter have been solved each by a single one of the following steps:

1. *By adding the same number to both members.*
2. *By subtracting the same number from both members.*
3. *By multiplying both members by the same number.*
4. *By dividing both members by the same number.*

The equations that follow may be solved by two or more of these steps taken separately.

EXERCISES

45. 1. Solve the equation $2x+20=80-4x$.

SOLUTION

The first step in solving an equation is to get the unknown terms into one member, usually the first, and the known terms into the other.

$$2x+20=80-4x.$$

Adding $4x$ to both members,

$$2x+4x+20=80+4x-4x,$$

or, uniting terms, $6x+20=80.$

Subtracting 20 from both members,

$$6x+20-20=80-20,$$

or, uniting terms, $6x=60.$

Dividing both members by 6, $x=10.$

VERIFICATION

We should always test the answer by finding whether the value obtained is such as to make the members of the original equation equal.

Thus, substituting $x=10$ in the given equation, we have

$$20+20=80-40,$$

or $40=40.$

Hence, 10 is the true value of x.

Solve and verify:

2. $7x+12=5x+16.$	6. $4x-11+2x=2x-5.$
3. $5x-20=2x+13.$	7. $3x+14+7x=78+2x.$
4. $4x-11=19-2x.$	8. $9x+23+2x=4x+37.$
5. $13x+4=5x+12.$	9. $5x-7+15x=13x+14.$

10. Solve the equation $\frac{3}{2}x = 15$.

FIRST SOLUTION

$\frac{3}{2}x = 15.$

Dividing both members by 3, $\frac{1}{2}x = 5.$

Multiplying both members by 2, $x = 10.$

SECOND SOLUTION

By multiplying by 2 *before* dividing by 3, fractions may be avoided.

$\frac{3}{2}x = 15.$

Multiplying both members by 2, $3x = 30.$

Dividing both members by 3, $x = 10.$

VERIFICATION. $\frac{3}{2}$ of $10 = 15.$

Solve and verify:

11. $\frac{3}{2}x = 9.$
12. $\frac{4}{3}x = 8.$
13. $\frac{3}{5}x = 21.$
14. $\frac{2}{3}x = 30.$
15. $\frac{5}{8}x = 15.$
16. $\frac{7}{3}x = 21.$

Solve by the method best adapted; verify results:

17. $9x - 17 = 23 + x.$
18. $2x + 3x - 2x = 21.$
19. $9x - 4x + 2x = 14.$
20. $8x + 5x - 5x = 48.$
21. $22 - 6x = 40 - 8x.$
22. $7x + 6x - 7x = 42.$
23. $\frac{5}{2}x = 10.$
24. $\frac{2}{3}x = 14.$
25. $\frac{4}{5}x = 28.$
26. $\frac{5}{6}x = 20.$
27. $\frac{7}{8}x = 63.$
28. $\frac{6}{7}x = 48.$
29. $2x - 4 + 6x = 22 - 15 + 21.$
30. $46 + 3x - 60 = 5x - 10 - 4x.$
31. $6x + 5x - 70 = 5x + 54 - 70.$
32. $5x + 16 - 6x = 16 + 24 - 6x.$
33. $9x + 15 - 2x = 32 + 4x - 11.$
34. $10x - 39 + 12x - 9x + 42 - 4x = 42 - 4x.$
35. $16x + 12 - 75 + 2x - 12 - 110 = 8x - 50 - 25.$
36. $3x - 18 + 27 + 10x - 11 = 25 + 4x - 7x + 12 + 3x.$
37. $18x + 16 = 8 + 12x + 8 - 13 + 25x - 9 + 100 - 25x.$

Algebraic Representation

46. 1. Express the sum of 2, $\frac{1}{3}$, and $\frac{1}{5}$; of x, $\frac{1}{2}y$, and $\frac{1}{4}z$.

2. What number is 4 less than 12? n less than 25?

3. Express the number that exceeds 5 by 3; a by b.

4. Represent in the shortest way the sum of five x's; the product of five x's.

5. Mary read 10 pages of a book. On what page did she begin to read, if she stopped at the top of page 21? of page a?

6. Express 10 dollars in terms of cents; 10 cents in terms of dollars; m dollars in terms of cents; m cents in terms of dollars.

7. A has 12 dollars and B, 8 dollars. How much will each have if A gives B 4 dollars? m dollars?

8. At 3 dollars per day, how much will a man earn in 4 days? in x days? At a dollars per day, how much will he earn in b days? in c days? in a days?

9. By what number must 25 be multiplied to produce 300? 10 to produce x? r to produce s?

10. What are the two odd numbers nearest to 5? If $n + 3$ is an odd number, what are the two odd numbers nearest to $n + 3$? the two even numbers?

11. How many square rods are there in a square field one of whose sides is 2 rods long? $(x + y)$ rods long?

12. How many square rods are there in a field 6 rods long and 4 rods wide? $(m + n)$ rods long and m rods wide?

13. If it takes 4 men 5 days to do a piece of work, how long will it take 1 man to do it? 2 men? x men? If it takes b men c days to do a piece of work, how long will it take 1 man? z men?

14. The number 25 may be written $20 + 5$. Write the number whose first digit is x and second digit y.

15. Represent $(a + b)$ times the number whose tens' digit is m and units' digit n.

Problems

47. **1.** What number increased by 6 is equal to 44?

SOLUTION

Let $x =$ the number.
Then, $x + 6 = 44$.
Solving the equation, $x = 38$, the number.

2. What number increased by 15 is equal to 51?

3. What number decreased by 32 is equal to 60?

4. What number multiplied by 3 is equal to 78?

5. What number divided by 8 is equal to 62?

6. If 20 is added to a certain number and 14 is subtracted from the sum, the result is 19. Find the number.

7. One half of a number, and 11 more, is equal to 37. Find $\frac{1}{2}$ of the number, then find the number.

8. If $\frac{3}{4}$ of a certain number is 18, what is the number?

9. The sum of two numbers is 55 and the larger is 4 times the smaller. What are the numbers?

SOLUTION

Let $x =$ the smaller number.
Then, $4x =$ the larger number,
and $x + 4x =$ the sum of the two numbers.
But $55 =$ the sum of the two numbers.
$\therefore x + 4x = 55$.
Solving the equation, $x = 11$, the smaller number,
and $4x = 44$, the larger number.

NOTE. — The sign $\therefore$ means 'therefore.'

10. Separate 116 into two parts, one of which shall be 3 times the other.

11. Separate 72 into two parts, one of which shall be $\frac{1}{3}$ of the other.

12. What number increased by $\frac{1}{2}$ of itself equals 54?

13. What number decreased by $\frac{1}{5}$ of itself equals 84?

14. Five times a number exceeds 3 times the number by 14. What is the number?

15. The double of a number is 64 less than 10 times the number. What is the number?

16. Four times a certain number exceeds 12 as much as 3 times the number is less than 72. What is the number?

17. Of the steam vessels built on the Great Lakes one year, 21, or 5 less than $\frac{1}{3}$ of all, were of steel. How many steam vessels were built on the Lakes that year?

SOLUTION

Let $x =$ the number of steam vessels built.
Then, $\frac{1}{3}x - 5 =$ the number of steel vessels.
But $21 =$ the number of steel vessels.
$\therefore \frac{1}{3}x - 5 = 21$.
Adding 5 to both members of the equation,
$\frac{1}{3}x + 5 - 5 = 21 + 5$,
or $\frac{1}{3}x = 26$.
Multiplying both members by 3,
$x = 78$, the number of steam vessels built.

NOTE. — The equation $\frac{1}{3}x - 5 = 21$ is called the **equation of the problem.**

General Directions for Solving Problems. — 1. *Represent one of the unknown numbers by some letter, as x.*

2. *From the conditions of the problem find an expression for each of the other unknown numbers.*

3. *Find from the conditions two expressions that are equal and write the equation of the problem.*

4. *Solve the equation.*

18. Two cars together contained 400 bales of cotton. If one car had 6 bales more than the other, how many had each?

19. The playgrounds of two cities occupy 183 acres. One city has 27 acres less than the other. How many acres has each?

20. The height of the big tree *Wawona* in California is 8 feet more than 9 times its diameter. If the height is 260 feet, what is the diameter of the tree?

21. Yellowstone Park contains 3400 antelope and deer. If the antelope number 200 less than twice the number of deer, how many deer are there in the Park?

22. A department store restaurant serves luncheon daily to 5000 people. If the number served lacks 1000 of being 3 times the number seated at once, find the seating capacity.

23. One year the Bureau of Engraving and Printing employed 2400 people. The number of women was 400 greater than the number of men. Find the number of each employed.

24. In a recent year the Lake Superior region furnished 38,400,000 tons of iron ore, or $\frac{4}{5}$ of all that was mined in the United States. How much was mined in the United States?

25. Denmark produces 44,000 tons of beet sugar annually. If this is 4000 more than $\frac{1}{2}$ the number of tons consumed, what is the annual consumption of beet sugar in Denmark?

26. One year the government spent $60,000 in operating a flag factory. The material cost $8000 less than 3 times the amount expended for labor. What was the cost of each?

27. Two power companies together use 27,200 cubic feet of water per second from Niagara Falls. Find the average discharge of the falls per second, if these companies use $\frac{2}{15}$ of it.

28. The whalebone in one whale was worth $\frac{5}{16}$ as much as that in another, and the value of the whalebone in the two was $525. Find the value of the whalebone in each.

29. In a fossil bed in Switzerland 470 species of insects were found and this was 30 less than $\frac{5}{7}$ of the number found in a bed in Colorado. Find the number in the latter bed.

30. The largest cask in the world contains a number of hogsheads that is 1 less than 25 times the number of feet in its diameter. If it contains 649 hogsheads, find its diameter.

31. The railroads consume $\frac{3}{10}$ of the total annual production of coal in the United States. Their annual expenses for coal are 240 million dollars with the average price $2 per ton. How many tons are produced in the United States each year?

32. The United States uses 101 million files each year. The number of files made in this country lacks 15 million of being 3 times the number of those imported. How many files are imported?

33. The Jamestown Exposition pier inclosed a rectangular lagoon, the length of which was 1000 feet more than its width. If its perimeter was 6800 feet, how long was it?

34. In one year the output of scrap mica was 5 tons more than twice the output of sheet mica and there were $430\frac{1}{2}$ tons more of the former than of the latter. Find the number of tons of each.

35. The largest concrete chimney in the world contains 1460 tons of steel and sand. The weight of the steel used was $\frac{3}{70}$ of the weight of the sand. Find the number of tons of each that were used.

36. Mt. Whitney, the highest point in the United States, is 14,500 feet above sea level. This is 700 feet more than 50 times the depth below sea level of Death Valley, the lowest point of dry land in the country. How far below sea level is Death Valley?

37. The lilies sent to the United States annually from Bermuda are worth $\frac{1}{20}$ as much as all our imported floral products. If the other floral products are worth $1,900,000, find the value of the lilies imported from Bermuda.

38. The distance from Cuba to Haiti is 31 miles less than the distance to Jamaica, and from Cuba to Yucatan, which is 130 miles, is 9 miles less than the sum of the distances to Haiti and Jamaica. Find the distance from Cuba to Jamaica.

39. In field and track events at the Olympic games in London, America won $35\frac{1}{2}$ points more than Great Britain and Sweden together, and Sweden won 54 points less than Great Britain. Find the score of each, if the total score of the three countries was $193\frac{1}{2}$ points.

REVIEW

48. 1. Tell how similar terms are added; subtracted. Tell what to do with dissimilar terms in addition; in subtraction.

2. Write a polynomial arranged according to the ascending powers of some letter; the descending powers.

3. State the law of exponents for multiplication; for division; the law of coefficients for each. $3^0 = ?$ $8^0 = ?$ $a^0 = ?$

4. What is an equation? Write one and point out the unknown numbers in it; the known numbers; its first member; its second member.

5. What is meant by 'solving an equation'? Give four methods by one or more of which equations may be solved. How may the value of an unknown number, obtained by solving an equation, be verified?

Solve and verify:

6. $3x = 21$.

7. $\frac{5}{6}x = 15$.

8. $7x - 3 + 4x = 21 - 2x + 2$.

9. $10 - 2x = 3x + 5 + 9 - 6x$.

10. Add $x + y + z$, $7x + 2z + 3y$, $4z + 5y$, and $9x + 3y + 2z$.

11. $11a + 5b + 2c - 4b + 2a - c + 4c - 9a + 5b - 3c + a = ?$

12. Subtract $5a^m + 7b^n + 18c$ from $7a^m + 25c + 8b^n + 8d$.

Expand:

13. $7p^3q^5r^7(pqr^2 + 4p^2qr + 2qr^3 + p^5 + 5p^3q^2r^4 + 3pq)$.

14. $(x^4 + 7x^3y + 4x^2y^2 + 3xy^3 + 2y^4)(x^3 + 4x^2y + 2y^3)$.

15. $(3z^4 + 4z^3w + 6z^2w^2 + 4zw^3 + 13w^4)(2z^2 + 4zw + 3w^2)$.

16. $(a^5b + 3a^4b^2 + 6a^3b^3 + 5a^2b^4 + 11ab^5)(a^3 + 3a^2b + 4ab^2)$.

Divide, and test each result:

17. $12l^6m^4n^2 + 18l^5m^2n^4 + 15l^4m^3n^3 + 3l^3mn^2$ by $3l^3mn^2$.

18. $35r^4 + 30r^3s + 69r^2s^2 + 12rs^3 + 22s^4$ by $5r^2 + 2s^2$.

19. $22x^2y^2 + 24xy^3 + 27x^3y^3 + 36x^2y^4 + 3x^3y$ by $3xy + 4y^2$.

20. $4a^5c + 8a^4bc + 11a^3b^2c + 24a^2b^3c + 24ab^4c + 7b^5c$ by $2a^2 + 3ab + b^2$.

POSITIVE AND NEGATIVE NUMBERS

49. The student of arithmetic knows the meaning of such an expression as $10 - 4$, but as yet an expression like $4 - 10$ has no meaning to him. It is the purpose of this chapter to extend the idea of number so that subtracting a larger number from a smaller one will have as much meaning as subtracting a smaller number from a larger one, to show that there is a practical demand for a new kind of number, and finally to show how operations involving this new kind of number are performed.

50. Suppose that at noon the temperature is 10° above 0 and that at 6 P.M. it has fallen 4°. The temperature is then 10° − 4°, or 6° above 0, but if it has fallen 15° instead of 4°, it is then 10° − 15°, and because the numbers on a thermometer extend below as well as above 0, we see that 10° − 15° means that the temperature is 5° below 0, 10° of the 15° of fall taking it to 0 and the other 5° of fall taking it to 5° below 0.

For convenience and brevity degrees 'above 0' are marked with the sign + and degrees 'below 0' with the sign −.

Such statements may be abbreviated algebraically, thus:

$$+10° - 4° = +6°,$$

and

$$+10° - 15° = -5°.$$

Similarly, if a ship now at 20° north latitude (latitude, +20°) sails south 30°, it will cross the equator (latitude, 0°) and be at 10° south latitude (latitude, −10°).

Again, a tourist in going from Lake Lucerne 1435 feet above sea level (altitude +1435 feet) to the Dead Sea 1295 feet below sea level (altitude −1295 feet) goes not only *to* 0 altitude (sea level), but *through* 0 altitude.