FIRST YEAR ALGEBRA

INTRODUCTION

1. In passing from arithmetic to algebra, the student will not find the change a very marked one. He will meet signs, definitions, principles, and processes with which he is already familiar. The fundamental principles of arithmetic and algebra are identical, but in algebra their application is broader.

Algebra uses the same number symbols as arithmetic, namely, 1, 2, 3, 4, 5, etc., but from time to time, as need for them arises, various new symbols will be introduced. While arithmetic, to a limited extent, uses letters to represent numbers, their use is a distinctive feature of algebra.

The terms, addition, subtraction, multiplication, division, fraction, etc.; the associated terms such as addend, subtrahend, multiplier, quotient, numerator; and the signs, $+$, $-$, $\times$, $\div$, $=$, have the same meanings that they have in arithmetic; but it will be seen that algebra gives to some of them additional meanings.

In algebra, multiplication is also indicated by the dot ($\cdot$) or by the absence of sign; thus, $a \times b$, $a \cdot b$, and ab all mean the same.

Division is often indicated by a fraction; thus, $a \div b$ and $\frac{a}{b}$ have the same meaning.

EXERCISES

2. Read, and tell the meaning of each of the following **algebraic expressions**:

1. $2+3$.
2. $a+b$.
3. $8-5$.
4. $x-y$.
5. 2×5.
6. $m\times n$.
7. $8\div 4$.
8. $w\div v$.
9. $4\cdot 5$.
10. $x\cdot y$.
11. pq.
12. $ab-rs$.
13. $3v+5z$.
14. $a+m-n$.
15. $\frac{m}{n}$.
16. $\frac{ab}{3x}$.
17. $\frac{a}{b}+\frac{r}{s}$.
18. $\frac{a-r}{b+s}$.

Indicate the

19. Sum of 5 and 2; of x and y.

20. * Difference of 9 and 6; of m and n.

21. Product of 3 and 4 in two ways; product of r and s in three ways.

22. Quotient of 8 divided by 5 in two ways; quotient of p divided by q.

23. Sum of 5 times d and 2 times c.

24. Difference of a times b and 2 times 4.

25. Product of $3m$ and n.

26. Quotient of $v-w$ divided by c times d.

27. Product of $2x+7$ and $3y-2$.

The product of $2x+7$ and $3y-2$ is indicated thus:

$$(2x+7)(3y-2).$$

NOTE.— **Parentheses**, ( ), are used to group numbers, when the numbers in each group are to be considered as a single number.

28. Product of $a-b$ and $5m+2$.

29. Product of a and $a+b$ divided by the product of b and $a-b$.

30. A boy had a apples and his brother gave him b more. How many apples had he then?

*In this book, the 'difference' of two numbers means the first mentioned less the second.

31. Edith is 14 years old. How old was she 4 years ago? a years ago? How old will she be in 3 years? in b years?

32. At x cents each, how much will 5 oranges cost?

33. If z caps cost 10 dollars, how much will 1 cap cost?

34. At y cents each, how many pencils can be bought for x cents?

35. George won a race by running the distance in t seconds. Represent Elmer's time, if he took 2 seconds longer.

36. James weighs p pounds. Represent Edward's weight, if he weighs 10 pounds less than James.

37. A boy who had p marbles lost q marbles and afterward bought r marbles. How many marbles did he then have?

38. If m represents the number of miles a boy can walk in a certain time, indicate the distance his father, who walks twice as fast, can walk in the same time.

39. Mary paid c cents for a pin and half as much for a belt. Represent the number of cents she paid for the belt.

40. What two whole numbers are nearest to 9? to x, if x is a whole number? to a, if a is a whole number?

41. If y is an even number, what are the two nearest even numbers? the two nearest odd numbers?

3. Unite **terms** as indicated by their signs:

20	2 tens	2×10	$2t$	$2x$	$2z$
$+40$	$+4$ tens	$+4\times 10$	$+4t$	$+4x$	$+4z$
$+30$	$+3$ tens	$+3\times 10$	$+3t$	$+3x$	$+3z$
90	9 tens	9×10	$9t$		

$2t+4t+3t=9t$. $2x+4x+3x=?$ $2z+4z+3z=?$

Such terms as $2x$, $+4x$, and $+3x$ are called **like**, or **similar**, **terms** because they have the same unit, x.

The multipliers, 2, 4, and 3 are called **coefficients** of x.

Such terms as $2t$, $+4x$, and $3z$ are **unlike**, or **dissimilar**, **terms** because they have different units, t, x, and z.

ALGEBRAIC SOLUTIONS

4. The numbers in this chapter do not differ in character from the numbers with which the student is already familiar in arithmetic.

The following solutions and problems, however, serve to illustrate how the solution of an arithmetical problem may often be made easier and clearer by the *algebraic* method, in which the numbers sought are represented by *letters*, than by the ordinary arithmetical method.

Letters that are used for numbers are usually called **literal numbers.**

5. Illustrative Problem. — A man had 400 acres of corn and oats. If there were 3 times as many acres of corn as of oats, how many acres were there of each?

ARITHMETICAL SOLUTION

A certain number = the number of acres of oats.
Then, 3 times that number = the number of acres of corn,
and 4 times that number = the number of acres of both;
therefore, 4 times that number = 400.
Hence, the number = 100, the number of acres of oats,
and 3 times the number = 300, the number of acres of corn.

ALGEBRAIC SOLUTION

Let x = the number of acres of oats.
Then, $3x$ = the number of acres of corn,
and $4x$ = the number of acres of both;
therefore, $4x = 400$.
Hence, $x = 100$, the number of acres of oats,
and $3x = 300$, the number of acres of corn.

Observe that in the algebraic solution x is used to stand for 'a certain number' or 'that number,' and thus the work is abbreviated.

6. A statement of the equality of two numbers or expressions is called an **equation.**

$5x = 30$ is an equation.

Problems

7. Solve the following problems:

1. A bicycle and suit cost $54. How much did each cost, if the bicycle cost twice as much as the suit?

2. Two boys dug 160 clams. If one dug 3 times as many as the other, how many did each dig?

3. Find a number whose double equals 52.

4. If 3 times a number equals 75, find the number.

5. A certain number added to 3 times itself equals 96. What is the number?

6. The average length of a fox's life is twice that of a rabbit's. If the sum of these averages is 21 years, what is the average length of a rabbit's life?

7. The battleship fleet that sailed for the Pacific consisted of 20 ships. The number of warships was 4 times the number of the auxiliary ships. How many warships were there?

8. The water and steam in a boiler occupied 120 cubic feet of space, and the water occupied twice as much space as the steam. How many cubic feet of space did each occupy?

9. One year the United States exported 24 million pounds of butter and cheese. If this included twice as much butter as cheese, how many pounds of each were exported?

10. Porto Rico and the Philippines together produce 400,000 tons of sugar each year. If the latter produces 3 times as much as the former, how much does Porto Rico produce?

11. Canada and Alaska together annually export furs worth 3 million dollars. If Canada exports 5 times as much as Alaska, find the value of Alaska's export.

12. The poultry and dairy products of this country amount to 520 million dollars a year, or 4 times the value of the potato crop. What is the value of the potato crop?

13. At Portland, Oregon, recently vessels were loaded with 25 million feet of lumber for home and foreign ports. Find the foreign shipment, if it was 4 times that to home ports.

14. In constructing the Galveston sea wall 10,000 loads of sand and crushed granite were used. If there were 3 times as many loads of sand as of granite, how many loads of each were used?

15. The Weather Bureau of the United States yearly saves the country 30 million dollars, or 20 times its cost. What is the annual cost of the Weather Bureau?

16. One year in continental Europe 6 million watches were made, and this number was $\frac{1}{2}$ of a million more than twice the number made in the United States. How many were made in this country?

SUGGESTION. $2x = 6 - \frac{1}{2}$.

17. Probably Ceylon has the oldest tree in the world, and its age is about 2200 years. If this is 70 years more than 6 times the age of the *Powhatan Oak* in Virginia, find the age of the latter.

18. The value of the *King's Cup*, the challenge trophy for yachting, is twice as great as that of the *Bennett Cup*, the prize for long-distance balloon racing. If the difference in value is $2500, find the value of each.

19. The owner of a piano found that the annual cost of keeping it in tune and insuring it against fire was $12.50, and that the cost of keeping it in tune was 9 times the cost of insuring it. Find the cost of each item.

20. The Quebec bridge that collapsed was 1800 feet long, and twice the length of the Forth bridge was $\frac{1}{10}$ of the length of that at Quebec. Find the length of the Forth bridge.

21. One year 1500 violins were made in the United States. Twice as many were made in New York as in Massachusetts, and these two states made half of all that were made in this country. How many violins were made in New York?

22. The sides of any square (Fig. 1) are equal in length. How long is one side of a square, if the perimeter (distance around it) is 36 inches?

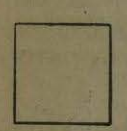

FIG. 1.

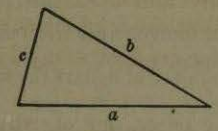

FIG. 2.

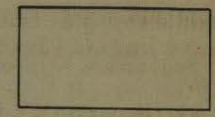

FIG. 3.

23. The length of each of the sides, a and b, of the triangle (Fig. 2) is twice the length of the side c. If the perimeter is 40 inches, what is the length of each side?

24. The opposite sides of any rectangle (Fig. 3) are equal. If a rectangle is twice as long as it is wide and its perimeter is 48 inches, how wide is it? how long?

25. Divide 21 into three parts, such that the first is twice the second, and the second is twice the third.

SUGGESTION. — Let x = the third part; then, $2x$ = the second part, and $2 \cdot 2x$ = the first part; that is, $x + 2x + 2 \cdot 2x = 21$.

26. Three newsboys sold 60 papers. If the first sold twice as many as the second, and the third sold 3 times as many as the second, how many did each sell?

27. The battleship *Connecticut* has twice as many 8-inch as 12-inch guns, and the sum of the two equals the number of its 7-inch guns. If it has in all 24 guns of these sizes, find the number of each.

28. One winter the Borough of Richmond had four falls of snow amounting in all to $16\frac{1}{2}$ inches. The second and third falls were each 4 times the first. Find the depth of the fourth fall, if it was twice the first.

29. In a recent year Massachusetts produced twice as many barrels of cranberries as New Jersey, and New Jersey 5 times as many as Wisconsin. Find the production of each of these states, if their total crop was 400,000 barrels.

30. A plumber and two helpers together earned $7.50 per day. How much did each earn per day, if the plumber earned 4 times as much as each helper?

31. James bought a pony and a saddle for $60. If the saddle cost $\frac{1}{5}$ as much as the pony, find the cost of each.

SUGGESTION. — Let x = the number of dollars the saddle cost.

32. Separate 72 into two parts, one of which shall be $\frac{1}{3}$ of the other.

33. Separate 78 into two parts, one of which shall be $\frac{1}{5}$ of the other.

34. A skating rink accommodated 4000 persons. If there were $\frac{1}{3}$ as many skaters as spectators, find the number of each.

35. The total production of sulphur averages 625,000 tons per year. How much is produced by the rest of the world, if it is $\frac{1}{4}$ the amount produced by Sicily?

36. The average height of the land above sea level is $\frac{1}{11}$ as great as the average depth of the ocean, and the sum of the two is 13,000 feet. Find the average height of the land and the average depth of the ocean.

37. The first issue of Christmas stamps by the Delaware Red Cross Society was $\frac{1}{2}$ as much as the second, which was $\frac{1}{2}$ as much as the third. If the three issues amounted to 350,000 stamps, how many were there in each issue?

38. Sand and clay road costs $\frac{1}{6}$ as much per mile as macadam. If the former costs $400 per mile, find the cost of the latter.

SOLUTION

Let x = the number of dollars macadam costs per mile.
Then, $\frac{1}{6}x = 400$.
Therefore, $x = 6$ times $400 = 2400$.
Hence, macadam road costs $2400 per mile.

39. The gold output of the United States for a recent year was 110 million dollars, or $\frac{1}{4}$ that of the entire world. What was the world's output for that year?

40. A man in New York rented his house and lived in an apartment costing him $2000 a year. This was $\frac{1}{3}$ as much as the rent of his house. For how much did his house rent?

41. The Pennsylvania Railroad station in New York is 780 feet long, and this is $404\frac{1}{3}$ feet more than $\frac{1}{2}$ the length of the Capitol at Washington. Find the length of the Capitol.

42. A basketball team won 16 games, or $\frac{2}{3}$ of the games it played. Find the number of games it played.

SOLUTION

Let x = the number of games it played.
Then, $\frac{2}{3}x = 16$,
and $\frac{1}{3}x = 8$.
Therefore, $x = 24$, the number of games it played.

43. The largest thermometer in the world has a glass tube 16 feet long. Find the length of the thermometer, if the tube is $\frac{4}{5}$ of the entire length.

44. What is the annual rainfall of Hawaii, if at least 56 inches, or $\frac{4}{5}$ of it, passes off without rendering any service?

45. Of the inhabitants of Guam, $\frac{9}{10}$, or 8100, can read and write. What is the population of the island?

46. The average annual fire loss in Berlin is $\frac{3}{10}$ of that in Chicago. If the fire loss in Berlin is $150,000, what is the fire loss in Chicago?

47. The largest stone ever quarried in the South was dressed down to weigh 60,000 pounds. If this was $\frac{3}{4}$ of its weight as originally blocked out, find its original weight.

48. Find the amount of lumber on hand in San Francisco at the time of the earthquake, if $\frac{2}{5}$ of it, or 36 million feet, were consumed by the fire that followed the earthquake.

49. The manufacturing industries of Great Britain use 150 million tons of coal per year. If this is $\frac{5}{6}$ of the total amount used, what is that country's annual consumption of coal?

50. The number of German-speaking people in the world is 75 million, or $\frac{3}{5}$ the number that speak English. What is the number of English-speaking people?

51. The United States sent to Germany one year 135,000 pairs of shoes. This was $\frac{3}{5}$ of the number sent the next year. How many pairs of shoes were sent the second year?

52. If $\frac{1}{5}$ of a number is added to the number, the sum is 12. What is the number?

SUGGESTION. $x + \frac{1}{5}x = 12$; that is, $\frac{6}{5}x = 12$.

53. If $\frac{1}{3}$ of a number is subtracted from twice the number, the difference is 35. What is the number?

SUGGESTION. $2x - \frac{1}{3}x = 35$; that is, $\frac{5}{3}x = 35$.

54. The total cost of the Pennsylvania Capitol was 13 million dollars. If the furnishings cost $2\frac{1}{4}$ times as much as the construction, what was the cost of each?

55. The retail dressmaking trade each day uses $\frac{1}{3}$ of the total daily output of spool silk. If the manufacturing trade uses the remainder, or 16,000 miles, how much does the dressmaking trade use per day?

56. Out of the average daily output of stamped envelopes $\frac{3}{10}$ are plain stamped. The remainder, 2,800,000, bear the return address. What is the daily output?

57. In one year, 5600 tons of dynamite were required for the Panama Canal. If the amount for the Culebra division was $1\frac{4}{5}$ as much as that for the rest of the canal, find the amount required for the Culebra division.

58. In the first twenty-one hours after the institution of regular wireless service, $6\frac{1}{2}$ times as many words were sent to Europe as were received, and the number sent was 11,000 more than the number received. Find the number sent; the number received.

59. The Pacific battleship fleet carried twice as much ham as it did salt pork, and $2\frac{1}{2}$ times as much beef as it did ham. The weight of the beef was 800,000 pounds more than that of the salt pork. Find the weight of each.

FACTORS, POWERS, AND POLYNOMIALS

8. Since the product of 2 and 6 or of 3 and 4 is 12, each of the numbers 2, 6, 3, and 4 is a **factor** of 12. So also, each of the numbers 3, a, b, $3a$, $3b$, and ab is a factor of $3ab$.

9. In algebra, as in arithmetic, such a product as $2 \times 2 \times 2 \times 2$, called a **power** of 2, may be more briefly written 2^4.

The small figure 4, placed at the right of, and a little above, the 2 to indicate the number of times 2 is used as a factor, is called an **exponent**.

Since a^1 means the same as a, the exponent 1 is usually omitted.

a^2 is read 'a second power' or 'a square'; a^3 is read 'a third power' or 'a cube'; a^4 is read 'a fourth power,' 'a fourth,' or 'a exponent 4.'

The terms 'coefficient' and 'exponent' should be distinguished.

Thus, $5a$ means $a + a + a + a + a$, but a^5 means $a \cdot a \cdot a \cdot a \cdot a$.

EXERCISES

10. Read, and tell the meaning of:

1. x^6.	4. x^2y^2.	7. $3zw^2$.	10. $9ab^3c^2d^4$.
2. y^4.	5. a^3b^3.	8. $4p^3q^5$.	11. $5p^2q^3s^4t^5$.
3. z^8.	6. r^2s^2.	9. $2m^3n^2$.	12. $7x^3ym^7n^4$.

Express in abbreviated form by using exponents:

13. $2 \cdot 2$.	16. $3aaa$.	19. $2 \cdot 2 \cdot 2 \cdot x \cdot x$.	
14. $3 \cdot 3 \cdot 3$.	17. $8lll$.	20. $7 \cdot 7 \cdot z \cdot z \cdot z \cdot z$.	
15. $5 \cdot 5 \cdot 5 \cdot 5$.	18. $9ssrrr$.	21. $3 \cdot 3 \cdot 3 \cdot a \cdot a \cdot b \cdot b \cdot b$.	

22. What is the coefficient of x in $3x$? in ax? in $3ax$?

NOTE. — A coefficient is *numerical*, *literal*, or *mixed* according as it is composed of figures, letters, or both.

When not otherwise specified 'coefficient' means numerical coefficient.

Since $1a$ means the same as a, the coefficient 1 is usually omitted.

23. What is the literal coefficient of t^2 in at^2? in gt^2? in n^3t^2? of y^3 in ny^3? in x^2y^3? in lmy^3?

Name the various factors of:

24. ax. 26. x^3. 28. $6n$. 30. $pqrs$.

25. $3mn$. 27. $5r^2s^2$. 29. $15z^2$. 31. $24vt$.

32. In each of the exercises 24–31, name the factors in sets such that the product of the factors in each set shall equal the given number.

11. An algebraic expression is called a **monomial, binomial,** or **trinomial** according as it has *one, two,* or *three* terms.

Thus, $3a$ is a monomial; $2x+y^3$, a binomial; and $x^2+2xy+y^2$, a trinomial.

The name **polynomial** is often applied to any algebraic expression of more than one term.

EXERCISES

12. From the algebraic expressions given below select the:

1. Binomials. 3. Monomials. 5. Similar terms.
2. Trinomials. 4. Polynomials. 6. Dissimilar terms.

$2ax$; $3x^2y$; $2a+3b$; $3x+2b$; $3ax+2y^2$; $6a-c+d$; $3a^2x^2-4ax+2d-y^2$; $2x^2y-xy+ax^2$.

7. Find the value of $3+4-2+3$; of $3\times4\div2\times3$.

SOLUTIONS. $3+4-2+3=7-2+3=5+3=8$;
$3\times4\div2\times3=12\div2\times3=6\times3=18$.

When only + and − occur in any expression, or only × and ÷, the operations are performed in order from left to right.

Find the value of:

8. $3-2-1+8-3+4$. 10. $10\div2\times8\div4\div2\times6$.
9. $5+1-4+3-2+6$. 11. $35\div7\div5\times3\times4\div2$.

12. Find the value of $7+10-6\div3\times4$.

SOLUTION. $7+10-6\div3\times4=7+10-2\times4=7+10-8=9$.

Unless otherwise indicated, as by the use of parentheses, *when* ×, ÷, *or both, occur in connection with* +, −, *or both, the indicated multiplications and divisions are performed first.*

Find the value of:

13. $5\times10-7$. 18. $6+2\times8-4\div2$.
14. $5\times(10-7)$. 19. $(6+2)\times8-4\div2$.
15. $2\times5+3\times4$. 20. $(6+2\times8-4)\div2$.
16. $(25-13)\div4\times2$. 21. $6+2\times(8-4)\div2$.
17. $16-2\times2\times12\div4$. 22. $6+2\times(8\div4\div2)$.

Read, and tell the meaning of each of these polynomials:

23. $2x^2+y^2$. 26. $a+d(ax-y)$. 29. $3x^2+2y-3z$.
24. $3x-4y$. 27. $3+4(y-3z)$. 30. $a^2x^2-3xy+2z^2$.
25. $4ab-c^3$. 28. $c(l^2+t^2)-4d$. 31. $5b^2y+x^2y^2+5cz^2$.

Represent algebraically:

32. The sum of five times a and three times the square of x.

33. Three times b less twice the fifth power of a.

34. The product of a, b, and $a-c$.

35. Three times x, divided by five times the sum of a, b, and c.

36. Seven times the product of x and y, increased by three times the cube of z.

37. Six times the square of m, increased by the product of m and n.

38. The product of a used five times as a factor, multiplied by the sum of b and c.

39. Twelve times the square of a, diminished by five times the cube of b.

40. Eight times the product of a and b, divided by four times the seventh power of c.

41. Six times the product of a second power and n, increased by five times the product of a and the second power of n.

42. The fourth power of the sum of a and b, increased by three times the product of the square of a and the square of b, diminished by the cube of d.

NUMERICAL SUBSTITUTION

13. When a particular number takes the place of a letter, or general number, the process is called **substitution.**

EXERCISES

14. 1. When $a=2$ and $b=3$, find the numerical value of $3ab$; of a^4.

SOLUTIONS. $3ab = 3 \cdot 2 \cdot 3 = 18$; also, $a^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$.

When $a=5$, $b=3$, $c=10$, $m=4$, find the value of:

2. $10a$.	6. $5m^2$.	10. am^4.	14. $\frac{1}{3}ab^2$.
3. $2ab$.	7. $2a^2b$.	11. $(ab)^2$.	15. $\frac{1}{2}bm$.
4. $3cm$.	8. $3bm^3$.	12. a^2b^2.	16. $\frac{1}{5}abc$.
5. $6bc$.	9. $4a^3b$.	13. a^bc.	17. $3b^2cm^2$.

18. When $m=0$ and $n=4$, find the value of $3m^2n$.

SOLUTION. $3m^2n = 3 \cdot 0^2 \cdot 4 = 3 \cdot 0 \cdot 4 = 0$.

NOTE. — When any factor of a product is 0, the product is 0; therefore, any power of 0 is 0.

When $a=4$, $b=2$, $r=0$, and $s=5$, find the value of:

19. $7b^2r$.	21. $3s^bb^a$.	23. $\frac{3}{8}a^3bs$.	25. $2^ab^3s^2r^4$.
20. $\frac{3a^2b}{sb}$.	22. $\frac{a^2bs}{abs^2}$.	24. $\frac{6a^sb^a}{b^2a^4}$.	26. $\frac{24a^bb^ss^a}{6a^3b^3s^3}$.

27. When $x=3$ and $y=2$, find the value of $(x+y)^2$; of $x^2+2xy+y^2$.

SOLUTIONS

$$(x+y)^2 = (3+2)^2 = 5 \cdot 5 = 25.$$
$$x^2+2xy+y^2 = 3 \cdot 3 + 2 \cdot 3 \cdot 2 + 2 \cdot 2 = 9+12+4 = 25.$$

28. Show that $2x+3x=5x$ when $x=2$; when $x=3$. Giving x any value you choose, find whether $2x+3x=5x$.

29. Show that $m(a+b)=ma+mb$ when $m=5$, $a=4$, and $b=3$. Find whether the same relation holds true for other values of m, a, and b.

30. Show that $(a-b)^2 = a^2-2ab+b^2$ when $a=4$ and $b=2$. Find whether this is true for other values of a and b.

When $a=5$, $b=3$, $m=4$, $n=1$, find the value of:

31. a^2+b^2.	33. n^5-1.	35. m^{a-b}.
32. $(a+b)^2$.	34. $(n-1)^5$.	36. $(bm)^{b-1}$.

37. $ab-bn+mb^2 \div 3mn^2$.

38. $(ab-bn+mb^2) \div 3mn^2$.

39. $2^am^2n^2 - abmn \div 4bn - m^3n^7$.

40. $ambn^2 - \frac{3}{4}b^2m + \frac{5}{8}m^2n^3 - \frac{1}{5}m^3$.

REVIEW

15. Read the following; classify each expression according to the number of terms it contains; find the number represented by each expression when $t=10$.

1. $6t$.	4. t^2.	7. t^3.
2. $7t$.	5. t^2+2t+1.	8. t^3+2t^2+5t+4.
3. $9t+9$.	6. $3t^2+6t+5$.	9. $5t^3+3t^2+8t+6$.

10. Write 25 as a polynomial in t, t representing 10; letting t represent 10, and, using exponents to represent powers of t, express in polynomial form:

732 523 893 4867 6248 72,565

11. What does $2a$ denote? a^2?

Illustrate the difference in meaning between $2a$ and a^2 when $a=1$; when $a=2$; when $a=3$; when $a=\frac{1}{2}$; when $a=\frac{1}{3}$.

For what value of a are $2a$ and a^2 equal?

12. Which is the greater, 2^3 or 3^2? 4^2 or 2^4? 2^5 or 5^2?

13. Compare also 2^3 and 2^2; $(\frac{1}{2})^3$ and $(\frac{1}{2})^2$; 1^3 and 1^2.

14. Find, for $x=1$, the value of:

$3x$ $4x^2$ $6x^3$ $8x^5-4x^4+2x^3-x+5$

Name the exponent of x in each term that contains x.

15. Name the coefficient of n in each of these monomials:

$2n$ n $\frac{1}{2}n$ bn $3b^2n$ a^2b^3n

16. Write three similar monomials; four dissimilar monomials.

17. If n is a whole number greater than 1 and a is any number, what is the meaning of a^n?

Find the value of each of the following expressions when $a=5$, $b=4$, $c=3$, $d=2$, $e=1$, and $n=3$.

18. $6ab$; $2cd$; $4nbd$; $\frac{1}{2}ea$; nd^{n+1}.

19. $3a^2b$; $3ab^2$; $3(a^b)^2$; d^2n^3; $(dn)^3$; d^{n-2}.

20. $a+b\div d-n\div e$.

21. $a(b-d)+a-n\div c$.

22. $10\div d+3\div n-e$.

23. $10\div(d+3)+ac\div n$.

24. $c^5+c^4+2c^3-2c^2-3c+3$.

25. $d^7+d^6+3d^5-5d^4+2d^3-4d^2+8d-1$.

26. For what value of x is $12x$ equal to 72?

Write '$12x$ is equal to 72' as an equation. Solve the equation.

Express in algebraic form; solve equations when you can:

27. Three times a certain number, x, is 21.

28. The sum of a certain number and three times the number is 40.

29. Six times a number, less 4 times that number, is 13.

30. The distance around a square lot, each side a feet long, is 1280 feet.

31. Half of a certain number is 17.

32. Twice a certain number, less $\frac{1}{3}$ of the number, equals 15.

33. Mary had m books and James had twice as many, the two together having 18 books.

34. John had 50 cents, spent c cents, and earned d cents. How much money had he then?

35. I bought 2 bottles of olives at b cents per bottle, 3 packages of crackers at p cents per package, and a small cheese for c cents. How much did I expend for all? How much money had I left out of a dollar?

FUNDAMENTAL OPERATIONS

16. In this chapter the student will use numbers he has used in arithmetic and letters to represent such numbers. He will notice that the processes of addition, subtraction, multiplication, and division here are performed as in arithmetic.

ADDITION

17. To add monomials.

1. How many are 2 plus 5? How many times a number are 2 times the number plus 5 times the number?

2. If n stands for a number, how many times n are 2 times n plus 5 times n? $2n+5n=?$

3. $2x+5x=?$ 4. $2r+5r=?$ 5. $2t+5t=?$

6. How many are $3+4+6$?

7. How many days are 3 days + 4 days + 6 days?

8. $3d+4d+6d=?$ 9. $3y+4y+6y=?$

EXERCISES

18. 1. Add $4a$ and $3a$.

PROCESS

$$\begin{array}{r} 4a \\ 3a \\ \hline 7a \end{array}$$

EXPLANATION. — Just as $3a$'s and $4a$'s are $7a$'s, so $3a+4a=7a$; that is, when the monomials are similar the sum may be obtained by adding the numerical coefficients and annexing to their sum the common literal part.

Add:

2. $\begin{array}{r} 3 \\ 6 \\ \hline \end{array}$ 3. $\begin{array}{r} 3x \\ 6x \\ \hline \end{array}$ 4. $\begin{array}{r} 7 \\ 1 \\ \hline \end{array}$ 5. $\begin{array}{r} 7m \\ m \\ \hline \end{array}$ 6. $\begin{array}{r} 3y \\ 4y \\ \hline \end{array}$